The Central Bank, the Treasury, or the Market: Which One Determines the Price Level?*

Jean Barthélemy Eric Mengus Guillaume Plantin

May 3, 2023

Abstract

This paper studies a political-economy model in which the price level is the outcome of dynamic strategic interactions between a fiscal authority, a monetary authority, and investors in government bonds and reserves. The "unpleasant monetarist arithmetic", whereby aggressive fiscal expansion forces the monetary authority to chicken out and to lose control of inflation, occurs only if the public sector lacks fiscal space, in the sense that public debt along the optimal fiscal path gets sufficiently close to the threshold above which the fiscal authority would find default optimal. Otherwise, monetary dominance prevails even though the central bank has neither commitment power nor fiscal backing.

*Barthélemy: Banque de France, 31 rue Croix des Petits Champs, 75001 Paris, France. Email: jean.barthelemy@banque-france.fr. Mengus: HEC Paris and CEPR, 1 rue de la Liberation, 78350 Jouy-en-Josas, France. Email: mengus@hec.fr. Plantin: Sciences Po and CEPR, 28 rue des Saints-Peres, 75007 Paris, France. Email: guillaume.plantin@sciencespo.fr. We thank, among many others, Vladimir Asriyan, Marco Bassetto, John Cochrane, Keith Kuester, Eric Leeper, Tom Sargent and François Velde as well as participants in many seminars and conferences for helpful comments. The views expressed in this paper do not necessarily reflect the opinion of the Banque de France or the Eurosystem.

1 Introduction

In the aftermath of the Covid crisis, the large and persistent fiscal measures in support of economic activity have led a number of observers to worry about the ability of central banks to fulfill the price-stability part of their mandates going forward (see Blanchard, 2021; Summers, 2021, among others). Vindicating these concerns, inflation has recently, and for the first time in decades, been significantly above target in both the US and the eurozone.

The underpinning of these concerns is primarily that fiscal and monetary authorities may sometimes have conflicting objectives, with the fiscal authority putting less weight on price stability than the monetary one.[1] This is a direct consequence from the independence of central banks with a prominent price-stability objective.[2] As is well understood since at least Alesina and Tabellini (1987), these conflicting objectives potentially lead to a non-cooperative game between fiscal and monetary authorities, and the list is long of examples in which they do not necessarily cooperate, and try instead to impose their views on each other.[3]

Ultimately, the risk is that despite formal central-bank independence, fiscal policy may make price stabilization difficult or even out of reach. Following Sargent and Wallace (1981)'s "unpleasant monetarist arithmetic", a large literature has studied how fiscal policy has the *ability* to constrain monetary policy. In Sargent and Wallace's seminal work, if the fiscal authority "moves first" in the sense that it commits at the outset to a path of deficits for the entire future, the monetary authority has no other option but to accommodate fiscal policy at the expense of controlling inflation in order to satisfy the public sector's budget constraint.

But to what extent is a fiscal authority actually *willing* to apply this arithmetic and impose its views on the monetary authority? If so, is there anything that the monetary

[1] See the recent speech by Powell (2023): "But restoring price stability when inflation is high can require measures that are not popular in the short term as we raise interest rates to slow the economy. The absence of direct political control over our decisions allows us to take these necessary measures without considering short-term political factors." See also Schnabel (2022): "In the current environment, there is a risk that monetary and fiscal policies may pull in opposite directions [...]."

[2] The leading rationale for central-bank independence is time-inconsistency problems as initially studied by Kydland and Prescott (1977) and Barro and Gordon (1983a). The delegation of monetary policy to an independent authority with a price-stability objective is generally thought to alleviate these problems (see Rogoff, 1985; Walsh, 1995; Svensson, 1997, among others).

[3] See, e.g., Mee (2019) for a historical analysis of the rise of an independent Bundesbank, Silber (2012) for the Volker era, and Bianchi et al. (2019) or Camous and Matveev (2021) for evidence that markets reacted to Trump's comments on monetary policy.

authority can do to deter it or at least to mitigate its costs, or is it always poised to accommodate fiscal expansion? Can their conflicting objectives even result either into sovereign default or into the reversal of central-bank independence to force debt monetization? How do financial markets assess the value of public liabilities given this "game of chicken" between two branches of the public sector?

To address these questions, this paper studies a political-economy model of the interactions between a fiscal and a monetary authority with distinct objectives – following the approach initiated by Tabellini (1986), Alesina (1987) or Alesina and Tabellini (1987). But, we depart from this approach by explicitly modeling the markets in which both authorities intervene – borrowing from the literature on fiscal-monetary interactions that followed Sargent and Wallace (1981) and Leeper (1991) and in which the price level is determined as the outcome of competitive markets. By combining these two approaches, we are able to both characterize the incentives and the means for the fiscal authority to impose fiscal dominance, and what the central bank can and is willing to do to prevent or mitigate it.

More precisely, we study a monetary authority whose objective is to keep the price level as close as possible to a given target. It is independent in the sense that it has a free hand at managing its balance sheet. The monetary authority issues reserves that are the unit of account of the economy—The price of a consumption unit in terms of reserves is the price level. It also decides on the nominal interest rate on reserves, on the investment of the proceeds from issuing reserves, and on possible transfers ("dividends") to the fiscal authority. As a simple and stark model of conflicting objectives, we assume that the fiscal authority seeks to spend optimally, but that it does not care about the price level.[4] It issues nominal bonds and uses the proceeds to spend or/and to repay all or part of maturing bonds. It can also raise distortive taxes. Potentially, the game starts with some initial legacy public liabilities – e.g., reserves or debt, possibly long term, issued in the past. Finally, Walrasian private investors form optimal portfolio of reserves and government bonds.

Critically, the fiscal authority cannot commit to repay its debt and, ex post, it may either outright default – "hard" default – or reverse central-bank independence to force

[4]This assumption, however extreme it can appear from a normative point of view, is consistent, from a positive point of view, with observed deficit biases for fiscal authorities. We connect our work with the literature explaining public debt patterns using political-economy arguments in the literature review section.

debt monetization – "soft" default.[5] In those cases, both the fiscal and the monetary authority incur costs. These costs are first exogenous disutilities in our simplest model. They endogenously arise from the trading strategies of investors in bonds and reserves later on, consistently with the idea of a reputation loss associated with default, being it hard or soft. In this sense, markets may actually play a central role in the determination of the price level.[6]

We solve for the subgame-perfect equilibria resulting from the interactions of fiscal and monetary authorities and the private sector. To do so, we adapt the equilibrium concept of Ljungqvist and Sargent (2018) in macroeconomic games to a situation with several large agents. Our focus is on the equilibrium price level. We deem "monetary dominance" the situation in which the equilibrium price level corresponds to the target of the monetary authority. "Fiscal dominance" is the alternative in which the price level exceeds this target, and reaches instead a higher level that is consistent with the solvency of the public sector.

The fiscal authority has an ex-post strict preference for inflation as it erodes the value of outstanding public liabilities, thereby allowing for more spending holding taxes fixed. Unlike in Sargent and Wallace (1981) and the literature thereafter, the fiscal authority must however find a way to commit to the type of fiscal expansion that would induce such an inflationary path. It must credibly establish that if the future price level is too low, it will prefer outright default or reversing central-bank independence to making good on its debt by raising taxes or/and cutting expenditures. Otherwise, the monetary authority would not accommodate the price level as it is willing to do so only to the extent that a default or the reversal of central-bank independence are costly for this authority. In our political-economy approach, the only way it can commit to such a future preference for default conditional on low inflation is by frontloading expenditures and financing them with a sufficiently large current debt issuance.

[5]If the threat to replace the central banker certainly plays a key role in fiscal dominance, that of an outright default may also be important to considered, as described in the case of the US debt ceiling by Leeper (2023).

[6]In particular, an exogenous cost of a (hard) default is not necessarily inconsistent with standard central banks' objectives. Such a default may well trigger financial disturbances that the central bank has to address because of a financial stability objective or because a default may jeopardize the transmission of monetary policy and have consequences on economic activity and inflation – see for example the minutes of the FOMC meeting on October 16, 2013 on the consequences of a default due to the debt ceiling: See p.15 of the minutes: "In such circumstances, the Committee might well want to take steps to address the market strains and so help support economic activity and keep inflation near its longer-term objective."

This commitment device is costly, however, in comparison with the smoother optimal fiscal path that takes price levels as given and on target. If such credible fiscal expansion is too unbalanced relative to the smoother optimal fiscal path, then the fiscal authority does not enter into it. In extensions, we show that large costs of taxation also make the inflationary fiscal expansion unpalatable, as this expansion typically involves larger future taxes than the optimal one. Also, since the inflating path involves more debt than the optimal one taking price levels as given, the higher interest rate that the market requires at such debt levels may also make this path unpalatable if it is sufficiently large relative to the fiscal authority's discount rate – in this latter case, monetary dominance may prevail even when interest rates are low in-equilibrium but expected to be high out-of-equilibrium.

In sum, the fiscal authority *can always* force the monetary one to inflate away legacy public liabilities by issuing enough public debt as soon as the monetary authority has some aversion to sovereign default or fears a potential reversal of its independence. However, the fiscal authority *wants* to do so only if the benefits of this inflationary fiscal expansion more than offset its costs. The benefits depend on the size of the legacy liabilities that are to be inflated away – a sufficiently large legacy debt leads the fiscal authority to engage in fiscal dominance, and on the maximum amount of inflation that the central bank is willing to tolerate to avert default. In contrast, and paradoxically, the costs to the fiscal authority from embarking on an unbalanced inflationary fiscal path are not necessarily tied to the cost of default incurred by the fiscal authority. In the absence of the other costs described above (taxation cost and a higher interest rate), the cost of default may be very large for the fiscal authority and yet not prevent it to push for fiscal dominance. The outcome of the political-economy game thus does not boil down to a simple comparison between the two authorities' costs of default.

Overall, monetary dominance prevails if the public sector has sufficient fiscal space, in the sense that at any point along the optimal fiscal path taking price levels as given, the fiscal authority would prefer to respond to an exogenous increase in public liabilities with an increase in taxes or/and a reduction in expenditures rather than with formal default or a reversal of central-bank independence. Conversely, if the optimal fiscal path gets sufficiently close to this default boundary, then the fiscal authority may deviate from it, and double down on debt in order to force the monetary authority to erode public liabilities through inflation.

Importantly, we show that the monetary authority has tools to prevent or, if not possible, to attenuate the costs of fiscal dominance. First of all, the central bank can partially control the size of legacy liabilities by maintaining the lowest possible volume of outstanding reserves. Second, even if the monetary authority is forced to deviate from its price level objective, it still has some tools to limit the costs of fiscal dominance. Critically, which tool the monetary authority finds best suited depends on the amount of legacy liabilities. When public liabilities are small enough, the central bank may find it useful to engage in preemptive inflation – even before the fiscal authority issues debt – with the objective to reduce the real value of legacy liabilities. By freeing up resources, this preemptive inflation limits the incentives of the fiscal authority to double down on debt issuance, as fiscal dominance would require the fiscal authority to issue a lot of new debt. When legacy liabilities are larger, the central bank may also inflate in the future, but at a smaller rate than what is implied by fiscal dominance. To commit to do so, the central bank increases the size of its balance sheet already in the present, which, as it cannot be easily narrowed down in the future, leads to inflation in the future. Such a latter situation resembles the one deemed "stepping on a rake" by Sims (2011), whereby the central bank loses the control of the price level. Otherwise, when the costs of inflating in the present are large enough, the monetary authority surrenders and lets the fiscal authority's budget constraint determine the future price level.

To be sure, our game is a very stylized representation of interactions between large branches of government in complex institutional settings. We do not expect to see any direct evidence that fiscal authorities deliberately and precisely design fiscal expansions as strategies to force monetary ones to deviate from their price-stability objectives. Instead, we may capture situations in which the fiscal authority "kicks the can down the road" by postponing the resolution of policy problems – a situation that can lead to "insidious fiscal dominance" to borrow the words by Leeper (2023) – or in which the fiscal authority, focused on another objective, fails to internalize the inflationary consequences of its own actions when designing bold fiscal expansions, e.g. due to bailouts in a financial crisis, big welfare programs or, even, wars. More generally, we believe that the forces that we capture in our political economy model manifest themselves in markets' and governments' expectations about the extent to which central banks would be willing to avoid a debt crisis in the face of fiscal expansions. These expectations have probably shifted significantly following the 2008 and Covid crises.

We first present our main insights in the simplest possible model with two dates. In this model, fiscal and monetary authorities incur exogenous costs in case of sovereign default or a reversal of central bank independence. These costs can be endogenized in infinite horizon, for example, due to market exclusion as in the sovereign debt literature following Eaton and Gersovitz (1981). In the last section of the paper, we study one particular situation in which public liabilities are Ponzi schemes—we aim to capture the idea of low rates as in Blanchard (2019) and Reis (2021). Then, endogenous default costs result from investors in bond and reserve markets downsizing the size of the Ponzi schemes that they believe—in a self-justified fashion—to be sustainable in case default occurs. In this case, the extent to which investors run not only on debt but also on reserves in case of sovereign default drives the monetary authority's willingness to accommodate fiscal expansion—a situation of "market dominance". The central bank is all the more willing to avoid default with some current inflation because it faces the risk of hyperinflation following formal default.

Related literature. Our paper is at the crossroads of the political-economy literature that investigates the games between multiple branches of government and of the less reduced-form literature investigating the interactions between monetary and fiscal policies.

We share with the first literature the idea that fiscal and monetary authorities may have ex-post conflicting objective (Alesina, 1987; Alesina and Tabellini, 1987; Tabellini, 1986, e.g.). More recent contributions include Dixit and Lambertini (2003) or the literature that explores disciplining mechanisms for the public sector in models following Barro and Gordon (1983a,b), such as Halac and Yared (2020). Our premises that fiscal authorities may prioritize spending over price stability also parallels the literature that explains the patterns of public debt accumulation using political economy frictions and a resulting deficit bias (see Halac and Yared, 2022; Yared, 2019, and the references herein). In particular, short-termism on the fiscal side due to political constraints may push the fiscal authority to neglect long-term objectives such as price stability, as also well summarized by Powell (2023). Also, such short-termism emphasized in this literature leads the fiscal authority to frontload expenditures and issue more debt, and we show that it is conducive to fiscal dominance. With respect to this literature, our contribution is to provide an explicit set of instruments to both the fiscal and the monetary authorities as well as a game-theoretic foundation to fiscal and monetary interactions. Our approach of the resulting macroeconomic game follows Chari and Kehoe (1990), Stokey (1991) and Ljungqvist and Sargent (2018) but extended to multiple large agents and markets.

This latter approach connects our paper to the literature studying the interactions between monetary and fiscal policies pioneered by Sargent and Wallace (1981) (see Leeper, 1991; Sims, 1994; Woodford, 1994, 1995; Cochrane, 2001, 2005; McCallum, 2001; Buiter, 2002; Niepelt, 2004; Jacobson et al., 2019; Camous and Matveev, 2022, among others). In particular, as in the fiscal theory of the price level, the monetary authority can adjust the price level to help the fiscal authority satisfy its budget constraint[7] and our approach to model markets follows Bassetto (2002) as, in our setting, price levels as well as debt prices are market-equilibrium objects. In addition, this literature has included additional modeling elements such as a richer debt maturity structure or nominal frictions to think about, e.g., the effect of fiscal policy not only on the price level but on inflation, as, for example, in the US in the 1970s (see, e.g., Bianchi et al., 2022). In contrast, we cast our "game of chicken" – to borrow Wallace's words to describe fiscal-monetary interactions – in a simple economy, that relates in particular to that in which Bassetto and Sargent (2020) study fiscal and monetary interactions. Our paper is also closely connected to the papers that identify fiscal requirements such that the central bank can attain its price stability objective, including fiscal rules (e.g. Woodford, 2001) or a ring-fenced balance sheet (e.g Sims, 2003; Bassetto and Messer, 2013; Hall and Reis, 2015; Benigno, 2020). Martin (2015) finds as we do that fiscal irresponsibility leads to long-term inflation. Our contribution with respect to this literature on fiscal-monetary interactions is to introduce explicit preferences for the fiscal and monetary authority and study the political economy game arising from the combination of objectives and tools of both authorities. In particular, this approach allows to provide a theory why the fiscal authority can credibly commit

[7] In Sargent and Wallace (1981), monetary policy accommodates by raising seignorage income despite the inflationary consequences — but public debt is real. In alternative models, such as the fiscal theory of the price level, and in this paper, an increase in the price level reduces the real value of nominal public debt. See Bassetto (2008) for a precise description of the connection between the fiscal theory of the price level and Sargent and Wallace (1981). See Reis (2017) for tools that the central bank has to increase fiscal resources.

to future fiscal policy—such commitment is an important ingredient in this literature to explain why fiscal policy can influence the price level.

Finally, our paper relates to the recent literature that compares formal sovereign default and soft default in the form of inflation (Bassetto and Galli, 2019; Galli, 2020). We cover the case in which distinct branches of government control each tool and act non-cooperatively. The infinite-horizon model offers a novel way of endogenizing the respective costs of each type of default.

