Public Liquidity Demand and Central Bank Independence*

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July 2, 2020

Abstract

This paper studies how a crisis that induces a large negative fiscal shock and a strong demand for safe stores of value affects the independence of a central bank vis-à-vis a fiscal authority that seeks to inflate away public liabilities. We find that the central bank can maintain price stability only if it can control the net increase in government debt held by the private sector. This occurs in turn in the presence of fiscal requirements and of a large demand for reserves that enables the central bank to sufficiently expand its balance sheet without triggering inflation.

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1 Introduction

Both the 2008 financial crisis and the COVID-19 pandemic have resulted in a massive negative shock to primary fiscal surpluses and in a surge in demand for safe stores of value. Public sectors have responded with equally massive issuances of sovereign debt and expansion of central banks' balance sheets. Monetary authorities have issued large amounts of reserves and invested the proceeds in government bonds as well as in private liabilities.

These unprecedented developments cast uncertainty on future monetary and financial stability. Have major central banks given up their independence by accommodating fiscal policies to an extent that will impair their future ability to ensure price stability, as in the unpleasant monetary arithmetic of Sargent and Wallace (1981)? Conversely, are the low rates on public liabilities enabling the public sector to issue bubbles that will repay themselves at zero fiscal and inflationary costs (see Blanchard, 2019)? Perhaps the public sector should even take more aggressive advantage of this high demand for safe storage, and directly finance deficits with the issuance of reserves, an arrangement deemed monetary-financed deficits or, sometimes, "helicopter money"?

Motivated by these questions, we offer a framework that is analytically tractable and yet sufficiently rich to jointly analyze many dimensions of the coordination (or lack thereof) of fiscal and monetary policies following large shocks to the economy.

We consider a public sector comprised of fiscal and monetary authorities. The fiscal authority operates transfers with and across the private sector and issues sovereign bonds. The monetary authority sets the nominal rate on reserves and issues them. Both authorities can transfer resources to each other and invest in each other's securities. They both are economic agents with well-defined objectives. The fiscal authority is biased towards subsidizing the most productive agents in the private sector whereas the central bank is biased towards price stability. The price level is the one that clears the market in which agents trade real resources for reserves.

The private sector is comprised of heterogeneous agents who reap gains from trades in a credit market that works seamlessly in normal times. There are episodic crises, however, during which the credit market shuts down. The private sector is willing to pay dear for public liabilities during such crises.

If crises are rare, the average return on public liabilities is high, and all public liabilities must be backed by fiscal surpluses. Sargent and Wallace (1981)'s unpleasant arithmetic applies, and all the public policies that share the same present value of fiscal surpluses must lead to the same unique equilibrium price level determined by the intertemporal budget constraint of the public sector.

More frequent crises lead to a situation that we deem one of "pleasant monetary arithmetic," whereby fiscal and monetary policies are no longer tightly interdependent this way. Taking fiscal policy as given, several price levels corresponding to various sizes of bubbles on public liabilities are feasible.

One could conclude that in the presence of such a pleasant monetary arithmetic, the central bank can independently control the price level and need not "chicken out" in the face of aggressive fiscal expansion.

To assess this intuition, we formally solve for the equilibria of Wallace's game of chicken between fiscal and monetary authorities. The fiscal authority would like the monetary one to chicken out and inflate away legacy public liabilities so as to free up resources for public subsidies. The central bank would conversely prefer fiscal consolidation so as to stabilize the price level. The cost for each authority of forcing the other to chicken out is that it may entail sovereign default, however.

We find that the authority that preempts private demand for public storages ultimately imposes its views to the other. The fiscal authority uses the share in the overall bubble on public storage it can preempt to issue debt that finances the current deficit. The monetary one issues reserves against its share in the bubble and uses the proceeds to buy bonds from the private sector. The relative firepower of each authority determines the net increase in the quantity of public debt in the hands of the private sector. Future inflation increases in this current net increase. In this sense, the authority that preempts liquidity forces the other to chicken out. Another way of summarizing the game of chicken is to acknowledge that the fiscal authority can force future monetary accommodation by being strategically fiscally irresponsible.[1]

A first implication from our setup is that low rates leading to a pleasant monetary

[1] In this sense we offer a strategic formalization of Sims (2016) who argues that: "The restraints on fiscal policy required by independence can be widely understood and implemented even without formal institutional limits – as in the case of the US Federal Reserve and the Bank of England, which emerge as among the least 'independent' central banks in the world in the Dincer and Eichengreen calculations. Some kinds of fiscal policy actions can force the hand of a central bank, even though it gets no orders from fiscal authorities and continues to pursue its goal of price stability."

arithmetic do not obsolete fiscal requirements for price stability such as caps on public debt and deficits. Second, the buyback of government debt by issuing reserves, as in quantitative easing, even though these liabilities are perceived as substitutes by the private sector, potentially modifies the future incentives of the fiscal and the monetary authorities. In particular, this modifies the central bank ability to maintain price stability. Third, a massive expansion of a central bank balance sheet is not a symptom that it will chicken out in the face of a shock to public finances. It is only if this expansion does not suffice to keep the amount of government bonds held by the private sector stable that this leads to future inflation.

Our rich description of the main tools available in practice to fiscal and monetary authorities in a unified framework also sheds light on issues such that price stability in the presence of a liquidity trap. We find that issuing large quantities of reserves and paying a low, possibly negative, interest rate on them ensure price stability despite a temporary liquidity trap. A lower bound on nominal rates may force the central bank to generate more future inflation than it would like otherwise in order to lift the current price level. A credible commitment to such inflation may involve that the central bank reduces its net wealth via helicopter money.

The paper is organized as follows. Section 2 outlines the model. Section 3 characterizes the space of feasible public policies and introduces our concept of pleasant monetary arithmetic. Section 4 solves for Wallace's game of chicken, identifying among feasible policies which ones correspond to subgame-perfect equilibria of the game. Section 5 investigates how a lower bound on interest rates modifies the central bank's policy. Section 6 spells out the policy implications from our analysis. Section 7 concludes.

Related literature. This paper is connected to the literature on the interactions between monetary and fiscal policies pioneered by Sargent and Wallace (1981) (see Leeper, 1991; Sims, 1994; Woodford, 1994, 1995; Cochrane, 2001, 2005; McCallum, 2001; Buiter, 2002; Niepelt, 2004; Jacobson et al., 2019; Brunnermeier et al., 2020, among others). Our characterization of a pleasant monetary arithmetic builds on Bassetto and Cui (2018), who show that low interest rates on public debt prevent fiscal policy from selecting a unique price level. The simple economy in which we cast our game of chicken relates in particular to one of the models in Bassetto and Sargent (2020), in which public liabilities also serve as liquidity vehicles and fund transfers that mitigate credit-market failures. Our paper is also closely connected to the papers that identify fiscal requirements such that the central bank can attain its price stability objective, including fiscal rules (e.g. Woodford, 2001) or a ring-fenced balance sheet (e.g Sims, 2003; Bassetto and Messer, 2013; Hall and Reis, 2015; Benigno, forthcoming).

With respect to this literature, an important contribution of our paper is to explicitly model the strategic interactions between fiscal and monetary authorities. That fiscal and monetary authorities may have ex-post conflicting objectives is a natural assumption. This has been in fact the main rationale behind setting up independent central banks. This is also motivated by the large set of evidence that authorities do not necessarily cooperate and, instead, try to impose their views to each other (see Bianchi et al., 2019, among others), even though coordination dominates (see Bianchi et al., 2020, for a recent contribution). In this respect, this makes our paper closer to an older literature (Alesina, 1987; Alesina and Tabellini, 1987; Tabellini, 1986, e.g.) that investigates the equilibria of games between multiple branches of government. More recent contributions include Dixit and Lambertini (2003) or Aguiar et al. (2015). The latter study fiscal and monetary policy in a monetary union with atomistic sovereigns that may default. Closer to our paper, Martin (2015) finds as we do that fiscal irresponsibility leads to long-term inflation. Our contribution with respect to this literature is to model the game of chicken between monetary and fiscal authorities when both components of the public sector are strategic, and to allow for the possibility that both authorities supply securities that are substitutes to the private sector.

We also relate to the literature on rational bubbles in two ways. First, informational asymmetries in credit markets create room for bubbles in our overlapping-generations example, as they do in Farhi and Tirole (2012) or Martin and Ventura (2012). Second, and more importantly, we also relate to the literature linking monetary policy to bubbles, including Gali (2014). In particular, Asriyan et al. (2019) consider a competition between private bubbles and a public one ("money"). By contrast, we study competition between distinct public bubbles, reserves and government bonds, to study fiscal-monetary interactions. Finally, the idea that public debt satisfies private liquidity demand goes back to at least Diamond (1965) and has been widely studied since (see Woodford, 1990; Holmström and Tirole, 1998, among others). Krishnamurthy and Vissing-Jorgensen (2012) showed in the data that public debt shared many of the properties of money.