The paper is organized as follows. Section 2 sets up our two-date model. Section 3 solves the baseline version of it to deliver our main insights in the simplest setup. Section 4 studies extensions of this baseline model. Section 5 introduces and solves an infinite-horizon version the model, mainly aiming at endogenizing the default costs incurred by public authorities in the two-date model. Section 6 concludes.

2 Two-Date Model: Setup

Our model features a fiscal authority and a monetary one that interact strategically. They also interact with the private sector in the markets for their respective liabilities. The monetary authority issues reserves that are the unit of account of the economy, and seeks to control the price level. The fiscal authority seeks to spend optimally and issues nominal bonds.

There are two dates indexed by $t \in \{0; 1\}$. There is a single consumption good. We describe in turn the private and public sectors.

Private sector. The private sector is comprised of a unit mass of agents, deemed "savers", who are each endowed with a large quantity of the consumption good at dates 0 and 1. They rank consumption streams (c_0, c_1) according to the criterion

$$c_0 + \frac{c_1}{r}, \tag{1}$$

where $r > 0$.

Public sector. The public sector features a monetary authority M and a fiscal authority F.

Monetary authority. The monetary authority issues reserves and announces the interest rate R_0 on them. Reserves trade for the consumption good in date-0 and date-1 markets for reserves. Reserves are the unit of account of the economy. We denote by P_t the price level—the price of the consumption good in terms of reserves in the date-t market for reserves. Let also X_t denote the quantity of outstanding reserves at the end of date t, and x_0 denote the endogenous quantity of goods that savers bid for reserves in the date-0 market for reserves. As detailed below, the terminal date-1 demand for reserves will be an exogenous quantity $\bar{x}$ in this two-date model.[8] We also assume that some legacy reserves $R_{-1}X_{-1} \geq 0$ are sold in the date-0 reserve market by some unmodelled agents—for example, by savers born at date -1 and seeking to consume at date 0.

M can also transfer resources to F ("pay a dividend"), and θ_t denotes the real date-t transfer from M to F. We do not make assumptions on the sign of θ_t, even if, as this will become clear, our timing assumption leads the monetary authority to make only positive transfers in and out of equilibrium.[9]

Fiscal authority. The fiscal authority issues one-period nominal bonds at date 0. A bond is a claim to one unit of account at date 1. Both savers and M can trade goods for bonds. Let B_0 denote the number of bonds issued by F at date 0, Q_0 the price at which they are sold (in terms of reserves), and b_0 and b_0^M the respective quantities of goods that savers and M respectively trade for bonds in the bond market.

The fiscal authority can tax savers' date-1 endowment. Collecting taxes $\tau_1 \geq 0$ comes at a utility cost $c(\tau_1)$ to F (as shown below in its preferences (6)).[10] F also consumes both at dates 0 and 1. Let g_t denote its date-t consumption.

Finally, F may default either by not repaying debt in full – a hard default – or by reversing central bank independence and monetizing the debt – a soft default. In the benchmark version of the model, we focus for simplicity on the hard default option and we relegate to Section 3.5 the latter possibility. F then decides on the haircut or loss given default $l_1 \in [0, 1]$ that it applies to its maturing bonds. A haircut l_1 means that bondholders receive $(1 - l_1)$ units of account per bond.

[8]This demand $\bar{x}$ will be endogenous in the infinite-horizon version of the model in Section 5. Notice that, as this will become clear, $\bar{x}$ can be arbitrarily small.

[9]See Del Negro and Sims (2015) or Reis (2015) for an analysis on the need of fiscal backing of the central bank, i.e., a negative transfer θ_t.

[10]We could also allow for taxation of the date-0 endowment, but this would slightly burden the analysis without generating additional insights.

2.1 Extensive-form game

The detail of the timing according to which the agents take the above actions is as follows. The game is one of public information, and so each action is conditional on the entire history, which we omit in the notations for simplicity. We discuss alternative timing assumptions in Section 3.5.

Date-0 market for reserves.

1. M selects total date-0 outstanding reserves $X_0 \geq R_{-1}X_{-1}$ by issuing new reserves $X_0 - R_{-1}X_{-1}$ on top of $R_{-1}X_{-1}$ sold by old savers, and announces the interest rate $R_0 \geq 0$ between dates 0 and 1 on them.[11]

2. Savers invest an aggregate quantity $x_0 \geq 0$ of consumption units in the market for reserves at the price level P_0.

Date-0 bond market.

3. F issues $B_0 \geq 0$ bonds.

4. M invests $b_0^M \in [0, (X_0 - R_{-1}X_{-1})/P_0]$ consumption units in the bond market.

5. Savers invest $b_0 \geq 0$ aggregate consumption units in the bond market at a bond price Q_0.

Date-0 spending.

6. F selects consumption g_0 such that

$$\frac{Q_0 B_0}{P_0} + \theta_0 = g_0, \tag{2}$$

where the dividend θ_0 paid by M is equal to its resources from the reserve market net of investment in the bond market:

$$\theta_0 = \frac{X_0 - R_{-1}X_{-1}}{P_0} - b_0^M. \tag{3}$$

[11] We could endow M with consumption units at date 0 that it could use to buy back and cancel all or part of the legacy reserves $R_{-1}X_{-1}$ without affecting the analysis. The remaining net legacy reserves would then be the variable of interest.

Date-1 reserve market.

7. M receives an exogenous terminal demand for reserves $\bar{x} > 0$ from unmodelled agents and issues $X_1 - R_0 X_0 \geq 0$ at a price level P_1.

Date-1 default, taxation, and spending.

8. F raises taxes τ, and decides on $l_1 \in [0, 1]$ and g_1 such that

$$g_1 = \tau + \theta_1 - \frac{(1 - l_1) B_0}{P_1}, \tag{4}$$

where the dividend θ_1 paid by M is equal to its proceeds from the date-1 reserve market and from bond repayment:

$$\theta_1 = \frac{X_1 - R X_0}{P_1} + \frac{(1 - l_1) b^M P_0}{Q_0 P_1}. \tag{5}$$

A strategy profile $\sigma = (R_0, X_0, x_0, P_0, B_0, b_0^M, b_0, Q_0, X_1, P_1, l_1, \tau_1)$ describes all the above actions for each agent given all possible histories.[12]

2.2 Objectives of F and M

The objectives that F and M respectively seek to maximize are respectively:

$$U^F = v(g_0) + \beta^F \left(v(g_1) - c(\tau_1) - \alpha^F \delta \right), \tag{6}$$

$$U^M = - \mid P_0 - P_0^M \mid - \beta^M \mid P_1 - P_1^M \mid - \beta^M \alpha^M \delta, \tag{7}$$

where $\delta = \mathbb{1}_{\{l_1 > 0\}}$, $\beta^F, \beta^M \in (0, 1)$, $\alpha^F, \alpha^M > 0$, v is an increasing function, and $P_0^M, P_1^M > 0$. In words, the variable δ is equal to 1 in case of an outright default on a government bond due at date 1, and to 0 otherwise. Thus, each authority $X \in \{F; M\}$ incurs a cost α^X in case of sovereign default. The fiscal authority also values spending and incurs costs of taxation but does not care about the price level, whereas the monetary authority also finds it costly to deviate from a given target P_t^M for the date-t price level.[13] Taxation costs can be interpreted as distortions that the fiscal authority cares about, or

[12] The strategy profile σ does not feature the variables θ_0, g_0, θ_1, and g_1 as they mechanically derive from the others from (2), (3), (4), and (5).

[13] Results would be similar with an inflation target.

more broadly as any political costs. Our results would carry over if we assumed that M and F both cared about price level and government expenditures, albeit with sufficiently different weights. The assumed stark difference in objectives simplifies the exposition.

We assume that holding (7) fixed, M prefers to maximize (6). Such lexicographic preferences only serve to eliminate equilibria that would crucially rely on M not caring at all about the government's consumption.

2.3 Equilibrium concept

Definition 1. ***(Equilibrium)*** *Given initial reserves $R_{-1}X_{-1}$, an equilibrium is a strategy profile σ such that:*

1. *Each action by F and M is optimal given history and its beliefs that the future actions are taken according to the strategy profile.*

2. *Saver $i \in [0,1]$ optimally invests $x^i = x_0$ in the reserve market given (R_0, X_0, x_0, P_0), and the strategy profiles for all future actions, and optimally invests $b^i = b_0$ in the bond market given $(R_0, X_0, x_0, P_0, B, b_0^M, b_0, Q_0)$, and the strategy profiles for all future actions.*

3. *The market for reserves clears at date 0, $P_0 x_0 = X_0$, at date 1, $P_1 \bar{x} = X_1$, and the market for bonds clears at date 0, $Q_0 B = P_0(b_0 + b_0^M)$.*

Our equilibrium concept is that of Ljungqvist and Sargent (2018), which adapts plain game-theoretic subgame perfection to the situation in which a "large" player interacts with Walrasian agents. We extend this concept to the case in which there are two such large players, a monetary and a fiscal authority. Very intuitively, F and M play against "the private sector", which responds to their supply of reserves and bonds with aggregate demands and prices in reserve and bond markets. In equilibrium, these "actions" of the private sector correspond to prices and aggregate quantities such that markets clear, and such that the behavior of each (price-taking) individual saver is optimal given prices and fiscal and monetary policies.

2.4 Interpretations

Comments on default costs. In the pioneering paper of Sargent and Wallace (1981), the preferences of the fiscal and monetary authorities are not spelled out. Yet it is implicit and important in their approach that the monetary authority has an arbitrarily large aversion to outright sovereign default. The monetary authority would otherwise not be willing to accommodate, no matter the inflationary consequences, whichever path of debt and deficits the fiscal authority announces.

The costs α^M and α^F are finite here, and are only two of the parameters that will determine whether fiscal or monetary dominance prevails.

In Section 3.5, we spell out a modified version of the model in which F takes back control of the price level to impose a soft default on its debt. We show that the costs related to such a reversal of central-bank independence have a similar role as α^F and α^M in the case of an outright default—the fiscal authority selecting the best option between hard and soft default. We also have the view that these costs, if finite, are not zero either: such a reversal may also lead to a reputation loss and a low future credibility of any attempt to make the central bank independent again. Also, in many countries, central-bank independence is enshrined in the law, thus amending it requires sufficient political consensus—following Riboni (2010) and Piguillem and Riboni (2015), building such a consensus is costly and then constitutes a commitment device for central-bank independence.

The costs of default α^F and α^M are exogenous in this two-date version of the model, savers will create fully endogenous default costs in the infinite-horizon analysis in Section 5 through market exclusion. Costs from formal default include in practice output losses due to financial-market exclusion or/and trade sanctions, legal and settlement costs, banking crises and more generally financial instability, as well as private costs—electoral or more generally political costs for the fiscal authority and career concerns for central bankers.

The reserve market at date 0 opens before debt issuance. Our main assumption on the timing is that the reserve market opens before the bond market. At date 0, the fiscal authority issues debt after the current price level is set. This assumed timing implies by construction that the date-0 debt issuance can only affect the date-1 price level. In contrast, as we detail in Section 3.5, if the reserve market opened after the bond market,

F would not be able to influence the date-1 price level but the date-0 one. More generally, current debt issuance affects only the price level formed in the subsequent reserve market, whether it is within the same date or at the following one, and our broad insights do not depend on a particular timing assumption.

3 Baseline Model

Subgame perfection boils down to sequential rationality with a finite horizon, and so we can solve this two-date model using backwards induction. This section does so using a baseline version of the model that showcases our central insight in the simplest fashion. We suppose:

Assumption 1. ***(Baseline model)***

- *There exists $\bar{\tau} \geq 0$ such that $c(\tau_1) = 0$ for $\tau_1 \leq \bar{\tau}$, and $c(\tau_1)$ is arbitrarily large for $\tau_1 > \bar{\tau}$.*
- *There exists $\underline{g} \geq 0$ such that $v(g) = g$ for $g \geq \underline{g}$, and $v(g)$ is arbitrarily small for $g < \underline{g}$.*
- $\alpha^F \geq \bar{x} + \bar{\tau} - \underline{g}$. (8)
- $\dfrac{R_{-1}X_{-1}}{P_0^M} \leq \dfrac{\bar{x}}{r}$ *and* $\bar{\tau} \geq (1+r)\underline{g}$. (9)

In words, the fiscal authority F essentially has fixed date-1 fiscal resources $\bar{\tau}$. It faces an incompressible level of expenditures $\underline{g}$. Condition (8) means that F stands ready to give up the consumption of date-1 real public resources net of incompressible expenditures $\bar{x}+\bar{\tau}-\underline{g}$, if this avoids default. Finally, condition (9) ensures that there is enough demand for reserves at date-1 to roll over legacy reserves $R_{-1}X_{-1}$ and set the price level at P_0^M. As detailed below, Assumption 1 simplifies the date-1 spending, taxation, and default decisions of the fiscal authority because it implies that F chooses to default if and only if making good on its debt requires spending less than $\underline{g}$. Beyond simplicity, Assumption 1 also allows to capture situations such as the one of "political dominance" as described by Leeper (2023) in the case of US debt ceiling, in which new debt could not be issued and taxes and expenditures could hardly adjust. In this view, default stems from an ex-post resource constraint rather than a preference.

We solve the game backwards. We relegate the full-fledged formal equilibrium derivation to the proofs of Propositions 1 to 4 below that spell out the results. The body of the paper offers instead an intuitive exposition of (what we think are) the most economically important features of the equilibrium. We focus on the following four stages. We first characterize how the fiscal authority F decides on taxation, spending, and default at the final stage of date 1, and then how the monetary authority M, rationally anticipating this, decides on date-1 monetary policy (Proposition 1). We then move on to date 0, studying date-0 debt issuance by the fiscal authority (Proposition 2). This is the keystone of the analysis, showing how date-0 public debt issuance may lead to what we will deem either fiscal or monetary dominance. Finally, we analyze optimal date-0 monetary policy (Propositions 3 and 4).

3.1 Date-1 taxation, spending, and default

At the terminal stage of date 1, it is optimal for F to raise taxes $\bar{\tau}$ that can be used for debt repayment or/and spending. Assumption 1 implies that F makes good on its debt if and only if this is compatible with spending above the incompressible level $\bar{g}$. F fully defaults otherwise so as to be able to consume at least $\bar{g}$. More precisely, given history $(R_0, X_0, x_0, P_0, B, b_0^M, b_0, Q_0, X_1, P_1)$, F finds it optimal to repay its debt if and only if

$$P_1(\bar{x} + \bar{\tau} - \underline{g}) \geq R_0 X_0 + B_0 - \frac{b_0^M P_0}{Q_0}. \tag{10}$$

Condition (10) admits a straightforward interpretation. The left-hand term is the nominal value of total public resources $\bar{x} + \bar{\tau}$ net of incompressible expenditures $\bar{g}$ at date 1. The right-hand term is the net total liabilities of the public sector at the opening of date 1, that is, the liabilities in the hands of the private sector, equal to the gross liabilities $R_0 X_0 + B_0$ minus holdings of government debt by the monetary authority $b_0^M P_0/Q_0$.

In sum, F never spends below $\underline{g}$, and defaults if and only if the solvency condition (10) fails to hold.

3.2 Date-1 monetary policy

Given history $(R_0, X_0, x_0, P_0, B, b_0^M, b_0, Q_0)$, date-1 monetary policy merely consists in selecting the amount $X_1 - R_0 X_0 \geq 0$ of new reserves issued in the date-1 reserve market. From market clearing $P_1 \bar{x} = X_1$, the monetary authority M can this way reach any date-1 price level P_1 above $R_0 X_0 / \bar{x}$.

In particular, M can always (but may not want to) set P_1 sufficiently large that the solvency constraint (10) holds so that F does not default. A larger price level P_1 frees up resources available for bond repayments by eroding the real value of maturing nominal bonds B_0—as in the fiscal theory of the price level—and also by reducing the real value of outstanding reserves $R_0 X_0$.

We denote by P^F the smallest price level such that this solvency constraint (10) holds:

$$P^F \equiv \frac{R_0 X_0 + B_0 - \frac{b_0^M P_0}{Q_0}}{\bar{x} + \bar{\tau} - \underline{g}}. \tag{11}$$

By definition, expenditures are at the incompressible level ($g_1 = \underline{g}$) as soon as $P_1 = P^F$ so that (10) holds with equality.

Let us also define

$$\underline{P}_1 \equiv \max \left\{ P_1^M ; \frac{R_0 X_0}{\bar{x}} \right\}. \tag{12}$$

If $P^F \leq \underline{P}_1$, then M optimally sets $P_1 = \underline{P}_1$ as it minimizes the departure from its target $\mid P_1 - P_1^M \mid$, possibly to 0 if $\underline{P}_1 = P_1^M$, without inducing default.

If $P^F > \underline{P}_1$, then M must trade off the distance to price-level target and sovereign solvency. If M lets F default then it incurs a cost α^M, but it can optimally set the date-1 price level at $\underline{P}_1$. If conversely M seeks to avert default, then it optimally does so by setting the date-1 price at the smallest level P^F at which this is possible, thereby reducing F's consumption to the incompressible level $\underline{g}$. As a result, M finds it optimal to prevent F from defaulting by setting $P_1 = P^F$ if and only if $P^F \leq \underline{P}_1 + \alpha^M$.