2 The environment

In this section, we introduce a model that features three main ingredients: the public sector has to absorb a fiscal shock in the form of an exogenous (nominal) amount that it becomes liable for; two public institutions, monetary and fiscal authorities, can issue liabilities, remunerated reserves and debt respectively, that are substitutes; and both authorities can trade their liabilities with and make transfers to each other.

2.1 Setup

Time is discrete and indexed by $t \in \mathbb{N}$. There is a single consumption good. The economy is populated by two types of private agents, savers and entrepreneurs. The public sector is comprised of a fiscal authority and a monetary one.

Private sector. At each date, a unit mass of entrepreneurs and a unit mass of savers are born. They live for two dates and value consumption only when old, at which time they are risk-neutral. They can store the consumption good with a linear return $e^{-\eta}$, where $\eta \in \mathbb{R}$.

Savers. Savers are endowed with units of the consumption good when young. Endowments are i.i.d. across savers of a given cohort. The date-t endowments' distribution has a unit mean and its support has a lower bound $\bar{\tau}_t \geq 0$.

Entrepreneurs. Young date-t entrepreneurs are penniless and endowed with a storage technology with a random linear return. The (gross) return has expected value e^{ρ}, where $\rho > \max\{-\eta; 0\}$, and its distribution has 0 in its support. Returns are perfectly correlated across entrepreneurs of the same cohort. Entrepreneurs are competitive.

Public sector. The public sector features a fiscal authority F and a monetary authority M.

Monetary authority. The monetary authority issues reserves and sets the (gross) nominal interest rate R_t on them. Reserves are claims of infinite maturity. A unit of reserves at date t is a claim to R_t units of reserves at date $t+1$. Reserves are the unit of account of the economy. We denote by P_t the date-t price of the consumption good in terms of reserves.

Fiscal authority. The fiscal authority implements taxes and transfers, and issues one-period nominal bonds.

Transfers. We denote by σ_t the date-t real (positive or negative) transfer from young entrepreneurs to F and by τ_t that from young savers.[2] We also denote by θ_t the date-t real (positive or negative) transfer from M to F.

Bonds. At each date t, the fiscal authority F issues one-period nominal bonds. Each bond is a claim to one unit of reserves at date $t + 1$.

Legacy liability of the fiscal authority. Savers born at date 0 also own a nominal claim on the fiscal authority of $L \geq 0$ units due at date 1.

Market for reserves. At each date t, the market for reserves opens up where the private sector, F, and M can trade reserves for the consumption good. All agents bid with goods or/and reserves. Only M can issue reserves ("sell reserves short"). We denote by x_t, x_t^F, and x_t^M the respective quantities of goods submitted by the private sector, F, and M, where $x_t, x_t^F, x_t^M \geq 0$. Let $\Delta_t \geq 0$ denote the quantity of new reserves issued by M in the date-t market, and $X_t \geq 0$ denote the outstanding reserves after the date-t market clears. M cancels the reserves that it buys back, and so

$$X_t = R_{t-1}X_{t-1} + \Delta_t - P_t x_t^M. \tag{1}$$

Without loss of generality, we suppose that Δ_t and x_t^M cannot be simultaneously strictly positive, and so the interventions of M in the market for reserves can interchangeably be summarized by stocks (X_t) or flows (Δ_t, x_t^M). We use X_t in the following. The market-clearing price level P_t solves

$$R_{t-1}X_{t-1} + \Delta_t = P_t(x_t + x_t^F + x_t^M). \tag{2}$$

The price level in this cashless economy, P_t, is thus defined as the inverse of the value of reserves as they serve as the unit of account.[3]

Primary public bond market. At each date t, F issues a quantity B_t of one-period nominal bonds. We denote by b_t the quantities of goods submitted by the private sector

[2] We omit transfers involving old agents to save on notations and because they will play no role in the subsequent strategic analysis in Section 4. The analysis is verbatim if we include them, though.

[3] See Reis (2015) for a discussion of this point. Insofar as reserves can be converted in currency, the value of reserves would also be the value of currency.

for bonds issued by F, where $b_t \geq 0$ and we denote by Q_t the nominal price at which the fiscal authority's bonds are traded at date t:

$$Q_t B_t = P_t b_t. \tag{3}$$

Secondary market for public bonds. The legacy liability L and the newly-issued debt B_t are traded together on a secondary market once the primary public bond market is closed. Both F and M can buy them back from the private sector at price Q_t. We denote by b_t^F and b_t^M the respective (positive) numbers of goods submitted by F and M to buy back the outstanding amount of debt, respectively, so that:

$$P_t(b_t^F + b_t^M) \leq Q_t(B_t + \mathbb{1}_{\{t=0\}} L). \tag{4}$$

Information structure. The public sector does not observe savers' endowments, entrepreneurs' return, nor consumption by either of them when old. There exists $T \in \mathbb{N}$ such that if t does not belong to $\{k(T+1), k \in \mathbb{N}\}$, then savers born at date t perfectly observe the return realized by date-t entrepreneurs at date $t+1$. Otherwise, they do not observe it.

Private credit market. That $\rho > -\eta$ implies that (risky) loans from savers to entrepreneurs unlock gains from trades. Such a private credit market works seamlessly for the cohorts that do not experience any informational asymmetries between lenders and borrowers. At dates that belong to $\{k(T+1), k \in \mathbb{N}\}$, however, the credit market collapses as entrepreneurs can always claim at the next date that their realized return is zero. Thus they cannot pledge any future output to savers.

Denoting by $1/\phi_t$ savers' return on private investment, this implies that $\phi_t = e^{\eta}$ when t belongs to $\{k(T+1), k \in \mathbb{N}\}$, and $\phi_t = e^{-\rho}$ otherwise.

2.2 Interpretation

Our aim with this setup is to capture two important features of episodes such as the 2008 financial crisis or the COVID-19 pandemic. First, financial markets stop functioning, which we model simply as a temporary informational friction. Interest rates are abnormally low during such market shutdowns. Second, the public sector inherits large unexpected explicit or/and implicit liabilities. We also offer a detailed description of the public-finance instruments that the public sector can avail itself of in the face of such shocks.

Credit market shutdown. We interpret the dates at which there are no informational asymmetries between savers and entrepreneurs as "normal times," and the ones, including $t = 0$, in which the credit market shuts down as "financial crises".[4] A crisis is a shutdown of the private credit market such that in the absence of transfers from the government to young entrepreneurs, there is no efficient investment. This market failure implies that the real interest rate is low during such episodes ($-\eta < \rho$).

Legacy debt L. Our favorite interpretation of L is that it results from the necessity to bail out the economy. Under this interpretation, L is not an explicit liability, but rather stands for the size of an (ex-post desirable) intervention needed by the private sector, and "default" on L would therefore correspond to an incomplete bailout. Another more standard interpretation of the legacy liability L is that the public sector has issued long-term debt in an non-modelled past (before date 0) and that L is the residual amount due at date 1.

Policy instruments. We endow both authorities with a rich set of policy tools that enable us to describe price-level determination and more generally monetary and fiscal interactions in great details, including in particular the unconventional measures of the post 2008 and post COVID-19 policy regimes. For instance, one can interpret the situation in which the monetary authority issues remunerated reserves X_t to buy government debt b_t^M as quantitative easing. The alternative situation in which $b_t^M = 0$ and the ressources that M collects in the market for reserves are passed to the government via monetary dividends $\theta_t > 0$ corresponds to helicopter money.

3 Feasible policies

In this section, we first define and characterize the set of *feasible* policies, that is, policies that satisfy the budget constraints of monetary and fiscal authorities and are consistent with a competitive equilibrium when the private sector correctly anticipates

[4] Note that the case $T = 0$ is essentially a situation of dynamic inefficiency à la Wallace (1980).

them. We then identify situations that we deem ones of "pleasant arithmetic," in which fiscal and monetary policies are not tightly interdependent: Multiple price levels are consistent with a given fiscal policy. We contrast these situations with that of "unpleasant arithmetic," in which the determination of the price level is tightly constrained by fiscal decisions. We finally show that any feasible policy without sovereign default features a lower bound on prices that depends on the public resources even under a pleasant arithmetic. This lower bound is going to be an important benchmark in the strategic interactions between the two authorities in Section 4.

3.1 Definition and characterization

A policy encompasses all the actions of the public sector. It is thus a sequence of vectors $((\sigma_t, \tau_t, \theta_t, B_t, X_t, x_t^F, b_t^F, b_t^M, R_t))_{t\in\mathbb{N}}$ that describes transfers, security issuances and investments by F and M, and the interest rate paid on reserves.