The following proposition summarizes this date-1 outcome.

Proposition 1. ***(Terminal date 1)*** *Given history $(R_0, X_0, x_0, P_0, B_0, b_0^M, b_0, Q_0)$, date 1 unfolds according to one of the three following situations.*

1. *Date-1 monetary dominance: If $P^F \leq \underline{P}_1$, M sets the date-1 price level at $\underline{P}_1$ by setting $X_1 = \bar{x}\underline{P}_1$. F fully repays maturing bonds: $l_1 = 0$, and consumes $g_1 \geq \underline{g}$, where the inequality is strict as soon as $P^F < \underline{P}_1$.*

2. *Date-1 fiscal dominance: If $\underline{P}_1 < P^F \leq \underline{P}_1 + \alpha^M$, M sets the date-1 price level at P^F. F fully repays maturing bonds: $l_1 = 0$, and spends at the incompressible level $g_1 = \underline{g}$.*

3. *Default: Otherwise, M sets the date-1 price level at $\underline{P}_1$. F fully defaults on B: $l_1 = 1$, and spends $g_1 = \bar{x} + \bar{\tau} - R_0X_0/\underline{P}_1 > \underline{g}$.*

Proof. See Appendix A.1. □

Figure 1 illustrates how the date-1 price level P_1 evolves as the (nominal) net public liabilities at the outset of date 1, $R_0X_0+B_0-b_0^M P_0/Q_0$, increase. As soon as the monetary authority M cares somewhat about sovereign solvency—that is, $\alpha^M > 0$, it chickens out and ensures that the price level is such that the fiscal authority is solvent. There is however a maximum nominal amount of net public liabilities beyond which M prefers to let F default.

The key result in Proposition 1 is that the situations of fiscal dominance in which M chickens out so that $P_1 = P^F > \underline{P}_1$ must be such that F cannot spend in excess of the incompressible level $\underline{g}$. If this were the case that $g_1 > \underline{g}$ and $P_1 > \underline{P}_1$ simultaneously along the equilibrium path, M would indeed strictly benefit from tightening monetary policy, thereby forcing F to reduce spending so as to avert default, a contradiction. We will now see that this feature of the equilibrium at date 1 will shape the date-0 debt policy of the fiscal authority. Provided the fiscal authority is sufficiently patient, it will face a dilemma between maximizing overall public spending at dates 0 and 1 by forcing the monetary authority to chicken out, versus being able to spend beyond the incompressible level at date 1.

Remark on "reserve overflow". In the case of monetary dominance or default, M might still have to set the price strictly above its date-1 target P_1^M when the reserves sold by old savers R_0X_0 are strictly larger than $\bar{x}P_1^M$, so that the price level must be at least equal to $R_0X_0/\bar{x} = \underline{P}_1 > P_1^M$. In this case, M has manufactured its own lower bound on the date-1 price level when deciding on (R_0, X_0) at date 0, thereby barring itself from reaching its date-1 price level target. We will see below that in the absence of a zero lower bound on the policy rate R_0, M can ensure that this does not occur along the equilibrium path. We will also see that there exist cases in which M deliberately uses this in order to commit to a date-1 price level that it finds ex-post excessive (see Proposition 4). Notice that, in this situation of reserve overflow, monetary policy may have perverse effects with a tightening (a higher R_0) leading to a higher price level.

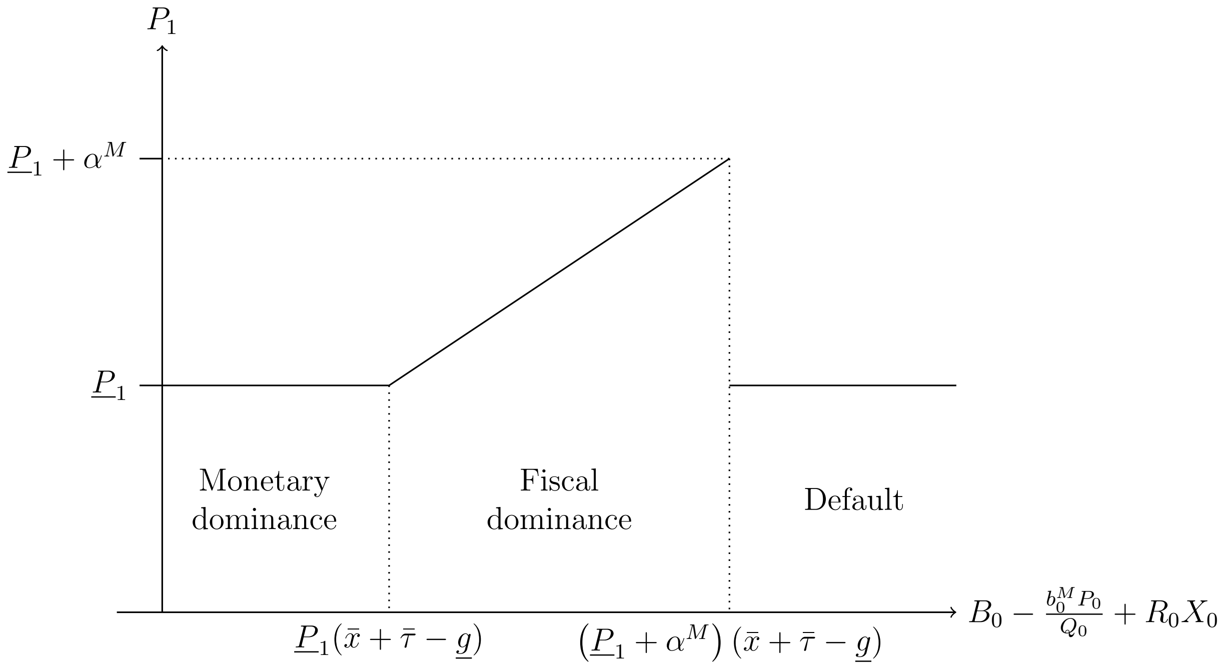

Figure 1: Date-1 price level P_1 as a function of net public liabilities held by the private sector $(B_0 - b_0^M P_0/Q_0 + R_0 X_0)$.

3.3 Date-0 bond market

Suppose now that the date-0 reserve market has generated history (R_0, X_0, x_0, P_0) and that the date-0 bond market opens. The fiscal authority must select a quantity $B_0 \geq 0$ of nominal debt to be issued, anticipating that this will lead to either monetary dominance, fiscal dominance, or default at date 1. Due to our timing assumption, date-0 debt issuance has no effects on date-0 prices.

We show that, unsurprisingly, F never finds it optimal to issue debt on which it defaults at date 1.[14] Thus the debt issued by F induces either monetary or fiscal dominance as a date-1 continuation equilibrium. There is a strictly positive gain for F from fiscal dominance over monetary dominance as soon as $R_0 X_0 > 0$ since a higher date-1 price level ($P^F > \underline{P}_1$ from Proposition 1) implies that reserves will have a strictly smaller real value under fiscal dominance, thereby allowing for more spending. There is also a potential cost of fiscal dominance due to the fact that F cannot spend beyond the incompressible amount $\underline{g}$ at date 1. This restriction on future spending is however costly to F only if it is sufficiently patient in the sense that $\beta^F r > 1$. In this case, one must assess the net benefits from fiscal over monetary dominance in order to determine F's issuance decision.

[14]See Appendix A.2.

We do so by comparing the maximum utility levels that F can reach conditional on fiscal and monetary dominance.[15]

Optimal debt issuance conditional on date-1 fiscal dominance. We show that any debt level B that leads to date-1 fiscal dominance entails that M invests its entire date-0 resources in the debt market—formally, $b_0^M = x_0 - R_{-1}X_{-1}/P_0$. The reason M maximizes the size of its balance sheet this way is that it minimizes the public liabilities in the hands of the private sector at date 1 and thus the departure from its price-level target. We also show that the date-1 price level that ensures sovereign solvency, P^F, increases in the amount B issued by F. This implies that F optimally issues B such that the date-1 price level is equal to $\underline{P}_1 + \alpha^M$, so that M is exactly indifferent between chickening out and letting F default. The utility of the fiscal authority viewed from the opening of the bond market is therefore

$$x_0 - \frac{R_{-1}X_{-1}}{P_0} + \frac{1}{r}\left(\bar{x} + \bar{\tau} - \frac{R_0 X_0}{\underline{P}_1 + \alpha^M} - \underline{g}\right) + \beta\underline{g}. \tag{13}$$

In words, F consumes at date 0 the resources $x_0 - R_{-1}X_{-1}/P_0$ that M collects in the reserve market and invests in bonds plus the present value of date-1 public resources net of reserve repurchases and incompressible expenditures. This corresponds to an amount of nominal debt held by the private sector such that M sets the date-1 price level at $\underline{P}_1 + \alpha^M$ and F consumes $\underline{g}$ at date 1. In the remainder of the paper, we deem this optimal amount of debt conditional on date-1 fiscal dominance the "Sargent-Wallace debt level".[16]

Optimal debt issuance conditional on date-1 monetary dominance. As already mentioned, the fiscal authority may find issuing a debt level such that monetary dominance prevails at date 1 optimal only if it is sufficiently patient in the sense that $\beta^F r > 1$. In this case, F optimally borrows b^*, the amount that fills the gap (if any) between the resources $x_0 - R_{-1}X_{-1}/P_0$ that F receives from the central bank at date 0 and its date-0

[15] The full formal treatment is in Appendix A.2.

[16] We use this denomination not because our model is stricto sensu the one in Sargent and Wallace (1981) but because it corresponds to a situation in which the fiscal authority forces the price level away from the central bank's objective to ensure solvency in equilibrium.

incompressible spending $\underline{g}$:

$$b^* \equiv \left(\underline{g} - x_0 + \frac{R_{-1}X_{-1}}{P_0}\right)^+. \tag{14}$$

F thus obtains utility

$$x_0 - \frac{R_{-1}X_{-1}}{P_0} + b^* + \beta\left(\bar{x} + \bar{\tau} - rb^* - \frac{R_0X_0}{\underline{P}_1}\right). \tag{15}$$

In the remainder of the paper, we deem this optimal amount of debt conditional on date-1 monetary dominance the "price-level taking debt level". Comparing (15) and (13) shows that F prefers the price-level taking debt level if and only if

$$\underbrace{(\beta^F r - 1)}_{\text{Unit cost of frontloading } g} \times \underbrace{\left(\bar{x} + \bar{\tau} - \underline{g} - rb^* - \frac{R_0X_0}{\underline{P}_1}\right)}_{\text{Net public resources}} \geq \underbrace{R_0X_0\left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1 + \alpha^M}\right)}_{\text{Fiscal-dominance gains}}. \tag{16}$$

This condition admits a simple interpretation. Relative to the price-level taking debt level, the Sargent-Wallace one generates additional resources from applying a higher inflation on the reserves R_0X_0 held by savers at date 1 (right-hand side of (16)). Generating these resources comes at the cost of frontloading the date-1 consumption of the government, however (left-hand side of (16)). The unit frontloading cost is $\beta^F r - 1$, and is actually a unit gain if $\beta^F r \leq 1$, in which case F always prefers the Sargent-Wallace debt level. This unit cost applies to the resources of the public sector $\bar{x} + \bar{\tau}$ net of the date-1 value of its liabilities, both explicit (reserves and bonds) and implicit (incompressible expenditures). F prefers the price-level taking debt level if this cost from the Sargent-Wallace debt level exceeds the benefits. The following proposition summarizes these results.

Proposition 2. ***(Debt issuance in the date-0 bond market)*** *Given (R_0, X_0, x_0, P_0), F issues one of either debt level:*

- ***Price-level taking debt level:*** *F issues bonds so as to optimize its consumption pattern taking the date-1 price level $\underline{P}_1$ as given: It raises an amount b^* of real resources. M's bond purchases are immaterial. There is no default at date 1.*

- ***Sargent-Wallace debt level:*** *F issues a larger amount in the bond market, front-loading consumption as much as possible ($g_1 = \underline{g}$) and issues enough debt to* *force a date-1 price level given by fiscal dominance. M buys back as many bonds as possible: $b_0^M = x_0 - R_{-1}X_{-1}/P_0$, but not the whole issuance. The date-1 price level is equal to $\underline{P}_1 + \alpha^M$. There is no default at date 1.*

F selects the "price-level taking" debt level whenever

$$\left(\beta^F r - 1\right)\left(\bar{x} + \bar{\tau} - \underline{g} - rb^* - \frac{R_0 X_0}{\underline{P}_1}\right) \geq R_0 X_0 \left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1 + \alpha^M}\right). \tag{17}$$

Proof. See Appendix A.2. □

The "Sargent-Wallace" debt level whereby F floods the bond market with paper so as to force M to "chicken out" and inflate away outstanding reserves at date 1 in order to ensure public solvency is related to that underlying the unpleasant monetarist arithmetic in Sargent and Wallace (1981). An important difference is that F creates a deficit that forces M to inflate away the value of public liabilities and, in particular, reserves, whereas, in Sargent and Wallace (1981), a deficit requires the monetary authority to generate seignorage income. Proposition 2 shows that issuing the Sargent-Wallace debt level need not be F's favorite strategy as this may induce an excessive distortion of its optimal spending relative to the gains from inflation. We are now equipped to solve for the first stage of the game: the date-0 market for reserves.

3.4 Date-0 reserve market

The date-0 reserve market opens before the bond market. In this market, the monetary authority selects a supply of reserves X_0 and a level of interest rate R_0. Savers' demand for reserves should be such that $1 = \frac{R_0}{r}\frac{P_0}{P_1}$, where P_1 is the expectation of the date-1 price level, given the continuation strategies described above. We describe the outcome in this market in two steps. Proposition 3 first characterizes situations in which monetary dominance prevails at both dates 0 and 1. Proposition 4 then tackles the situations in which M cannot reach this outcome.

As we show in the proof of these propositions, the monetary authority can select the price level P_0 on the date-0 reserve market in all these case. Importantly, it does so using the optimal portfolio decisions by savers. Intuitively, M can pin down a unique demand for reserves x_0 and, thus, a unique price level $P_0 = X_0/x_0$ with an appropriate choice of R_0 and X_0. A demand below (above) this target level x_0 would raise (reduce) the price level P_0, thereby raising (reducing) the real return on reserves away from r, which would contradict savers' optimal portfolio choice.[17]

Proposition 3. ***(Characterization of monetary dominance)*** *The equilibrium is such that price levels are on target at dates 0 and 1 ($P_0 = P_0^M$ and $P_1 = P_1^M$) if and only if*

$$\left(\beta^F r - 1\right)\left(\bar{x} + \bar{\tau} - (1+r)\underline{g} - \frac{R_{-1}X_{-1}}{P_0^M}\right) \geq \frac{rR_{-1}X_{-1}}{P_0^M}\frac{\alpha^M}{P_1^M + \alpha^M}. \tag{18}$$

If this holds, M issues no or sufficiently small new reserves, and announces a rate $R = rP_1^M/P_0^M$. The game then unfolds as in the price-level taking debt level situation in Proposition 2 with $P_1 = P_1^M$.

Proof. See Appendix A.3. □

Condition (17) driving the bond issuance of F suggests that M must keep the quantity of reserves R_0X_0 with which it starts out date 1 sufficiently low if it wants to impose monetary dominance at date 1. Accordingly, condition (18) states that M can enforce monetary dominance at dates 0 and 1 ($P_0 = P_0^M$ and $P_1 = P_1^M$) if the legacy reserves $R_{-1}X_{-1}$ are sufficiently small other things being equal. In this case, by issuing no new reserves $X_0 - R_{-1}X_{-1}$, or a sufficiently small amount of them, M makes the gains from the Sargent-Wallace debt level sufficiently small that F does not issue it. M is indifferent between several level of reserves below a threshold (unless (18) binds) because reserves and bonds are perfect substitutes, and so the resources that M raises and transfers to F to fund g_0 can be raised by F at the same cost in the bond market.

In addition to low legacy public liabilities $R_{-1}X_{-1}$, the other interesting feature that drives monetary dominance is the existence of a large fiscal space $\bar{x} + \bar{\tau} - (1+r)\underline{g}$. In this case, F needs to engineer a very large distortion of its public finances in the form of large current borrowing and spending in order to be credibly ready to default in the future. It is important at this point to recall that the analysis is carried out under condition (8) ensuring that F does not contemplate default as long as it can consume at least $\underline{g}$ without taxing more than $\bar{\tau}$. Thus the case in which F has a lot of fiscal space is also implicitly one in which F has a sufficiently large aversion to default.

[17]See Appendix A.3 for the details of the proof. Such implementation is consistent with the approach by Bassetto (2005) who argues that equilibrium selection by policies should derive from optimal private agent decisions – and not on the commitment to violate ex post feasibility constraints.

If F has limited fiscal space or/and there are large legacy liabilities so that inequality (18) fails to hold, then F may find it preferable to double down and worsen its situation so as to force help from the monetary authority by issuing the Sargent-Wallace debt level. The following proposition describes date-0 monetary policy in this case.