Definition 1. ***(Feasible policy)*** *A policy is feasible if and only if there exists a competitive equilibrium in the private sector given this policy: a sequence of prices $(P_t, Q_t)_{t\in\mathbb{N}}$ such that all private agents optimize, are indifferent between all available storages,*[5] *markets clear and the budget constraints of the fiscal and the monetary authorities are satisfied.*

The budget constraints that face M and F in the absence of default are respectively:

$$\frac{X_t - R_{t-1}X_{t-1}}{P_t} + \frac{P_{t-1}}{Q_{t-1}P_t}b_{t-1}^M = \theta_t + b_t^M; \tag{5}$$

$$\frac{Q_tB_t - (B_{t-1} + \mathbb{1}_{\{t=1\}}L)}{P_t} + \frac{P_{t-1}R_{t-1}}{P_t}x_{t-1}^F + \frac{P_{t-1}}{Q_{t-1}P_t}b_{t-1}^F = x_t^F + b_t^F - \sigma_t - \tau_t - \theta_t. \tag{6}$$

Equation (5) states that M uses proceeds from net reserves issuance and past bond investments to pay a dividend to F and purchase new bonds in the secondary market. Relation (6) similarly states that F uses the proceeds from the primary bond market (net of bond repayments) and past investments in reserves and bonds to fund new investments in these instruments and transfers to the private sector and M.

The following related definition will prove useful in Section 4. It allows for default on the legacy liability L.

[5] This indifference condition implicitly rules out corner equilibria that are possible given the fully linear model. Such equilibria would be eliminated by the assumption of a strictly concave storage technology that would complicate the analysis.

Definition 2. ***(Feasible policy with default)*** *A policy is feasible with default if and only if there exists $h \in [0,1]$ such that the policy is feasible when the legacy liability L is replaced by $(1-h)L$.*

Notice that we rule out default on the debt endogenously issued by F as this would amount to a simple redefinition of its promises in this perfect-foresight environment. We will of course allow for default on any claim as a deviation in the strategic environment in Section 4. The following proposition shows that the set of allocations to the private sector induced by all feasible policies is spanned by a subset of "simple" policies that we characterize.

Proposition 1. ***(Characterization of feasible policies)*** *Any feasible policy induces a unique sequence of prices $(P_t, Q_t)_{t\in\mathbb{N}}$. Any allocation induced by a feasible policy is also induced by a simpler feasible policy such that:*

$$\forall t \geq 0,\ x_t^F = b_t^F = 0 \text{ and } \forall t > 0,\ b_t^M = 0. \tag{7}$$

Such policies $((\sigma_t, \tau_t, \theta_t, B_t, X_t, R_t))_{t\in\mathbb{N}} \cup \{b_0^M\}$, that we deem "simple", are in turn feasible if and only if there exists $(P_t, Q_t)_{t\in\mathbb{N}}$ such that for any date $t \geq 0$:

$$\sigma_t \leq 0, \tau_t \leq \bar{\tau}, \tag{8}$$

$$\frac{P_{t+1}}{R_t P_t} = \frac{Q_t P_{t+1}}{P_t} = \phi_t, \tag{9}$$

$$\frac{X_t}{P_t} + \frac{Q_t B_t}{P_t} \leq 1 - \tau_t + \mathbb{1}_{\{t=0\}} b_0^M, \tag{10}$$

$$P_0 b_0^M \leq Q_0(B_0 + L), \tag{11}$$

$$\frac{Q_t B_t - (B_{t-1} + \mathbb{1}_{\{t=1\}} L)}{P_t} = -\sigma_t - \tau_t - \theta_t, \tag{12}$$

$$\frac{X_t - R_{t-1} X_{t-1}}{P_t} + \frac{P_0}{Q_0 P_1} \mathbb{1}_{\{t=1\}} b_0^M = \theta_t + \mathbb{1}_{\{t=0\}} b_0^M. \tag{13}$$

Proof. See Appendix A. □

Proposition 1 has two parts.

The first one is a Modigliani-Miller type of result echoing ones by Wallace (1981) or Chamley and Polemarchakis (1984) on the irrelevance of open-market operations: trades of bonds and reserves by F and M can be replicated by a simple stream of transfers between them. For example, standard open-market operations, whereby M sells reserves to the private sector to fund the purchase of bonds in the secondary market from savers who bought them in the primary market at date t, correspond to a payment of M towards F at date t that replicates the bond purchase followed by a date-$(t+1)$ transfer to M from F that replicates the bond repayment. As a result, all feasible allocations are implementable merely with transfers $((\sigma_t, \tau_t, \theta_t))_{t\in\mathbb{N}}$, security issuances $((B_t, X_t))_{t\in\mathbb{N}}$, and a nominal interest rate R_t. The possibility that the public sector can buy debt in the secondary market at date 0, which the inclusion of b_0^M warrants, is needed only insofar as there is some legacy debt L held by the private sector.

The second part of the proposition states conditions under which such a simple policy $((\sigma_t, \tau_t, \theta_t, B_t, X_t, R_t))_{t\in\mathbb{N}} \cup \{b_0^M\}$ is feasible. These conditions are quite intuitive. Conditions (8) result from informational asymmetries. Entrepreneurs can always claim a zero-return so as to avoid taxation. Given the distribution of endowments, each date-t saver can always claim that her endowment is $\bar{\tau}_t$. Conditions (9) ensures that savers are indifferent between holding reserves, public bonds, and using private storage solutions. Condition (10) ensures that (unit) aggregate savings exceed the equilibrium supply of public securities. Condition (11) makes sure that the buyback by M is feasible given the outstanding debt due at date 1. Condition (12) is the equilibrium budget constraint of F, and (13) that of M.

3.2 The monetary arithmetic

Proposition 2 links the date-1 price level and fiscal policy when the monetary arithmetic is pleasant or unpleasant.

Proposition 2. ***(Unpleasant vs pleasant arithmetic)***

Unpleasant arithmetic. *If $T > \eta/\rho$, then all the feasible policies that share the same fiscal component $((\sigma_t, \tau_t))_{t\in\mathbb{N}}$ pin down a unique date-1 price level P_1 that solves*

$$\frac{\phi_0 L}{P_1} = \sigma_0 + \tau_0 + \sum_{t\geq 1} \left(\Pi_{t'=0}^{t-1} \phi_{t'}\right)(\sigma_t + \tau_t). \tag{14}$$

Furthermore, all simple feasible policies such that $X_t > 0$ for some t must be such that $\theta_{t'} < 0$ for at least one t'.

Pleasant arithmetic. *If $T \leq \eta/\rho$, then there exists feasible policies that share the same fiscal component $((\sigma_t, \tau_t))_{t\in\mathbb{N}}$ but correspond to different date-1 price levels P_1. There exists simple feasible policies such that $X_t > 0$ and $\theta_t \geq 0$ for all t.*

Proof. See Appendix B. □

The price level under unpleasant and pleasant arithmetic. If $T > \eta/\rho$, then crises are rare, and the average return on public liabilities is too high to allow the public sector to rollover fiscally unbacked claims. The present value of outstanding public liabilities must equal that of fiscal surpluses in any equilibrium. The existence of a legacy nominal liability due at date 1 therefore implies that the present value of fiscal surpluses pins down the date-1 price level. A monetary authority that takes fiscal policy as given and cares about sovereign default does not choose the price level P_1 when the arithmetic is unpleasant.

Conversely, if crises are more frequent ($T \leq \eta/\rho$), reserves (as any other assets) command a lower average return, and so the central bank can indefinitely rollover several levels of reserves without any fiscal backing. Thus, taking fiscal policy as given, several monetary policies are feasible that are consistent with sovereign solvency and lead to several date-1 price levels.

The fiscal theory of the price level. Note that our results under unpleasant arithmetic (e.g., equation (14)) are consistent with important insights from the literature on the fiscal theory of the price level. In particular, first, as in Niepelt (2004), the absence of a nonzero exogenous nominal government liability ($L = 0$) also leads to the absence of constraints on the price level—or, equivalently, there is no such determination of the price level by fiscal surpluses with only endogenously issued debt. Second, the price level in equation (14) corresponds to the inverse of the price of reserves – and not only that of the price of government debt as criticized by Buiter (2002) – due to the possibility of issuance of reserves and transfers between the government and the central bank (see also the discussion in Bassetto, 2016).

Pleasantness and the sign of transfers. Under an unpleasant arithmetic, any simple feasible policy with an active market for reserves must be such that the monetary authority receives fiscal backing at some point—that is, a positive net transfer from F—

as it cannot indefinitely rollover reserves. In a more general action space in which M can store using government bonds, this means that the investment of M in new bonds has to be at some date smaller than the repayments on its maturing bonds, so that F makes a net payment to M. By contrast, under a pleasant arithmetic, M can be in the position to always make transfers to F ($\theta_t \geq 0$). That the central bank never requires any net infusion from the government this way is commonly put forward as an important criterion for independence.