Proposition 4. ***(Optimal monetary policy without monetary dominance)*** *Suppose that condition* (18) *in Proposition 3 does not hold. M adopts one of the following three strategies in the reserve market:*

1. *M announces a rate $R_0 = r(P_1^M + \alpha^M)/P_0^M$ and is indifferent between several levels of newly created reserves (including 0). The date-0 price level is P_0^M and then the game unfolds according to the Sargent-Wallace debt level situation with $P_1 = P_1^M + \alpha^M$.*

2. *M announces a rate $R_0 = rP_1^M/P_0$, where $P_0 > P_0^M$ and issues no new reserves ($X_0 = R_{-1}X_{-1}$). Then the game unfolds according to the price-taking debt level situation with $P_1 = P_1^M$.*

3. *M announces a rate $R_0 = rP_1/P_0$, where $P_0 \geq P_0^M$ and $P_1 > P_1^M$. It issues reserves $P_0\bar{x}/r - R_{-1}X_{-1} \geq 0$. Then the game unfolds according to the price-taking debt level situation with $P_1 = R_0X_0/\bar{x} > P_1^M$ (reserve overflow).*

Furthermore, strategy 1 prevails if $\beta^M r \leq 1$, and strategy 2 prevails if $\beta^M r > 1$ and $R_{-1}X_{-1}$ is sufficiently small other things being equal.

Proof. See Appendix A.3. □

In strategy 1, M "surrenders" and does not try to deter F from issuing the Sargent-Wallace debt level. This is the only strategy in which M is indifferent between several reserve issuance levels whose range is detailed in the proof of Proposition 4.

In strategies 2 and 3, by contrast, M deters F with a strategic use of both interest rate and quantity of reserves. In strategy 2, M reduces the real value of legacy reserves at date 0 by announcing a low interest rate that sets P_0 above target, and issues no new reserves. Formally, M sets the date-0 price level at the smallest value such that (18) holds. This strategy both reduces the basis $R_{-1}X_{-1}/P_0$ to which the Sargent-Wallace induced rate of "seigniorage" α^M/P_1^M applies (right-hand side of (18)), and creates fiscal space that F must eliminate at a cost to create such seigniorage (left-hand side of (18)). In sum, strategy 2 is one of preemptive inflation meant to avoid larger future inflation.

In strategy 3, M combines shrinking this way the basis $R_{-1}X_{-1}/P_0$ to which the rate of seigniorage α^M/P_1^M applies by setting $P_0 \geq P_0^M$ together with a reduction in this seigniorage rate by setting $P_1 > P_1^M$. Committing to a date-1 price level above target requires however that M creates its own future lower bound by issuing new reserves at date 0. This expansion of reserves is costly for the same reasons why not issuing new reserves is optimal in strategy 2.

Which of these three strategies is optimal depends on the parameters in a generally complex fashion. The analysis is tractable in two important cases stated in the proposition. First, M has no choice but going for strategy 1 when $\beta^F r \leq 1$. In this case, F finds frontloading consumption optimal even when holding the date-1 price level fixed. It is thus always happy to issue enough nominal debt against this date-0 consumption that it can get additional resources along the way by forcing M to go beyond its target at date 1.

Second, strategy 2 of preemptive inflation is optimal if $\beta^F r > 1$ and $R_{-1}X_{-1}$ is sufficiently small. Compare it first to strategy 1. The latter comes at a fixed utility cost $\beta^M \alpha^M$ for M. Conversely, the cost of strategy 2 is linearly increasing in $R_{-1}X_{-1}$. In particular if $R_{-1}X_{-1}$ is sufficiently small other things being equal that the economy is not too far off from condition (18), the rise in P_0 that warrants the price-level taking strategy is sufficiently small that it induces a disutility $P_0 - P_0^M < \beta^M \alpha^M$. Compare now strategies 2 and 3. The latter consists in raising P_1 on top of raising P_0. This requires the issuance of new reserves such that $X_0 = P_0 \bar{x}/r$ in order to create a reserve overflow at date 1. This level of new reserves creates a fixed cost—making condition (17) harder to satisfy, smaller than the benefits from being able to raise P_1 a little bit over P_1^M, which is all that is needed for $R_{-1}X_{-1}$ sufficiently small.

Remark. Notice that strategy 3 resembles a situation deemed "stepping on a rake" by Sims (2011). In this case, as we previously noted, any tightening in monetary policy (a higher R_0) would have the perverse effect of increasing the price level at date 1 as this corresponds to situation of reserve overflow. Notice, however, that this is not the only situation in which monetary policy can accept some form of fiscal dominance.

3.5 Discussion

Reversing central-bank independence and soft default. In the benchmark model, the fiscal authority can only threaten the monetary authority with a hard default at date 1. In this section, we allow the fiscal authority to take control of the price level directly by intervening in the reserve market. Our main finding is that the fiscal authority always uses its best option between hard and soft default as a threat.

Let us slightly modify the baseline model and allow the fiscal authority to issue reserves $X_1^F \geq 0$ at date 1 once observed X_1.[18] The market clearing condition for reserves at date 1 writes:

$$X_1^F + X_1 = P_1 \bar{x}.$$

Yet, such an issuance implies a fixed cost γ^F to the fiscal authority. When the fiscal authority intervenes on the reserve market, we assume that the payoff of the monetary authority is $-\gamma^M$, which does not depend on the price level—capturing that the incumbent central banker is replaced by a government's crony and no longer cares about policy outcomes. The payoffs are thus modified as follows:

$$U^F = v(g_0) + \beta^F \left(v(g_1) - c(\tau) - \alpha^F \delta - \gamma^F \epsilon \right), \tag{19}$$

$$U^M = - \mid P_0 - P_0^M \mid - \beta^M \left(\left(\mid P_1 - P_1^M \mid + \alpha^M \delta \right) (1 - \epsilon) + \gamma^M \epsilon \right), \tag{20}$$

with $\epsilon = 1$ when the fiscal authority takes control of monetary policy and $\epsilon = 0$ otherwise. The rest of the model remains unchanged. For simplicity, we assume $\underline{g} = 0$ and $\gamma^F \geq \bar{x} + \bar{\tau}$—that is, F only intervenes in the reserve market due to resource constraint not because of F's preference.

Let us focus on date-1 decisions. When intervening on the reserve market, the fiscal authority seeks to set the price level $P_1 \geq \frac{X_1}{\bar{x}}$ so as to maximize:

$$g_1 = \tau + \bar{x} - \frac{(1 - l_1) B_0 + R_0 X_0 - (1 - l_1) b_0^M P_0 / Q_0}{P_1} - \alpha^F \delta$$

[18] We model the reversal of central-bank independence in this manner for tractability. However, the idea that the Treasury can print money and force this way monetary policy is not a pure abstraction and can potentially be linked to the proposal in the US to issue a trillion-dollar coin or to the one in the euro area to issue zero-coupon perpetual bonds.

The solution is $P_1 = +\infty$. To implement this price level, the fiscal authority floods the market with reserves. Subsequently, F optimally chooses $l_1 = 0$ and $\tau = \bar{\tau}$. This situation describes one of soft default—debt is fully inflated away—with full reimbursement of debt. In real terms, however, the outcome is the same as under a full default.

As a result, F intervenes on the reserve market if and only if $\gamma^F \leq \alpha^F$ and if M does not issue enough reserves to prevent from hard default, that is:

$$\frac{X_1}{\bar{x}}\left(\bar{\tau} + \bar{x}\right) \leq B_0 + R_0 X_0 - b_0^M P_0/Q_0. \tag{21}$$

When $\alpha^F < \gamma^F$, F never takes the control of the reserve market and the threat is immaterial. Otherwise, M will choose the lowest X_1 that ensures a price level above $\underline{P}_1$, that is, such that $X_1 \geq \underline{P}_1 \bar{x}$, that is below $\underline{P}_1 + \gamma^M$ and does not satisfy (21). If net public liabilities in the hand of the private sector are too high (that is, if inequality (21) is satisfied for $X_1 = (\underline{P}_1 + \gamma^M)\bar{x}$), then X_1 is immaterial for M and M prefers resigning and being replaced. Overall, the outcome of this game is very similar to that in the baseline model except that what matters is γ^F instead of α^F.

The rest of the game follows Section 3 with either hard or soft default at date 1 depending on the relative values of α^F and γ^F.

Remark. Which option between a soft and a hard default is the most expensive one? On many dimensions, this question goes much beyond the scope of the paper. However, it is worth mentioning that an outright default may be easier to implement and cheaper than trying to take back control of monetary policy for countries within monetary unions, as this may mean leaving the common currency. In contrast, a hard default typically does not require a decision by the legislative branch, and may thus decided solely by the executive branch. On the other hand, the absence of formal independence may ease the possibility to reverse central-bank independence. A political consensus against central-bank independence may have the same effect.

Ex-ante fiscal gains from the unpleasant arithmetic. It is worthwhile stressing that F does not derive ex-ante gains from issuing the Sargent-Wallace debt level when it does so in equilibrium. When it finds it optimal to do so ex-post, it is anticipated in the reserve and bond markets, so that all public liabilities command the same real return r. F on the other hand incurs the costs from excessive borrowing when $\beta^F r > 1$. In this case, F

would be happy to avail itself of a commitment device to not issue at the Sargent-Wallace level, such as a credible fiscal requirement putting an upper bound on the amount of debt it can issue.

There are also parameter values such that F derives ex-ante gains from its ex-post optimal behavior. These correspond to the equilibria in which M deters the Sargent-Wallace debt level with an increase in P_0—in strategy 2 and possibly (but not necessarily) in strategy 3. This erodes the value of the legacy liabilities, thereby generating additional public resources for consumption. Furthermore, F does not borrow inefficiently in this case and thus extracts these benefits at no cost.[19]

What if the bond market opens before the reserve market? Suppose that the bond market opens and clears before that for reserves at date 0. The insights are broadly similar to that when M issues reserves first.[20] The main difference is that F cannot benefit from forcing a date-1 price level above target by borrowing a lot at date 0 since this would be anticipated in both date-0 bond and reserve markets. F may however still find it worthwhile forcing M to set the date-0 price level at $P_0^M + \alpha^M$ so as to reduce the date-0 real value of legacy reserves $R_{-1}X_{-1}$. This is so again when the associated gain more than offsets the cost from excessive date-0 borrowing. But then, the interesting analysis of optimal monetary policy in anticipation of this behavior—the equivalent of Propositions 3 and 4—would have to take place in the date-(-1) reserve market at which these reserves are issued. In sum, our analysis shows that current debt issuance can affect the price-level determination that follows, whether it is within the same date or at the following one. More generally, the exact intradate timing of the game would play no significant role in a version of the model with a large number of dates.

What if F is financially constrained? Condition (8) implies that F is financially unconstrained in the sense that it can borrow against its entire future resources $\bar{x} - R_0X_0/P_1 + \bar{\tau} - \bar{g}$. Thus the default boundary that it must reach when entering into the Sargent-Wallace debt level is equal to the point at which it would be forced to either raise taxes above $\bar{\tau}$ or cut expenditures below $\underline{g}$ in order to make good on its debt. This situation in which borrowing constraints play no role is a natural first step. The main insights are

[19] This may, however, be anticipated in the unmodelled date-(-1) reserve market in which $R_{-1}X_{-1}$ is issued.

[20] The full analysis is available upon request.

identical, however, if F is financially constrained. Suppose that condition (8) is replaced with

$$r\underline{g} \leq \alpha^F < \bar{\tau} - \underline{g}, \tag{22}$$

so that F cannot borrow against its entire future resources, but can borrow enough to fund date-0 incompressible expenditures $\underline{g}$. In this case, the default boundary is hit when F owes real debt α^F at date 1, as it finds default preferable to cutting spending by α^F in this case. The counterpart of condition (16) under which F prefers the price-level taking debt level is in this case[21]

$$\underbrace{(\beta^F r - 1)}_{\text{Unit cost of frontloading } g} \times \underbrace{\left(\frac{\alpha^F}{r} - b^*\right)}_{\text{Amount to be frontloaded}} \geq \underbrace{\beta^F R_0 X_0 \left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1 + \alpha^M}\right)}_{\text{Fiscal-dominance gains}}. \tag{23}$$

The only difference with condition (16) is that the Sargent-Wallace debt level no longer corresponds to borrowing against the entire date-1 resources net of incompressible expenditures, but only against the default boundary α^F.[22] Condition (23) shows that a higher cost of default makes the Sargent-Wallace debt level more costly and thus less appealing to F. As will be shown in Section 4, this result that a larger cost of default α^F makes fiscal dominance less appealing to F other things being equal generally holds when the fiscal authority faces a smooth, convex cost of taxation.

Return on central bank investments. One can interpret $\bar{x}$ as including not only an exogenous demand for reserves but also the return on investments that M funded with the proceeds from issuing X_{-1} at date -1. This implies that monetary dominance benefits from a high expected return viewed from date 0. This shapes the risk-taking incentives of M when investing at date -1. In particular, if fiscal dominance is very likely viewed from date -1 conditionally on investing in safe assets, M may be tempted to opt for assets with riskier returns to increase the probability of monetary dominance. Such gambling for resurrection behavior would parallel that of investors subject to limited liability constraints as studied in the finance literature (see Allen and Gale, 2000, among

[21] We omit the derivation for brevity, it is available upon request.

[22] The date-0 expenditures b^* are substracted from this level because F has to borrow to fund them anyway in the price-level taking strategy.

others).

4 Extensions

This section discusses two extensions of the baseline model. We first open up the possibility that the (real) return that savers require on reserves and bonds depends on the volume of public liabilities that they must hold (Section 4.1). We then posit smooth convex costs of taxation (Section 4.2). These extensions confirm the broad insights from the baseline model. They also suggest that the cost of inducing fiscal dominance is in general larger than in the baseline model because setting public debt at a level that induces M to chicken out may come both with an increase in the interest rate and with higher taxes down the road. These effects are shut down in the baseline model for expositional simplicity.

4.1 Variable interest rate

This section studies an extension of the baseline model in which the issuance of public liabilities affects the interest rate. Formally, we modify the baseline model as follows.

Assumption 2. ***(Variable-rate model)***

- *Savers are endowed with one consumption unit at date 0, and with a large quantity of them at date 1. Their preferences are given by* $u(c_0) + c_1/r$*, where* u' *exists and is a decreasing strictly convex bijection mapping* $(0, 1]$ *into* $[u'(1), +\infty)$*.*
- *We drop condition* (9)*.*
- *As in the baseline model, taxes come at no cost up to the threshold* $\bar{\tau} \geq 0$*, and at an arbitrarily large cost beyond it.*
- *For notational simplicity, we assume that* $\underline{g} = 0$*.*

Here the public sector lifts the (real) interest rate when issuing liabilities simply because it reduces savers' date-0 consumption. This impact on the interest rate could stem in practice from other mechanisms such as the crowding out of private investment.[23]

[23] Crowding out of private investment was actually the force at play in an earlier draft.

Condition (9) is no longer relevant as it involves a fixed assumed discount rate r. For all $x \in [0,1)$, we define

$$r(x) \equiv ru'(1-x). \tag{24}$$

The equilibrium derivation by backward induction goes as follows. First, it is easy to see that the analysis of date 1 following a history $(R, X_0, x, P_0, B, b^M, b, Q_0)$ is verbatim that of the baseline model summarized in Proposition 1. The reason is simply that the interest rate no longer plays a role once public liabilities have been issued at date 0. Spending at the end of date 0 by F is also identical for the same reason.

Consider now the date-0 bond market given history (R_0, X_0, x, P_0). For the same reason as in the baseline model, there is no default along the equilibrium path, and so F anticipates that its bond issuance will lead either to monetary or fiscal dominance at date 1. As in the baseline model, we solve for the optimal debt level conditional on each of these date-1 outcomes.

Monetary dominance. The fiscal authority F seeks to optimally consume taking the date-1 price level as given, and thus issues the "price-level taking" debt level $B_0 = \underline{P}_1 r(1 - b^{PT} - x)b^{PT}$, where[24]

$$b^{PT} \equiv \arg\max_b \{g_0 + \beta^F g_1\} \tag{25}$$

$$\text{s.t. } g_0 = x_0 + b - \frac{R_{-1}X_{-1}}{P_0}, \tag{26}$$

$$g_1 = \bar{x} + \bar{\tau} - \frac{R_0 X_0}{\underline{P}_1} - r(1-x-b)b, \tag{27}$$

$$0 \leq b < 1 - x, 0 \leq g_1. \tag{28}$$

Date-0 consumption g_0 stems from raising b from savers and receiving $x_0 - R_{-1}X_{-1}/P_0$ from M, and date-1 consumption g_1 is what is left of resources $\bar{x} + \bar{\tau}$ once public liabilities have been repaid. Unlike in the baseline model, the convexity of the interest rate schedule $r(.)$ leads to a strictly concave problem. We let (g_0^{PT}, g_1^{PT}) denote the consumption stream of F solving this program. This corresponds (in the case of an interior solution) to the

[24]As in the baseline model (See proof of Proposition 2), b^M is payoff irrelevant in the case of date-1 monetary dominance, and we set it to 0 without loss of generality.

blue point (g_0^{PT}, g_1^{PT}) in Figure 2.[25]

Fiscal dominance. A second option for the fiscal authority is to issue debt so that there is fiscal dominance at date 1: The date-1 price level P_1 satisfies $P_1 = P^F > \underline{P}_1$, where P^F is given by (11). Fiscal dominance implies that F cannot consume at date 1 from Proposition 1 given $\underline{g} = 0$. Thus, denoting (g_0^{SW}, g_1^{SW}) the optimal consumption pattern that F can obtain conditionally on date-1 fiscal dominance, it must be that $g_1^{SW} = 0$ and that g_0^{SW} maximizes date-0 consumption over all the debt levels leading to date-1 fiscal dominance. The proposition below states that the fiscal authority, as in the baseline model, selects the "Sargent-Wallace" debt level such that the date-1 price level is $\underline{P}_1 + \alpha^M$, the largest value of P^F that does not trigger default.

The following proposition summarizes these results.

Proposition 5. ***(Debt issuance in the date-0 bond market)*** *Given* (R_0, X_0, x_0, P_0)*, F issues one of either debt level:*

- ***Price-level taking debt level:*** *F issues bonds so as to optimize its consumption pattern taking the date-1 price level $\underline{P}_1$ as given.*
- ***Sargent-Wallace debt level:*** *F issues a larger amount in the bond market, front-loading consumption as much as possible ($g_1^{SW} = 0$) so as to force a date-1 price level $\underline{P}_1 + \alpha^M$.*

F selects the "price-level taking" debt level whenever

$$\Delta \equiv g_0^{PT} + \beta^F g_1^{PT} - g_0^{SW} \geq 0. \tag{29}$$

Proof. See Appendix A.2. □

This Sargent-Wallace debt level and the associated government consumption is depicted by the red point on Figure 2. That $g_1^{SW} = 0$ of course means that this point is on the x-axis. The gain in terms of resources for the public sector associated with a price level P^F larger than $\underline{P}_1$ implies that this red point is to the right of the intersection of the x-axis with the feasibility frontier in the case of the price-level taking debt level.

[25] We are grateful to Vladimir Asriyan for suggesting this graphical representation of our results.

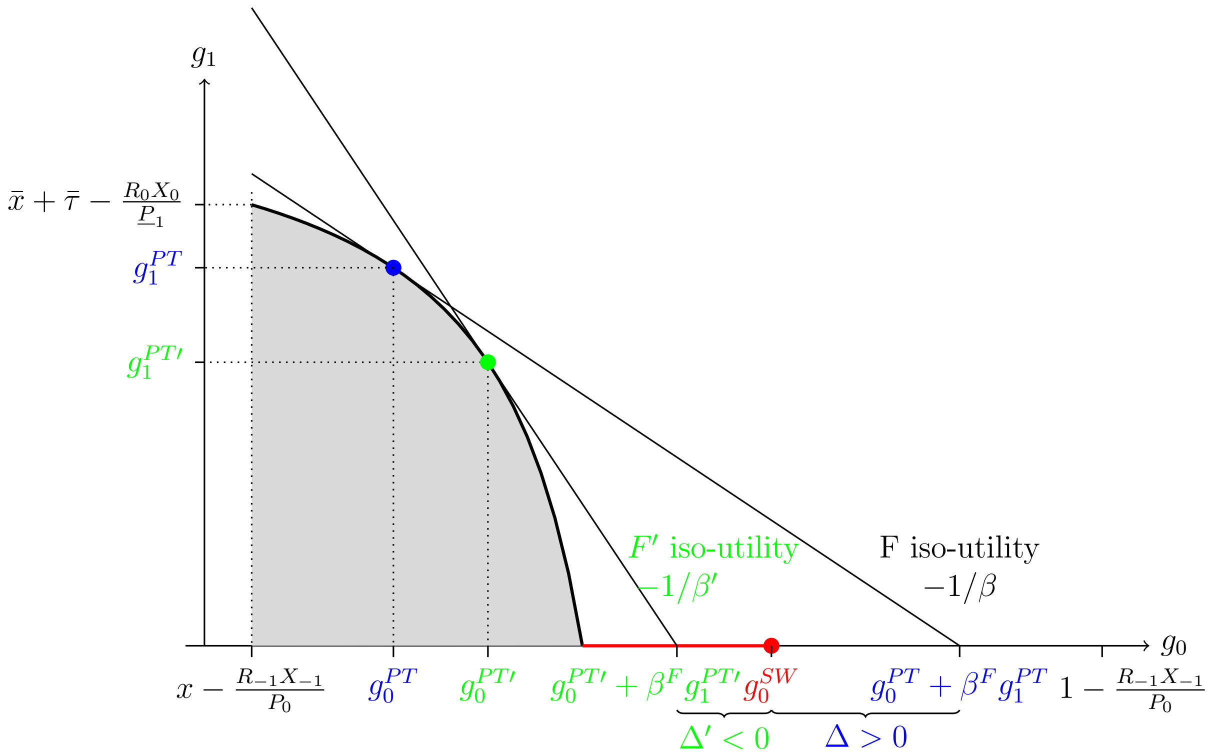

Figure 2: Problem faced by F on the date-0 debt market.
The red circle corresponds to consumption associated with Sargent-Wallace debt issuance. The blue circle corresponds to consumption pattern associated with the price level taking debt level with high β^F and the green circle with low $\beta' < \beta^F$.

The value of Δ defined in (29) drives F's issuance decision, and can simply be observed on Figure 2: it corresponds to the (signed) distance along the x-axis between the red point which corresponds to the payoff from the Sargent-Wallace debt level and the intersection between the x-axis and the iso-utility associated with the consumption pattern (g_0^{PT}, g_1^{PT}) obtained through the price-taking debt level. Figure 2 displays two situations, one in which F prefers the price-level taking debt level, and one in which F is more impatient $(\beta' < \beta^F)$ and prefers the Sargent-Wallace debt level. In sum, the sign of Δ measures as in the baseline model the cost of distorted spending net of the gains from inflating reserves $R_0 X_0$.

Date-0 reserve issuance. The final step is the determination of the action of M in the date-0 market for reserves. The following proposition is the counterpart when the interest rate is variable of Proposition 3 that spelled out the conditions for monetary dominance at all dates when the rate is fixed. We denote $(g_0^{PT}(0), g_1^{PT}(0))$ the solution to F's optimal spending problem under monetary dominance (25) when $x_0 = R_{-1}X_{-1} = R_0 X_0 = 0$. Notice that this solution is mathematically well defined but not economically so as M

needs arbitrary small reserves to pin down the price level.

Proposition 6. ***(The determinants of monetary dominance)***

If $g_1^{PT}(0) > 0$, there exists a threshold $\overline{RX} > 0$ such that if $R_{-1}X_{-1} \leq \overline{RX}$, the unique equilibrium is such that the price level is on target at each date— $P_0 = P_0^M$ and $P_1 = P_1^M$, and such that M minimizes the amount of reserves in circulation ($X_0 = R_{-1}X_{-1}$).

If $g_1^{PT}(0) = 0$, any equilibrium is such that F issues the Sargent-Wallace debt level implying $P_1 = \underline{P}_1 + \alpha^M$. M (and thus F) is indifferent across several levels of reserves X_0.

Proof. See Appendix A.5. □

Proposition 6 offers two insights. First, it exhibits conditions under which M reaches its price-level objective at each date. As in the baseline model, the first of these conditions is that legacy reserves be sufficiently small. The second one is that F finds frontloading consumption sufficiently costly in the sense that $g_1^{PT}(0) > 0$.

The second insight is that this latter condition is actually necessary: The fiscal authority always enters into the Sargent-Wallace debt level when it fails to hold. The situation in which $g_1^{PT}(0) = 0$ is therefore the counterpart of $\beta^F r \leq 1$ in the baseline model, as F enjoys (ex-post) benefits but incurs no cost form the Sargent-Wallace debt level in both cases.

Equilibrium interest-rate level versus demand curve for public securities. The most interesting difference between the baseline model and this variable-rate extension is that monetary dominance can prevail at any equilibrium value of the interest rate level, including when it is smaller than $1/\beta^F$. If the (out-of-equilibrium) debt issuance required to force M to chicken out triggers a sufficiently large increase in the interest rate, then monetary dominance can prevail even when the interest rate observed in equilibrium is arbitrarily low.

4.2 General cost of taxation

Another simplification in the baseline model is a marginal cost of taxation that jumps from 0 to an arbitrarily large value at $\bar{\tau}$, leading to a trivial taxation decision by the fiscal authority. This section posits smooth convex taxation costs. We maintain the modelling of savers in the baseline model leading to a fixed interest rate, but substitute Assumption 1 with the following set of assumptions:

Assumption 3. ***(General cost of taxation)***

- *The cost of taxation c is such that c' exists and is an increasing bijection over $[0;+\infty)$.*
- $\frac{R_{-1}X_{-1}}{P_0^M} \leq \frac{\bar{x}}{r}$.
- *For notational simplicity, we assume that $\underline{g} = 0$.*

Besides introducing a general convex cost of taxation, we also for brevity posit that M has sufficient resources $\bar{x}$ relative to the legacy reserves $R_{-1}X_{-1}$ to implement $P_0 = P_0^M$ if it wishes so, and we set the incompressible level of expenditures $\underline{g}$ to zero.

The full-fledged analysis of this model is more cumbersome than that of the baseline one and we relegate it to Appendix B. The reason is that the final decision of the fiscal authority is now a joint, history-dependent default and taxation decision, whereas taxes are unconditionally and simply set at $\bar{\tau}$ in the baseline model. Here we only present a broad intuition for the main insight from this extension: Monetary dominance prevails if the cost of default of F, α^F, is sufficiently large other things being equal. Thus, it may prevail even if $\beta^F r < 1$ and F finds it optimal to borrow against its entire future resources ($g_1 = 0$). To see this, it is useful to study how F optimally borrows conditionally on inducing date-1 monetary dominance ($P_1 = \underline{P}_1$). Among all "price-level taking" debt levels, the optimal one is $B_0 = \underline{P}_1 r b^{PT}$, where b^{PT} solves:

$$b^{PT} = \arg\max_{b,\tau \geq 0} \left\{ g_0 + \beta^F g_1 - \beta^F c(\tau) \right\} \tag{30}$$

$$\text{s.t. } g_0 = x_0 + b - \frac{R_{-1}X_{-1}}{P_0}, \tag{31}$$

$$g_1 = \bar{x} + \tau - \frac{RX_0}{\underline{P}_1} - rb, \tag{32}$$

$$c(\tau) - \tau - c(\tau^*) + \tau^* \leq \alpha^F - rb, \tag{33}$$

$$g_1 \geq 0. \tag{34}$$

Again, date-0 consumption g_0 stems from raising b from savers and receiving $x_0 - R_{-1}X_{-1}/P_0$ from M, and date-1 consumption g_1 is what is left of resources $\bar{x} + \tau$ once public liabilities have been repaid. Condition (33) ensures that F finds it optimal to make good on its debt at date 1. The tax level τ^* is defined as

$$\tau^* \equiv \arg\max_{\tau}\{\tau - c(\tau)\} = (c')^{-1}(1), \tag{35}$$

and thus corresponds to the taxes that F optimally raises at date 1 if it does not need to tax more to be solvent.

To stack the deck against monetary dominance, suppose that $\beta^F r < 1$ so that F optimally pledges its entire date-1 resources ($g_1 = 0$). These date-1 resources depend on the choice of taxes τ, which depends in turn on whether the incentive-compatibility constraint (33) binds or not.

First, τ may be determined by setting $g_1 = 0$ in (32) and by a binding incentive-compatibility constraint (33). In this case, we show that monetary dominance cannot prevail because if it were so, it would be strictly dominant for F to issue more nominal debt, thereby forcing M to inflate reserves away at date 1. M could not induce any fiscal consolidation by F as a response as F would credibly rather default given that (33) binds.

Second, it may also be that (33) is slack and that the taxes are $\tau = (c')^{-1}(1/\beta^F r)$. Notice that this situation prevails as the cost of default α^F is sufficiently large other things being equal. We also show in the appendix that the Sargent-Wallace debt level becomes prohibitively costly in this case in which α^F becomes arbitrarily large, at least under the assumption that F does not face a Laffer curve for tax revenues – see remark below. As a result, for α^F sufficiently large other things being equal, F prefers the price-taking debt level, even if $r < 1/\beta^F$ so that it borrows against its entire fiscal space ($g_1 = 0$), a situation that cannot occur neither in the baseline model nor in the variable-rate extension.

Remark. This latter result stands in sharp contrast with the baseline model in which the fiscal cost of default did not have an influence on the outcome of the game between F and M. The main reason is the absence of a Laffer curve for tax revenues as, here, F can always increase taxes τ, even if at a high welfare cost $c(\tau)$. With such a Laffer curve, there would exist a point after which F cannot increase tax revenues anymore, as in the baseline model, and, in which case, the cost to implement the Sargent-Wallace debt level would not be a function of the fiscal cost of default.

5 Infinite-horizon model

This section studies an infinite-horizon version of the model in which infinitely lived fiscal and monetary authorities interact with a private sector populated by overlapping generations of savers each identical to that in the two-date model. The motive behind this OLG modelling choice is our intent to focus on a dynamically inefficient economy to capture a low interest rate situation as in Blanchard (2019). The infinite horizon would not add significant broad insights to that from the two-date setup in the dynamically efficient case. By contrast, when the public sector finances its resources with Ponzi schemes, market forces become a central driver of the price level.[26] This section illustrates this by showing that the key exogenous variables $(\bar{x}, \bar{\tau}, \alpha^M, \alpha^F)$ of the two-date baseline model can all be endogenized as part of the private sector's strategy in the infinite-horizon model.[27]

5.1 Setup

Time is discrete and indexed by $t \in \mathbb{N}$.

Private sector. At each date t, a unit mass of savers are born. They live for two dates and have preferences $c_t + c_{t+1}/r_t$, where $r_t > 0$. They each receive an endowment of the consumption good when young.[28] This economy is dynamically inefficient in the sense that the endowment of cohort $t+1$ is at least r_t times that of cohort t.[29]

Public sector. The public sector is populated by infinitely-lived monetary and fiscal authorities very much identical to that in the two-date model, except that the fiscal one has no taxation power (more on this below). The extensive form of the game at each date t is similar to that of date 0 in the two-date game. We detail it again as follows.

Date-t market for reserves.

[26] Our understanding is that these insights would extend to dynamically efficient economies where public liabilities are rational bubbles due to financial frictions as, among others, in Martin and Ventura (2012) or Farhi and Tirole (2012a).

[27] The costs in case of a soft default, as in Section 3.5 may also be endogenized following the same approach.

[28] They may also receive consumption units when old but this is immaterial.

[29] For example, the endowment is constant across cohorts and $r_t \leq 1$.

1. M selects total date-t outstanding reserves $X_t \geq R_{t-1}X_{t-1}$ by issuing new reserves $X_t - R_{t-1}X_{t-1}$ on top of $R_{t-1}X_{t-1}$ sold by old savers, and announces the interest rate $R_t \geq 0$ between dates t and $t+1$.

2. Young savers invest an aggregate quantity $x_t \geq 0$ of consumption units in the market for reserves at the price P_t.

Date-t bond market.

3. F issues $B_t \geq 0$ bonds.

4. M invests $b_t^M \in [0, (X_t - R_{t-1}X_{t-1})/P_t]$ consumption units in the bond market.

5. Young savers invest $b_t \geq 0$ aggregate consumption units in the bond market at the price Q_t.

Date-t spending and default.

6. F decides on the haircut $l_t \in [0,1]$ on legacy debt B_{t-1} and consumption g_t such that

$$g_t = \theta_t - \frac{(1-l_t)B_{t-1}}{P_t} + \frac{Q_t}{P_t}B_t, \tag{36}$$

where the dividend θ_t paid by M is equal to

$$\theta_t = \frac{X_t - RX_{t-1}}{P_t} - b_t^M + \frac{(1-l_t)b_{t-1}^M P_{t-1}}{Q_{t-1}P_t}. \tag{37}$$

Figure 3 summarizes these three stages.

A date-t strategy profile $\sigma_t = (R_t, X_t, x_t, P_t, B_t, b_t^M, b_t, Q_t, l_t)$ describes all the above date-t actions of each agent given all possible history. A strategy profile for the game $\sigma = (\sigma_t)_{t\in\mathbb{N}}$ is the sequence of date-t strategy profiles.