3.3 Price levels under pleasant arithmetic.

It would be incorrect to infer from the loosening of the interdependence between fiscal surpluses and price level and from the financial autonomy of the central bank that a pleasant monetary arithmetic automatically reinforces central-bank independence. Independence must be assessed in an environment in which it matters, for example because F and M have distinct objectives (at least ex-post). Before offering such a model in Section 4, we first introduce a useful indexation of feasible policies by their associated private demands for public liquidity vehicles.

For the remainder of the paper, we assume that the monetary arithmetic is pleasant ($T \leq \eta/\rho$) and that $\bar{\tau}_t = 0$ for all t. This latter assumption is for analytical simplicity only. It implies that F cannot raise taxes, and so private liquidity demand $((b_t, x_t))_{t\in\mathbb{N}}$ is the sole source of income for the public sector. The following definition identifies the policies that are compatible with a given pattern of private liquidity demand $((b_t, x_t))_{t\in\mathbb{N}}$.

Definition 3. ***(Compatibility of a policy with a given liquidity demand)*** *A policy is compatible with the sequence $(b, x) = ((b_t, x_t))_{t\in\mathbb{N}} \in (\mathbb{R}_+{}^2)^{\mathbb{N}}$ if and only if it is feasible with default, and is associated with a competitive equilibrium in which the respective date-t private demands for government bonds and reserves are b_t and x_t, respectively.*

Whereas the proof of Proposition 1 consists in taking a policy as given and constructing a competitive equilibrium associated with it, this definition goes the other way round. It starts from a candidate path of private demand for public liquidity (b, x) and identifies the policies that are compatible with this pattern arising in equilibrium.

Proposition 3. ***(All Ponzi schemes admit compatible policies)*** *Suppose that*

$(b, x) = ((b_t, x_t))_{t\in\mathbb{N}} \in ({\mathbb{R}_+}^2)^{\mathbb{N}}$ satisfies at any date $t \geq 0$,

$$\phi_t^{-1}(b_t + x_t) \leq b_{t+1} + x_{t+1} \leq 1, \tag{15}$$

then the sequence (b, x) admits compatible policies. In such a case, compatible policies generate date-1 price levels P_1 that span $(0, +\infty)$. Compatible policies that do not feature default generate date-1 price levels that span $[L/(b_1 + x_1), +\infty)$.

Reciprocally, if the sequence $(b, x) \in ({\mathbb{R}_+}^2)^{\mathbb{N}}$ admits compatible policies, then condition (15) is satisfied at any date $t \geq 1$ and $\phi_0^{-1} x_0 \leq b_1 + x_1$.

Proof. See Appendix C. □

Condition (15) simply states that (b, x) admits compatible policies if the aggregate private demand for public storages $b_t + x_t$ is a feasible bubble pattern—a Ponzi scheme. Given such a scheme, all date-1 price levels are spanned by compatible policies. Only date-1 prices above $L/(b_1 + x_1)$ warrant sovereign solvency, though. Note that this latter result is also obtained by Bassetto and Cui (2018) in a different setup.

The proof of Proposition 3 provides additional informations on the policy actions leading to a given price level. More precisely, the price below which default cannot be avoided implicitly depends on the date-0 buyback b_0^M as follows:

$$P_1 \leq \frac{(b_1 + x_1) - \phi_0^{-1}(b_0 + x_0 - b_0^M)}{L},$$

where b_0^M is smaller than $b_0 + x_0$. The resources b_0^M used to buy back bonds are not used for transfers to young entrepreneurs at date 0, and increase the consolidated resources of the public sector at date 1. Thus, the lowest price level consistent with no default is reached when the buyback is maximum.

4 Wallace's game of chicken

This section solves an explicit model of Wallace's "game of chicken" in the economy outlined thus far. We only need to augment the description of the model with that of the objectives of F and M. We also detail the intra-date timing of their policy actions. Whereas timing was immaterial in the previous sections that focussed on feasible policies, it is important in a strategic context.