Objectives of F and M. For all $t \in \mathbb{N}$, the respective date-t objectives of F and M are:

$$U_t^F = \sum_{s\geq t}(\beta^F)^{s-t}v(g_s),\ U_t^M = -\sum_{s\geq t}\left(\beta^M\right)^{s-t} \mid P_s - P_s^M \mid, \tag{38}$$

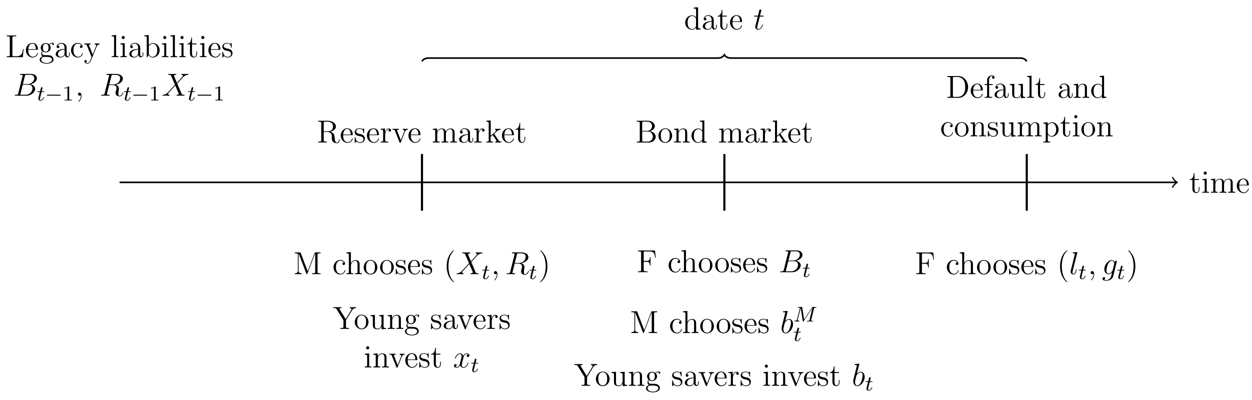

Figure 3: Intradate timing of the game.

where $\beta^F, \beta^M \in (0,1)$, there exists $\underline{g} > 0$ such that $v(g) = g$ if $g \geq \underline{g}$ and $v(g) = -\infty$ otherwise, and $P_s^M > 0$ for all s.

As in the two-date model, F values spending and is subject to an incompressible level of expenditures $\underline{g}$, whereas M values the price level being on (an exogenously given) target. Unlike in the two-date model, the public authorities incur no exogenous costs of default. We will focus on equilibria in which the private sector's strategy endogenously creates such costs.

Equilibrium concept. The equilibrium concept is the same as that in the two-date game—subgame perfection with large and small agents:

Definition 2. ***(Equilibrium)*** *An equilibrium is a strategy profile σ such that:*

1. *Each action by F and M is optimal given history and its beliefs that the future actions are taken according to the strategy profile.*

2. *Date-t young saver $i \in [0,1]$ optimally invests $x_t^i = x_t$ in the reserve market given history up to date $t-1$, (R_t, X_t, x_t, P_t), and the strategy profiles for all future actions, and optimally invests $b_t^i = b_t$ in the bond market given history up to date $t-1$, $(R_t, X_t, x_t, P_t, B_t, b_t^M, b_t, Q_t)$, and the strategy profiles for all future actions.*

3. *At each date, the market for reserves clears, $P_t x_t = X_t$, and so does the bond market, $Q_t B_t = P_t(b_t + b_t^M)$.*

This infinite-horizon section focuses exclusively on situations, ruled out by a finite horizon, in which public liabilities are self-sustained Ponzi schemes. Accordingly and for analytical simplicity, we deprive the public sector from any resources other than that generated by such schemes. We abstract in particular from taxation. Our main goal is to show that the important exogenous variables of the baseline model can arise as equilibrium objects of this infinite-horizon setting. More precisely, we endogenize the respective real resources $\bar{x}$ and $\bar{\tau}$ of M and F at date 1 and their respective costs of default α^M and α^F as resulting from their continuation utilities in the infinite-horizon game after dates 0 and 1 have been played.

Consider thus $\bar{x}, \bar{\tau}, \alpha^M, \alpha^F \geq 0$ that satisfy the conditions (8) and (9) of the baseline model. We have:

Proposition 7. ***(Endogenous payoffs of the baseline model)*** *If $\beta^F r_t \leq 1$ for all $t \geq 1$, there exists an equilibrium σ such that date 0 is strategically equivalent to date 0 in the baseline model with parameters $\bar{x}, \bar{\tau}, \alpha^M, \alpha^F \geq 0$ and interest rate r_0. In other words, the continuation profiles $(\sigma_t)_{t\geq 1}$ generate the same payoffs as that of the baseline model.*

Proof. See Appendix A.6. □

The construction of the equilibrium that endogenizes the exogenous variables of the baseline model, somewhat involved, is detailed in the proof of Proposition 7. Yet the main forces at play are simple: The private sector imposes discipline on the public one by reducing the size of public liabilities in case of default, thereby inducing both reduced public spending and inflation. Dynamic inefficiency is crucial to make such market behavior subgame perfect.

Consider first the fiscal authority. The date-1 default cost α^F imposed by the market to the fiscal authority F is simply a reduction α^F/β^F in the date-2 present value of the Ponzi scheme that the market is willing to sustain on public debt in the event of a date-1 default relative to the case in which F has made good on its date-1 liabilities. The date-1 resources $\bar{\tau}$ are the maximum debt capacity that the market grants to F at date 1. From date 2 on, the private sector discourages default by credibly threatening to stop rolling over debt in case of past credit event. This is effective as the fiscal authority would then be unable to finance its incompressible expenditures.

The cost α^M to the monetary authority M in case of sovereign default is also a form of partial market exclusion, albeit more subtle. In case of default, savers invest only $R_1 X_1/(P_2^M + \alpha^M/\beta^M)$ in the date-2 reserve market. This forces a reserve overflow no matter the date-1 monetary policy (R_1, X_1), leading in turn to a date-2 price level off target by α^M/β^M.

Under this microfoundation of $\bar{x}, \bar{\tau}, \alpha^M, \alpha^F$, F's ability to induce M to inflate away public liabilities is thus driven by the extent to which savers run not only on bonds but also on reserves in the event of sovereign default. The monetary authority is willing to preemptively generate itself the inflation that a run on its currency would generate anyway following a credit event. Thus, in an economy in which the private sector can swiftly switch out of the local currency and "dollarize" in case of a debt crisis (high α^M), the monetary authority would be eager to prevent such crises by monetizing sovereign debt even if this comes at a sizeable inflation cost. On the polar opposite, if the private sector has an incompressible demand for reserves whose level is not too far below that of the legacy reserves $R_{-1}X_{-1}$ (low α^M), then the central bank can discourage any fiscal attempt at a Sargent-Wallace expansion. It is credible at doing so because there will be no run on its liabilities in the (out-of-equilibrium) event of a sovereign default.

We find it interesting to fully micro-found our baseline model by means of the infinite-horizon one using market-discipline arguments. We offer in particular a simple formalization of the broad idea that a central bank with a pure price-stability mandate may still care about sovereign solvency because default affects the transmission of monetary policy. This is a useful contribution because such an impact of sovereign default on price stability has seldom been modelled to our knowledge. Yet, the study of fiscal and monetary interactions hinges on the assumption that sovereign solvency matters to the monetary authority, albeit often implicitly so as in the pioneering work of Sargent and Wallace (1981).

6 Concluding remarks

This paper formalizes Wallace's "game of chicken" as a full-fledged political economy model of strategic dynamic interactions between fiscal and monetary authorities, and investors in their liabilities. We find that a monetary authority that lacks both commitment power and fiscal support may still be in the position of imposing its objectives. Monetary dominance prevails when the implementation of the inflationary fiscal expansion envisioned by Sargent and Wallace (1981) is too costly to the fiscal authority. This may in turn occur because, in the absence of commitment power, inflationary fiscal expansion requires a large initial debt issuance. The benefits from future inflation may be smaller than the costs from repaying this debt if the interest on it, or/and taxation costs are sufficiently large.

We believe that our framework opens up many avenues for future research on strategic fiscal and monetary interactions, including in particular the four following ones. First, we posit in this first pass that all public liabilities are perfect substitutes. A natural extension is one in which they provide different liquidity services. Second, we restrict the analysis to a perfect-foresight environment, and a study of shocks is in order. Based on our perfect-foresight analysis, we conjecture that the fiscal authority endogenously amplifies shocks above a certain size by doubling down with a Sargent-Wallace expansion when the fiscal situation becomes sufficiently dire. The prudential management of the central bank's balance sheet in anticipation of these amplified shocks is an interesting question. Third, we focussed on the case in which the agent whose solvency the monetary authority cares about is the government. Yet, we could also consider the case in which such important borrowers belong to the private sector (e.g., financial institutions). The monetary authority would then presumably have to manage a collective moral hazard problem related to that in Farhi and Tirole (2012b). The alternative to monetary dominance would in this case be the so-called financial dominance rather than the fiscal one. Fourth, to become potentially more quantitative, our model may be enriched along several additional dimensions, for example with informational or nominal frictions or a richer debt maturity structure.

References

ALESINA, ALBERTO (1987): "Macroeconomic policy in a two-party system as a repeated game," *Quarterly Journal of Economics*, 102, 651–678.

ALESINA, ALBERTO AND GUIDO TABELLINI (1987): "Rules and discretion with noncoordinated monetary and fiscal policies," *Economic Inquiry*, 25, 619–630.

ALLEN, FRANKLIN AND DOUGLAS GALE (2000): "Bubbles and crises," *Economic Journal*, 110, 236–255.

BARRO, ROBERT J. AND DAVID B. GORDON (1983a): "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy*, 91, 589–610.

BARRO, ROBERT J AND DAVID B GORDON (1983b): "Rules, discretion and reputation in a model of monetary policy," *Journal of monetary economics*, 12, 101–121.

BASSETTO, MARCO (2002): "A Game-Theoretic View of the Fiscal Theory of the Price Level," *Econometrica*, 70, 2167–2195.

——— (2005): "Equilibrium and government commitment," *Journal of Economic Theory*, 124, 79–105.

——— (2008): "Fiscal Theory of the Price Level," in *The New Palgrave Dictionary of Economics*, London: Palgrave Macmillan UK, 1–5.

BASSETTO, MARCO AND CARLO GALLI (2019): "Is Inflation Default? The Role of Information in Debt Crises," *American Economic Review*, 109, 3556–84.

BASSETTO, MARCO AND TODD MESSER (2013): "Fiscal Consequences of Paying Interest on Reserves," *Fiscal Studies*, 34, 413–436.

BASSETTO, MARCO AND THOMAS J. SARGENT (2020): "Shotgun Wedding: Fiscal and Monetary Policy," Working Paper 27004, National Bureau of Economic Research.

BENIGNO, PIERPAOLO (2020): "A Central Bank Theory of Price Level Determination," *American Economic Journal: Macroeconomics*, 12, 258–283.

Bianchi, Francesco, Renato Faccini, and Leonardo Melosi (2022): "A Fiscal Theory of Persistent Inflation," Working Paper 30727, National Bureau of Economic Research.

Bianchi, Francesco, Howard Kung, and Thilo Kind (2019): "Threats to Central Bank Independence: High-Frequency Identification with Twitter," NBER Working Papers 26308, National Bureau of Economic Research, Inc.

Blanchard, Olivier (2019): "Public Debt and Low Interest Rates," *American Economic Review*, 109, 1197–1229.

——— (2021): "In defense of concerns over the $1.9 trillion relief plan," *Peterson Institute for International Economics*, 18.

Buiter, Willem H. (2002): "The Fiscal Theory of the Price Level: A Critique," *Economic Journal*, 112, 459–480.

Camous, Antoine and Dmitry Matveev (2021): "Furor over the Fed: A President's Tweets and Central Bank Independence," *CESifo Economic Studies*, 67, 106–127.

——— (2022): "The Central Bank Strikes Back! Credibility of Monetary Policy under Fiscal Influence," *The Economic Journal*, 133, 1–29.

Chari, V V and Patrick J Kehoe (1990): "Sustainable Plans," *Journal of Political Economy*, 98, 783–802.

Cochrane, John H. (2001): "Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level," *Econometrica*, 69, 69–116.

——— (2005): "Money as Stock," *Journal of Monetary Economics*, 52, 501–528.

Del Negro, Marco and Christopher A. Sims (2015): "When does a central bank?s balance sheet require fiscal support?" *Journal of Monetary Economics*, 73, 1–19.

Dixit, Avinash and Luisa Lambertini (2003): "Interactions of commitment and discretion in monetary and fiscal policies," *American Economic Review*, 93, 1522–1542.

Eaton, Jonathan and Mark Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 48, 289–309.

Farhi, Emmanuel and Jean Tirole (2012a): "Bubbly Liquidity," *Review of Economic Studies*, 79, 678–706.

——— (2012b): "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *American Economic Review*, 102, 60–93.

Galli, Carlo (2020): "Inflation, Default Risk and Nominal Debt," .

Halac, Marina and Pierre Yared (2020): "Inflation Targeting under Political Pressure," *Independence, Credibility, and Communication of Central Banking, ed. by E. Pastén and R. Reis, Santiago, Chile, Central Bank of Chile*, 7–27.

——— (2022): "A Theory of Fiscal Responsibility and Irresponsibility," NBER Working Papers 30601, National Bureau of Economic Research, Inc.

Hall, Robert E and Ricardo Reis (2015): "Maintaining Central-Bank Financial Stability under New-Style Central Banking," CEPR Discussion Papers 10741, C.E.P.R. Discussion Papers.

Jacobson, Margaret M., Eric M. Leeper, and Bruce Preston (2019): "Recovery of 1933," NBER Working Papers 25629, National Bureau of Economic Research, Inc.

Kydland, Finn E and Edward C Prescott (1977): "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 85, 473–491.

Leeper, Eric M. (1991): "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics*, 27, 129–147.

——— (2023): "Fiscal Dominance: How Worried Should We Be?" Policy briefs, Mercatus Center.

Ljungqvist, Lars and Thomas J. Sargent (2018): *Recursive Macroeconomic Theory, Fourth Edition*, no. 0262038668 in MIT Press Books, The MIT Press.

Martin, Alberto and Jaume Ventura (2012): "Economic Growth with Bubbles," *American Economic Review*, 102, 3033–3058.

Martin, Fernando M. (2015): "Debt, inflation and central bank independence," *European Economic Review*, 79, 129 – 150.

McCallum, Bennett T. (2001): "Indeterminacy, bubbles, and the fiscal theory of price level determination," *Journal of Monetary Economics*, 47, 19–30.

Mee, Simon (2019): *Central Bank Independence and the Legacy of the German Past*, Cambridge University Press.

Niepelt, Dirk (2004): "The Fiscal Myth of the Price Level," *Quarterly Journal of Economics*, 119, 277–300.

Piguillem, Facundo and Alessandro Riboni (2015): "Spending-Biased Legislators: Discipline Through Disagreement," *Quarterly Journal of Economics*, 130, 901–949.

Powell, Jerome H. (2023): "Panel on "Central Bank Independence and the Mandate—Evolving Views"," At the Symposium on Central Bank Independence, Sveriges Riksbank, Stockholm, Sweden, January 10.

Reis, Ricardo (2015): "Comment on: "When does a central bank's balance sheet require fiscal support?" by Marco Del Negro and Christopher A. Sims," *Journal of Monetary Economics*, 73, 20–25.

——— (2017): "Can the Central Bank Alleviate Fiscal Burdens?" CEPR Discussion Papers 11736, C.E.P.R. Discussion Papers.

——— (2021): "The constraint on public debt when r," CEPR Discussion Papers 15950, C.E.P.R. Discussion Papers.

Riboni, Alessandro (2010): "Committees As Substitutes For Commitment," *International Economic Review*, 51, 213–236.

Rogoff, Kenneth (1985): "The Optimal Degree of Commitment to an Intermediate Monetary Target," *Quarterly Journal of Economics*, 100, 1169–89.

Sargent, Thomas J. and Neil Wallace (1981): "Some unpleasant monetarist arithmetic," *Quarterly Review*.

Schnabel, Isabel (2022): "Finding the right mix: monetary-fiscal interaction at times of high inflation," Keynote speech by Isabel Schnabel, Member of the Executive Board of the ECB, at the Bank of England Watchers' Conference, London, 24 November.

Silber, W.L. (2012): *Volcker: The Triumph of Persistence*, Bloomsbury Publishing.

Sims, Christopher A. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4, 381–399.

——— (2003): "Fiscal Aspects of Central Bank Independence," Princeton University.

——— (2011): "Stepping on a rake: The role of fiscal policy in the inflation of the 1970s," *European Economic Review*, 55, 48–56.

Stokey, Nancy L. (1991): "Credible public policy," *Journal of Economic Dynamics and Control*, 15, 627–656.

Summers, Lawrence H. (2021): "The Biden stimulus is admirably ambitious. But it brings some big risks, too," *The Washington Post*, february 4.

Svensson, Lars E O (1997): "Optimal Inflation Targets, "Conservative" Central Banks, and Linear Inflation Contracts," *American Economic Review*, 87, 98–114.

Tabellini, Guido (1986): "Money, debt and deficits in a dynamic game," *Journal of Economic Dynamics and Control*, 10, 427–442.

Walsh, Carl E (1995): "Optimal Contracts for Central Bankers," *American Economic Review*, 85, 150–67.

Woodford, Michael (1994): "Monetary policy and price level determinacy in a cash-in-advance economy," *Economic theory*, 4, 345–380.

——— (1995): "Price-level determinacy without control of a monetary aggregate," *Carnegie-Rochester Conference Series on Public Policy*, 43, 1–46.

——— (2001): "Fiscal Requirements for Price Stability," *Journal of Money, Credit and Banking*, 33, 669–728.

Yared, Pierre (2019): "Rising Government Debt: Causes and Solutions for a Decades-Old Trend," *Journal of Economic Perspectives*, 33, 115–140.

A Proofs

A.1 Proof of Proposition 1

Step 1: Date-1 taxation, spending, and default. At the terminal stage of date 1, it is dominant for the fiscal authority F to raise taxes $\bar{\tau}$ as this comes at no cost and generates resources that can be used for debt repayment or/and spending. The expression of F's terminal consumption as a function of all other actions given by (4) and (5), together with $X_1 = P_1\bar{x}$, shows that F can avoid default while spending at least $\underline{g}$ and taxing $\bar{\tau}$ if and only if (10) holds.

Condition (8) implies that F does not default if (10) holds because the default cost exceeds the resulting additional spending $(B_0 - b_0^M P_0/Q_0)/P_1$. If (10) fails to hold, then F defaults, which warrants $g_1 = \bar{\tau} + \bar{x} - R_0X_0/P_1 > \underline{g}$ because $\bar{\tau} > \underline{g}$ from condition (9) and $\bar{x} = X_1/P_1 \geq R_0X_0/P_1$.

In sum, F never spends below $\underline{g}$, and F defaults if and only if the solvency condition (10) fails to hold.

Step 2: Date-1 price level. In the date-1 reserve market, the monetary authority M can set any date-1 price level $P_1 \geq R_0X_0/\bar{x}$, by issuing $X_1 - R_0X_0 = \bar{x}P_1 - R_0X_0$ new reserves. If $P^F \leq \underline{P}_1$, then M optimally sets $P_1 = \underline{P}_1$ as it minimizes the departure from its target $\mid P_1 - P_1^M \mid$ (possibly to 0 if $\underline{P}_1 = P_1^M$) without triggering default. If $P^F > \underline{P}_1$, if M lets F default then it incurs a cost α^M and can and optimally does set the date-1 price level at $\underline{P}_1$. If conversely M seeks to avert default, then it optimally does so by setting the date-1 price at P^F, thereby reducing F's consumption to the incompressible level $\underline{g}$. As a result, M prevents F from defaulting by setting $P_1 = P^F$ if and only if $P^F \leq \underline{P}_1 + \alpha^M$.

A.2 Proof of Proposition 2

Step 1: Date-0 government consumption does not depend on b_0^M. The date-0 transfer to the fiscal authority F from the monetary authority M is $\theta_0 = x_0 - R_{-1}X_{-1}/P_0 - b_0^M$, equal to the resources from reserve issuances $x_0 - R_{-1}X_{-1}/P_0$ net of bond purchases b_0^M. F consumes these resources on top of the amount $b_0 + b_0^M$ collected in the bond market. F thus consumes $g_0 = x_0 + b_0 - R_{-1}X_{-1}/P_0$, independent of the resources spent by the monetary authority to purchase bonds b_0^M.

Step 2: No default in equilibrium. In the market for government bonds, F issues B bonds, M invests b_0^M, and then savers invest b_0. From Proposition 1, these actions lead to one of the following date-1 situations: monetary dominance, fiscal dominance, or default. Default cannot be an equilibrium outcome. Since default is total ($l = 1$) when it occurs, savers' rationality would imply $b_0 = 0$ in case of date-1 default, and F would receive (at best) only resources from M in the bond market against an empty promise. But then F would be strictly better off not issuing bonds ($B_0 = 0$) and receiving these resources as a dividend from M, as this averts default leaving g_0 and g_1 unchanged.

Step 3: Bond market equilibrium given B_0 and b_0^M. In the absence of default, if F issues B_0 bonds and M then invests b_0^M, savers' optimal portfolio choice and market clearing yield a bond price Q_0 and savers' investment b such that

$$r = \frac{P_0}{P_1 Q_0} \text{ and } Q_0 B_0 = P_0(b_0^M + b_0), \tag{39}$$

where P_1 is given by Proposition 1.

We now derive optimal date-0 debt issuance B_0 by F as follows. We first study which debt level grants F the highest date-0 utility among all the levels that lead to date-1 monetary dominance. We then describe the optimal debt level among those that generate date-1 fiscal dominance. Finally, we compare these two conditionally optimal debt levels.

Step 4: Optimal debt policy conditional on date-1 monetary dominance. Suppose first that the bond issuance B_0 by F leads to strict monetary dominance at date 1

$(P_1 = \underline{P}_1 < P^F)$. Optimal debt issuance by F requires

$$\max_B g_0 + \beta^F g_1 \tag{40}$$

$$\text{s.t. } \underline{g} \leq g_0 = b_0 + x_0 - \frac{R_{-1}X_{-1}}{P_0}, \tag{41}$$

$$\underline{g} < g_1 = \bar{x} + \bar{\tau} - \frac{B - \frac{b_0^M P_0}{Q_0} + R_0 X_0}{\underline{P}_1}, \tag{42}$$

$$B_0 - \frac{b_0^M P_0}{Q_0} = r b_0 \underline{P}_1. \tag{43}$$

Date-0 consumption (41) stems from Step 1, date-1 consumption (42) from Proposition 1 (with a strict inequality because we consider strict monetary dominance $P_1 = \underline{P}_1 < P^F$), and condition (43) from the bond-market equilibrium relations (39). Notice that combining these latter two equations, this program depends on B_0 and b_0^M only through (43). This is because M pays as date-0 dividends whichever amount it does not invest in the bond market, and pays as date-1 dividends whichever bond repayment it collects. Thus F can choose the real amount borrowed from savers b by correctly anticipating b^M when selecting the nominal amount B_0, and the value of b_0^M does not affect any agent's payoff. We therefore restrict without loss of generality the analysis to $b^M = 0$. Notice also that condition (9) ensures that there exists $b_0 \geq 0$ satisfying (41) and (42).

It cannot be that $\beta^F r < 1$, otherwise F would seek to minimize its date-1 consumption to $g_1 = \underline{g}$ from (40), contradicting strict monetary dominance. Thus a necessary condition for strict monetary dominance is $\beta^F r \geq 1$. If $\beta^F r \geq 1$, F maximizes its utility conditional on strict monetary dominance (strictly so if $\beta^F r > 1$) by borrowing b^* defined in (14), the smallest amount necessary to consume $\underline{g}$ at date 0, and this yields F a utility (15).

Step 5: Optimal debt policy conditional on date-1 fiscal dominance. Suppose now that the bond issuance B leads to date-1 fiscal dominance: $P_1 = P^F$ and $g_1 = \underline{g}$. In this case, combining the definition of P^F given by (11) and the equilibrium determination of the bond price (39) yields a date-1 price level

$$P_1 = P^F = \frac{B_0 + R_0 X_0}{\bar{x} + \bar{\tau} - \underline{g} + r b_0^M}. \tag{44}$$

The date-1 price is thus decreasing in b_0^M, and so it must be that M optimally invests as much as possible in the date-0 bond market, that is, $b_0^M = x_0 - R_{-1}X_{-1}/P_0$. This implies in turn that the date-1 price level is strictly (and linearly) increasing in B_0. Conditionally on date-1 fiscal dominance, F's utility is

$$x_0 - \frac{R_{-1}X_{-1}}{P_0} + \frac{1}{r}\left(\bar{x} + \bar{\tau} - \underline{g} - \frac{R_0 X_0}{P^F}\right) + \beta \underline{g}, \tag{45}$$

strictly increasing in P^F. Thus F issues B so that P^F takes the largest possible value that M prefers to forcing default, $\underline{P}_1 + \alpha^M$.

Comparing (15) and (45) then yields condition (16).

A.3 Proof of Propositions 3 and 4

If M announces a rate R_0 and issues new reserves $X_0 - R_{-1}X_{-1}$, savers' optimal portfolio choice and market clearing define the date-0 price level P_0 and demand for reserves x_0 as the unique solution to

$$R_0 = \frac{rP_1}{P_0} \text{ and } P_0 x_0 = X_0, \tag{46}$$

where P_1 is given by the continuation described in Propositions 2 then 1.

Since condition (17) cannot hold if $\beta^F r \leq 1$, M cannot avoid the Sargent-Wallace debt level in this case. It can announce $R_0 = r(P_1^M + \alpha^M)/P_0^M$ and issue any level of reserves $X_0 - R_{-1}X_{-1} \in [0, P_0^M \bar{x}/r - R_{-1}X_{-1}]$ so that the date-0 price level is P_0^M, and the economy unfolds such that F issues the Sargent-Wallace debt level. The date-1 price level is $P_1^M + \alpha^M$ because the upper bound $P_0^M \bar{x}/r$ on X_0 rules out a date-1 reserve overflow.

Suppose now that $\beta^F r > 1$. Using relations (46) to eliminate R and x from condition (17) ensuring that F issues the price-taking debt level yields

$$(\beta^F r - 1)\left(\bar{x} + \bar{\tau} - \underline{g} - r\left(\underline{g} + \frac{R_{-1}X_{-1} - X_0}{P_0}\right)^+ - \frac{rX_0}{P_0}\right) \geq \frac{\alpha^M r X_0}{P_0(\underline{P}_1 + \alpha^M)}. \tag{47}$$

M can reach $P_0 = P_0^M$ and $P_1 = P_1^M$ by announcing $R_0 = rP_1^M/P_0^M$ and setting X_0 below the minimum X_m of two values. First, in order to avoid date-1 reserve overflow, it must be that $X_0 \leq P_0^M \bar{x}/r$, which is compatible with $X_0 \geq R_{-1}X_{-1}$ from assumption

(9). Second, X_0 must also be smaller than the maximum value such that (47) holds with $P_0 = P_0^M$ and $\underline{P}_1 = P_1^M$. It is easy to check that this is compatible with $X_0 \leq R_{-1}X_{-1}$ if and only if (18) holds. Furthermore, in this case of monetary dominance at each date, reserves X_0 are below not only this minimum X_m but also smaller than $P_0^M \underline{g} + R_{-1}X_{-1}$ as M's lexicographic preferences lead it to ensure that F does not consume more than the incompressible minimum at date 0.

If (18) does not hold, monetary dominance at each date is not possible. M can in this case let F issue the Sargent-Wallace debt level and warrant, acting as in the above case $\beta^F r \leq 1$, that $(P_0, P_1) = (P_0^M, P_1^M + \alpha^M)$, in which case its utility is $-\beta^M \alpha^M$.

Another option is to discourage F from issuing the Sargent-Wallace debt level by manipulating price levels. First M can ensure that $P_1 = P_1^M$ by setting $R_0 = rP_1^M/P_0^*$ and $X_0 = R_{-1}X_{-1}$, where P_0^* is the smallest date-0 price level ensuring that (47) holds with $X_0 = R_{-1}X_{-1}$ and $\underline{P}_1 = P_1^M$. It is easy to see that P_0^* is linearly increasing in $R_{-1}X_{-1}$ and tends to P_0^M as $R_{-1}X_{-1}$ gets close to the largest level warranting monetary dominance. Thus the disutility from this strategy vanishes as $R_{-1}X_{-1}$ tends to this level. Second M may want to manipulate both P_0 and $\underline{P}_1$ as the right-hand side of (47) decreases in $\underline{P}_1$. Formally, P_0 and P_1 solve in this case:

$$\min_{P_0, P_1} P_0 + \beta^M P_1 \tag{48}$$

$$\text{s.t.} \quad \left(\beta^F r - 1\right) \left(\bar{\tau} - \underline{g} - r \left(\underline{g} - \frac{\bar{x}}{r} + \frac{R_{-1}X_{-1}}{P_0} \right)^+ \right) \geq \frac{\alpha^M \bar{x}}{P_1 + \alpha^M}. \tag{49}$$

This strategy cannot dominate that consisting in raising only P_0 as $R_{-1}X_{-1}$ tends to the level warranting monetary dominance because it requires issuing a strictly positive quantity of new reserves creating a cost of deterring the Sargent-Wallace debt level that is bounded away from 0.

A.4 Proof of Proposition 5

The only part of the proposition that is not established in the body of the paper is that the optimal debt issuance conditional on date-1 fiscal dominance leads to $P_1 = \underline{P}_1 + \alpha^M$. Suppose that F issues B leading to date-1 fiscal dominance ($P_1 = P^F$).

We first show that M optimally sets $b_0^M = x_0 - R_{-1}X_{-1}/P_0$ in response to such a B_0

to minimize $P_1 = P^F$. The conditions for bond-market equilibrium:

$$Q_0 B_0 = P_0(b_0 + b_0^M) \text{ and } \frac{P_0}{P_1 Q} = r(1 - b_0 - x_0) \tag{50}$$

together with the definition of P^F (11) yield

$$P^F = \frac{B_0 + R_0 X_0}{\bar{x} + \bar{\tau} + r(1 - b_0 - x_0) b_0^M}, \tag{51}$$

and

$$\frac{B_0}{B_0 + R_0 X_0}(\bar{x} + \bar{\tau}) = b_0 r(1 - b_0 - x_0) + \left(1 - \frac{B_0}{B_0 + R_0 X_0}\right) b_0^M r(1 - b_0 - x_0). \tag{52}$$

Condition (52) implies that given B_0, $r(1 - b_0 - x_0)b_0^M$ must increase with b^M. Suppose otherwise: Then b_0 must be decreasing as b_0^M increases. In this case, $r(1-b_0-x_0)b_0$ is also decreasing in b_0^M. But then the left-hand term of (52) is independent from b_0^M whereas the right-hand term is decreasing in b_0^M, a contradiction since no equilibrium would form as b_0^M increases. Condition (51) then implies that M finds it optimal to maximize b_0^M in order to minimize P^F.

Using $b_0^M = x_0 - R_{-1}X_{-1}/P_0$, one can rewrite (52) as

$$b_0 = \frac{B_0(\bar{x} + \bar{\tau})}{(B_0 + R_0 X_0) r(1 - b_0 - x_0)} - \frac{(x_0 - \frac{R_{-1}X_{-1}}{P_0}) R_0 X_0}{B_0 + R_0 X_0}, \tag{53}$$

and simple algebra shows that this implies that b_0 increases with respect to B_0. Since F consumes $x_0 - R_{-1}X_{-1}/P_0 + b_0$, it chooses the maximum B_0 that is compatible with absence of default. That $P^F = R_0 X_0/(\bar{x} + \bar{\tau} - r(1 - b_0 - x_0)b_0)$ implies in turn that P^F increases in B_0 (taking into account that b_0 increases in B_0), and so B_0 is such that

$$P_1 = \underline{P}_1 + \alpha^M. \tag{54}$$

A.5 Proof of Proposition 6

From the previous proof, the real proceeds from the Sargent-wallace debt level b^{SW} solve:

$$b^{SW} = \frac{1}{r(1-x_0-b^{SW})}\left(\bar{x}+\bar{\tau}-\frac{R_0X_0}{\underline{P}_1+\alpha^M}\right). \tag{55}$$

As a result, F's utility differential Δ between the "price-level taking" debt level (such that $P_1 = \underline{P}_1$) and the "Sargent-Wallace" debt level (such that $P_1 = \underline{P}_1 + \alpha^M$) is:

$$\Delta = x_0 - \frac{R_{-1}X_{-1}}{P_0} + b^{PT} + \beta\left(\bar{x}+\bar{\tau}-r(1-x_0-b^{PT})b^{PT}-\frac{R_0X_0}{\underline{P}_1}\right) \tag{56}$$

$$-\,(x_0 - \frac{R_{-1}X_{-1}}{P_0} + b^{SW}) \tag{57}$$

$$= \underbrace{b^{PT}[1-\beta^F r(1-x_0-b^{PT})] - b^{SW}(1-\beta^F r(1-x_0-b^{SW})}_{A} \tag{58}$$

$$\underbrace{-\,\beta^F R_0X_0\left(\frac{1}{\underline{P}_1}-\frac{1}{\underline{P}_1+\alpha^M}\right)}_{B}. \tag{59}$$

This latter expression of Δ illustrates the costs and benefits from the price-level taking issuance versus the Sargent-Wallace issuance. Term A measures the difference in utility from allocating consumption over time in different ways across debt levels. The sign of A is ambiguous as the allocation is suboptimal under the Sargent-Wallace issuance but the total to be allocated is larger due to the lower value of reserves. Term B is positive. It is the benefit from eroding the value of reserves R_0X_0 with inflation.

First stage of date 0**.** Market clearing in the reserve market reads:

$$X_0 = P_0x, \tag{60}$$

and savers' rationality implies

$$\frac{RP_0}{P_1} = r(1-b-x). \tag{61}$$

Given the continuation of the game derived above, relations (60) and (61) form a system in (x_0, P_0) as a function of (R_0, X_0) with a unique solution. We solve for the equilibrium in the two cases covered by Proposition 6: i) $g_1^{PT}(0) > 0$ and $R_{-1}X_{-1}$ sufficiently small; ii) $g_1^{PT}(0) = 0$.

Suppose first that $g_1^{PT}(0) > 0$ and take $R_{-1}X_{-1}$ sufficiently small other things being equal. In this case, M sets $X_0 = R_{-1}X_{-1}$ and announces $R_0 = r(1 - X_0/P_0^M - b^{PT})P_1^M/P_0^M$. ($R_{-1}X_{-1}$ sufficiently small implies that there is no date-1 reserve overflow when M keeps reserves at the minimum level this way.) This corresponds to an equilibrium in which savers invest X_0/P_0^M in the market for reserves and b^{PT} in that for bonds, and the price level is on M's target at each date. The reason is that for $R_{-1}X_{-1}$ sufficiently small, b^{PT} is interior as it converges to $b^{PT}(0)$, and so term A in Δ is positive, bounded away from 0, whereas the gains B are sufficiently small. In particular, the lexicographic preferences of M imply that minimizing x_0 this way is optimal because this minimizes the distortions in F's choice of b_0 given that prices are on target.

Suppose then that $g_1^{PT}(0) = 0$. In this case, it is always optimal for F to issue the Sargent-Wallace level in the bond market since A is always negative no matter M's actions in the date-0 reserve market: The increase in date-1 resources induced by the lower value of reserves in the Sargent-Wallace debt level relaxes the binding constraint $g_1 \geq 0$ in the consumption-smoothing one. As a result, $\underline{P}_1 + \alpha^M$ is the lowest price that M can hope for at date 1. Since the largest one that it prefers to default is $\underline{P}_1 + \alpha^M$, this has to be the date-1 price. Accordingly, monetary policy in the date-0 reserve market is as follows. Let y_0 implicitly defined by

$$y_0 r(1 - y_0) = \bar{x} + \bar{\tau}, \tag{62}$$

and

$$\underline{P}_0 \equiv \max\left\{P_0^M; \frac{R_{-1}X_{-1}r(1-y_0)}{\bar{x}}\right\} \tag{63}$$

M announces a rate $R_0 = r(1 - y_0)(P_1^M + \alpha^M)/\underline{P}_0$ and issues $X_0 \in [R_{-1}X_{-1}, \bar{x}\underline{P}_0/r(1 - y_0)]$. This sets the date-0 price at $\underline{P}_0$ and $x_0 = X_0/\underline{P}_0$. M in particular may be indifferent across several levels of reserves X_0 because any resources that it leaves on the table are borrowed against by F in the bond market, and the utilities of both authorities are unchanged across these levels.

A.6 Proof of Proposition 7

We prove the proposition in two steps. First, we construct a subset of equilibria indexed by sequences of savings in reserves and bonds. In this subset, equilibria are such that price levels are on target and F does not default. Second, we build an equilibrium that has the properties of the proposition by selecting, from the subset of equilibria that we have constructed in the first step, continuation equilibria contingent on F's date-1 default decision.

Step 1. *Let* $(\bar{x}_t, \bar{b}_t)_{t\geq 0}$ *such that* $\bar{x}_0 \geq R_{-1}X_{-1}/P_0^M$, $\bar{b}_0 \geq 0$, $\bar{x}_0 + \bar{b}_0 \geq \underline{g} + R_{-1}X_{-1}/P_0^M$, *and for all* $t \geq 0$:

$$\bar{x}_{t+1} = r_t \bar{x}_t, \;\; \bar{b}_{t+1} = r_t \bar{b}_t + \underline{g}. \tag{64}$$

There exists an equilibrium without default and such that for all $t \geq 0$, $P_t = P_t^M$, $x_t = \bar{x}_t$, *and* $b_t = \bar{b}_t$.

Proof. Define for all $t \geq 0$:

$$P_{t+1}^* = \frac{R_t X_t}{\bar{x}_{t+1}} \tag{65}$$

The strategy profile is the following. At each date $t \geq 0$:

- M announces a rate $R_t = r_t P_{t+1}^M / P_t^M$.
- M issues $X_t = R_{t-1} X_{t-1}$ if $t > 0$ and $X_0 = P_0^M \bar{x}_0$.
- The date-t price level P_t and demand for reserves x_t solve $X_t = P_t x_t$ and $P_t R_t = P_{t+1}^* r_t$ if $R_t > 0$, and $x_t = 0$ otherwise.
- F issues $P_{t+1}^* r_t \bar{b}_t$.
- M does not invest in the bond market ($b_t^M = 0$).
- If $B_t > P_{t+1}^* r_t \bar{b}_t$ then savers shun the bond market ($b_t = 0$). So do they if $t > 0$ and at some $0 \leq t' < t$, F has defaulted ($l_{t'} > 0$). Otherwise the demand b_t and price

Q_t for bonds solve:

$$Q_t B_t = P_t(b_t + b_t^M), \tag{66}$$

$$r_t P^*_{t+1} Q_t = P_t. \tag{67}$$

- F sets $l_t = 0$ as long as this is compatible with $g_t = \theta_t + b_t^M - B_{t-1}/P_t \geq \underline{g}$, where $\theta_t = (X_t - R_{t-1}X_{t-1} + b_{t-1}^M P_{t-1}/Q_{t-1})/P_t - b_t^M$, and defaults otherwise.

We now show that this strategy profile corresponds to an equilibrium with outcome $(x_t, b_t, P_t) = (\bar{x}_t, \bar{b}_t, P_t^M)$ and no default.

Notice first that this strategy profile yields this outcome. First, $X_t = P_t x_t$ and $P_t = P^*_{t+1} r_t / R_t = X_t/\bar{x}_t$ imply $x_t = \bar{x}_t$, and together with $X_0 = P_0^M \bar{x}_0$ this implies in turn that $P_t = P_t^M$. This outcome in the reserve market implies in turn that $b_t = \bar{b}_t$ and that there is no default.

Second, we show that each agent acts optimally given the others' strategies. First, Savers act optimally given F and M's strategies and market outcomes since they earn r_t on date-t public securities.

Second, given F and the market's strategies, M's strategy is optimal. M reaches its price target at each date. It cannot generate more resources at date t while being on these targets, because the market's strategy in the reserve market implies $x_t = \bar{x}_t$ no matter the values of $R_t > 0$ and X_t from $X_t/x_t = P_t = P^*_{t+1} r_t / R_t = X_t/\bar{x}_t$ as seen above.

Third, F's strategy is optimal given that of M and savers. It dominates any alternative that generates expenditures below $\underline{g}$ at any date or default. On the debt market, F cannot issue more than $P^*_{t+1} r_t \bar{b}_t$ as savers would credibly shun the bond market forever in this case. Thus the highest possible real ressource extracted on the debt market is $b_t = \bar{b}_t$ due to the date-t market's strategy and future strategies.

Step 2. We now construct an equilibrium that has the properties claimed in the Proposition. First, strategies from date 2 on depend on whether there has been default at date 1 (B_0 and l_1 strictly positive) or not.

In the absence of date-1 default, the date-2 continuation equilibrium is as in Step 1 taking date 2 as the initial date with $\bar{x}_2^{ND} = \bar{x} r_1$ and $\bar{b}_2 = \bar{b}_2^{ND}$ taken above a lower bound specified below. The only difference is that we add the condition that date-t savers shun the date-t bond market if F raised more than $\bar{\tau}$ at date 1. This pins down the date-1 debt capacity of F at $\bar{\tau}$.

In case of date-1 default, then the date-2 continuation equilibrium is such that $\bar{x}_2^D = R_1 X_1/(P_2^M + \alpha^M/\beta^M)$, implying that P_2 cannot be smaller than and is in equilibrium equal to $P_2^M + \alpha^M/\beta^M$. Accordingly, M announces a rate $r_2 P_3^M/(P_2^M + \alpha^M/\beta^M)$ at date 2. Furthermore $\bar{b}_2^D = \bar{b}_2^{ND} - \alpha^F/\beta^F - r_1 B_1(1/P_2^M - 1/[P_2^M + \alpha^M/\beta^M)] \geq \underline{g} + r_1\bar{\tau}$, and this latter inequality puts a lower bound on $\bar{b}_2^{ND}$. Finally, we add again the condition that date-t savers shun the bond market if F raised more than $\bar{\tau}$ at date 1.

This profile from date 2 on implies that F always raises exactly $\bar{\tau}$ in the date-1 bond market, and faces a cost of default in the form of a loss in date-2 resources whose date-1 present value is α^F. M faces a date-2 run on its reserves in case of date-1 default, with a cost α^M viewed from date 1. Overall, F, M, and savers face the same date-1 payoffs viewed from date 0 as in the baseline model.

B General cost of taxation

This appendix solves the equilibrium by backward induction.

B.1 Date-1 taxation and default decisions

The program that F solves after the date-1 reserve market has cleared is

$$\max_{l\in[0,1],\tau\geq 0} \left(\bar{x} + \tau - \frac{RX_0 + (1-l)B}{P_1} + \frac{(1-l)b^M P_0}{P_1 Q}\right) - c(\tau) - \mathbb{1}_{\{l>0\}}\alpha^F, \tag{68}$$

$$\text{s.t. } \bar{x} + \tau - \frac{RX_0 + (1-l)B_0}{P_1} + \frac{(1-l)b^M P_0}{P_1 Q} \geq 0. \tag{69}$$

The fixed default cost implies that as in the baseline model, F either repays B in full ($l = 0$) or fully defaults ($l = 1$). Let us introduce

$$\tau^* \equiv \arg\max\{\tau - c(\tau)\} = (c')^{-1}(1) \tag{70}$$

the taxes that F optimally raises at date 1 if it does not need to tax more to be solvent.

F then prefers to repay its bond if and only if

$$\bar{x} + \tau_1 - \frac{RX_0 + B}{P_1} + \frac{b^M P_0}{P_1 Q} - c(\tau_1) \geq \bar{x} + \tau^* - \frac{RX_0}{P_1} - c(\tau^*) - \alpha^F, \tag{71}$$

where τ_1 is the optimal level of taxes conditional on repayment, defined as

$$\tau_1 \equiv \max\left\{\frac{RX_0 + B}{P_1} - \frac{b^M P_0}{P_1 Q} - \bar{x}; \tau^*\right\}. \tag{72}$$

Rearranging (71) as follows offers a natural interpretation:

$$\underbrace{c(\tau_1) - \tau_1 - c(\tau^*) + \tau^*}_{\text{Relative disutility of taxation}} \leq \underbrace{\alpha^F - \left(\frac{B}{P_1} - \frac{b^M P_0}{P_1 Q}\right)}_{\text{Net cost of default}}. \tag{73}$$

Taxes τ_1 when making good on debt are by definition (72) weakly higher than that when defaulting, equal to τ^*. The relative net utility cost of taxation (differential taxation cost minus proceeds on the left-hand side of (71)) is then positive. On the right-hand side of (71), the net utility cost of default is the fixed cost α^F net of the gains from defaulting on the debt held by private agents $B_0/P_1 - b^M P_0/P_1 Q$. Overall, F repays B when the disutility from taxation when repaying relative to that when defaulting $(c(\tau_1) - \tau_1 - c(\tau^*) + \tau^*)$ is low, the fixed cost of default (α^F) is large, or public debt net of central bank's holdings $(B_0/P_1 - b^M P_0/(P_1 Q))$ is small.

B.2 Date-1 price level

Other things being equal, an increase in the date-1 price level P_1 reduces both the relative cost of taxation when repaying and the gains from defaulting, and so it makes repayment more appealing to F. The cost of taxation decreases in P_1 because so does τ_1 from (72). The gain from default decreases in P_1 because so does the real repayment due.

As in the baseline model, we let P^F denote the minimum price level that ensures that F is willing to repay—the minimum value of P_1 such that condition (71) holds as an equality (with the convention $P^F = 0$ if it holds for every $P_1 > 0$). Notice that an explicit formula for P^F such as (11) in the baseline model is out of reach.

Notice also that net public debt and reserves do not affect P^F symmetrically here as they do in the baseline model in which their sum determines P^F (see expression

(11)). Here more debt not only increases the distortionary cost of taxes in the case of repayment—as is symetrically the case for more reserves, but it also increases the gain from defaulting. This latter effect is absent in the baseline model in which the assumed discontinuity in the marginal cost of taxation implies that the fiscal authority has a strict preference for not defaulting at $P_1 = P^F$.

As in the baseline model, M compares P^F to $\underline{P}_1 = \max\left\{RX_0/\bar{x}, P_1^M\right\}$ and to $\underline{P}_1+\alpha^M$. This leads to monetary dominance when $P^F \leq \underline{P}_1$, in which case the price level at date 1 is $P_1 = \underline{P}_1$, to fiscal dominance when $\underline{P}_1 < P^F \leq \underline{P}_1 + \alpha^M$, in which case $P_1 = P^F$, and to default otherwise, in which case $P_1 = \underline{P}_1$.

B.3 Date-0 bond market

Date-0 government consumption is verbatim that in the baseline model (with $\underline{g} = 0$), and so we turn to the date-0 bond market. For the same reason as in the baseline model, there is no default in equilibrium, and a given debt issuance by F leads either to monetary or fiscal dominance at date 1. As in the baseline model, we study optimal debt issuance conditional on either date-1 outcome.

Monetary dominance. Among all "price-level taking" debt levels, the optimal one is $B = \underline{P}_1 r b^{PT}$, where b^{PT} solves:

$$\max_{b\geq 0} \left\{g_0 + \beta^F g_1 - \beta^F c(\tau)\right\} \tag{74}$$

$$\text{s.t. } g_0 = x + b - \frac{R_{-1}X_{-1}}{P_0}, \tag{75}$$

$$g_1 = \bar{x} + \tau - \frac{RX_0}{\underline{P}_1} - rb, \tag{76}$$

$$c(\tau) - \tau - c(\tau^*) + \tau^* \leq \alpha^F - rb, \tag{77}$$

$$g_1 \geq 0. \tag{78}$$

As in the baseline model, purchases of bonds by M are immaterial under monetary dominance, and so we assume without loss of generality $b^M = 0$ in this program. The optimal debt level critically depends on the level of the interest rate r. When $\beta^F r \geq 1$, F does not borrow and the level of taxes is at its unconstrained maximum $\tau = \tau^*$. When $\beta^F r < 1$, F borrows as much as it can against its date-1 resources : b is selected so that

$g_1 = 0$. The date-1 taxes driving these date-1 resources are the minimum of two values, either $(c')^{-1}(1/\beta^F r)$ or the solution in τ to $\{(76);(77)\}$ with $g_1 = 0$ in (76). In the former case, which prevails if α^F is sufficiently large other things being equal, F strictly prefers to make good on its debt at date 1 whereas it is indifferent in the latter case in which $\tau = \tau_1$ defined in (72).

Fiscal dominance. Suppose now that F issues debt B so that the date-1 outcome is fiscal dominance. In this case, the date-1 taxes are given by τ_1 defined in (72), the date-1 price-level P^F, and savers' investment in the date-0 bond market b solve the three equations:

$$\tau_1 = \max\left\{\frac{RX_0}{P^F} + rb - \bar{x}; \tau^*\right\}, \tag{79}$$

$$c(\tau_1) - \tau_1 - c(\tau^*) + \tau^* = \alpha^F - rb, \tag{80}$$

$$B = r\left(b + x - \frac{R_{-1}X_{-1}}{P_0}\right)P^F. \tag{81}$$

The first two equations state that F must be indifferent between defaulting or making good on B at at date 1, and the third one expresses bond-market clearing. These equations take into account that investors in bonds correctly anticipate that $P_1 = P^F$, and that M optimally invests as much as possible in the bond market ($b^M = x - R_{-1}X_{-1}/P_0$).

The solution to this system is such that P^F and b increase with respect to B whereas τ_1 decreases. Suppose otherwise that b decreases in B. Equation (81) implies that P^F must increase, but then τ_1 must decrease from (79) and increase from (80), a contradiction. So, b increases in B, (80) implies that τ_1 decreases, and (79) in turn that P^F increases.

Since increasing B both raises P^F, thereby eroding the value of reserves RX_0, and reduces taxes τ_1, F finds it optimal, as in the baseline model, to set B as large as possible up to the point at which $P^F = \underline{P}_1 + \alpha^M$.

B.4 Date-0 reserve market

The generic result shown in the baseline model that M seeks to discourage F from issuing the Sargent-Wallace debt level by keeping the amount of circulating reserves sufficiently low still holds. The detailed analysis of monetary policy carried out in the case of the baseline model is however more cumbersome in this case. For brevity, we skip it here, and only state the most interesting result showing that the central role of the interest rate in the baseline model owed to the very simple assumed cost of taxation.

Proposition 8. ***(A sufficiently large α^F warrants monetary dominance.)*** *If other things being equal α^F is sufficiently large, then the price level is on target at every date ($P_0 = P_0^M$ and $P_1 = P_1^M$).*

Proof. Suppose that M announces $R = rP_1^M/P_0^M$ and $X_0 = R_{-1}X_{-1}$, and that savers invest x in the reserve market. We show that for α^F sufficiently large, F chooses the price-level taking strategy in the bond market.

Notice first that for α^F sufficiently large other things being equal, constraint (77) is slack at the solution to (74). Thus in the monetary-dominance strategy, the outcome no longer depends on the value of α^F past a threshold.

An inspection of {(79); (80); (81)} shows that holding $P^F = \underline{P}_1 + \alpha^M$ fixed, τ_1, b, and B grow without bounds as so does α^F other things being equal. The properties of the cost of taxation c implies that the utility that F derives from the Sargent-Wallace debt level thus tends to $-\infty$ as α^F grows.

Overall this means that for α^F sufficiently large, F issues the monetary-dominance debt level. This implies in turn that the date-0 reserve market clears at $P_0 = P_0^M$ and $x = R_{-1}X_{-1}/P_0^M$, and that $P_1 = \underline{P}_1 = P_1^M$ from $\bar{x} \geq rR_{-1}X_{-1}/P_0^M$. □