4.1 Setting and equilibrium definition

Objectives of F and M. The respective date-t objectives of F and M are:

$$U_t^F = -\sum_{t' \geq 0} \beta^{t'} (\sigma_{t+t'} + \alpha_F \Delta_{t+t'}), \tag{16}$$

$$U_t^M = -\sum_{t' \geq 0} \beta^{t'} (\mid P_{t+t'} - P_{t+t'}^M \mid + \alpha_M \Delta_{t+t'}), \tag{17}$$

where $\beta \leq e^{-\rho}$, $\alpha_F, \alpha_M \geq 0$, and $P_t^M > 0$. The variable Δ_t is equal to 1 in case of an outright default on a government bond due at date t, and to 0 otherwise.

In words, each authority $X \in \{F; M\}$ incurs a cost α_X in case of sovereign default.[6] The fiscal authority also values subsidies to young entrepreneurs (but does not care about the price level), whereas the monetary authority also finds it costly to deviate from a given target P_t^M for the date-t price level (but does not care about transfers).

We also assume the following lexicographic preferences for M. Holding (17) fixed, M prefers to maximize the current transfer to young entrepreneurs. Such preferences are realistic, quite in line with the ECB's mandate for example.

Intuitively, such preferences for F and M set the stage for a game of chicken. F would like M to accommodate and inflate away the legacy liability L so that it can deploy more resources towards subsidies to entrepreneurs. M would conversely prefer fiscal consolidation by F at the expense of such subsidies so as to ensure price stability. The cost for each authority of forcing the other to chicken out is that it may entail sovereign default, however. We focus for brevity on the case in which

$$\alpha_F = +\infty. \tag{18}$$

In words, F is willing to do whatever it takes to avoid sovereign default. This presumably stacks the deck in favor of central-bank independence.

[6]Costs from outright default are exogenous here. They include in practice output losses due to financial-market exclusion, trade sanctions, banking crises and more generally financial instability, as well as private costs—electoral or more generally political costs for the fiscal authority and career concerns for central bankers.

Where do these objectives come from? For brevity, we simply posit that the public sector is comprised of two distinct authorities with different objective functions. Yet a simple time-inconsistency argument could micro-found the delegation of price-level determination to a biased monetary authority. Suppose that the social welfare function puts more weight on entrepreneurs than on savers, but that the government lacks commitment. In this case, such a government would be tempted to inflate away savers' public claims ex-post. Savers would anticipate this, and this would inefficiently shut down bond markets ex-ante. Delegation to an entity with a mandate for a stable price level mitigates this problem.

Intra-date timing. At each date t,

1. M announces a rate on reserves R_t.
2. The market for reserves opens up and clears.
3. The primary market for government bonds opens up and clears.
4. The secondary market for government bonds opens up and clears.
5. M and F decide on transfers to each other, and on transfers to the private sector. One authority makes a take-it-or-leave-it offer to the other.[7]
6. F repays maturing bonds if it can and is willing to do so.

The important feature of this timing is that M has the last word in the bond market at each date. Provided it has resources, it can intervene in the secondary market in order to control the amount of bonds in the hands of the private sector after F has issued new bonds. The analysis will show that it is a necessary, but not a sufficient condition for central-bank independence in this setup.

Note also that this timing implies that only the private sector invests real resources in the market for reserves since the public sector has no real resources to invest at the opening of each date ($x_t^F = x_t^M = 0$). This assumption is innocuous in this section in

[7]Which one does so turns out to be immaterial for equilibrium determination and it could for example be randomly drawn at each date. The reason is that transfers are decided once the price level is determined. As a result of the lexicographic preferences of M, F and M have aligned interests at this stage 5.

which deviations from price targets are only about inflating rather than deflating the economy and only the latter requires the public sector to buy back reserves.[8]

Before defining and characterizing equilibria, we introduce a last ingredient that would have been irrelevant in Section 3 and will play a major role in the strategic analysis.

Assumption 1. ***(Fiscal requirements)*** *The public sector cannot pledge at date t a fraction of its date-$t+1$ resources that exceeds $\lambda \in (0,1]$.*

This assumption is of course void when $\lambda = 1$. When $\lambda < 1$, it imposes a constraint on the aggregate issuance of reserves and bonds. We will see that the constraint binds only for F when it does, so that it is in effect a fiscal requirement.

In order to formally define our equilibrium concept, the following definition first introduces a natural notion of deviation in our environment in which the actions taken by F and M must be associated with a competitive equilibrium.

Definition 4. ***(Date-t deviation by F or M)*** *Consider a given policy $\mathcal{P}$ that is feasible with default. A policy $\mathcal{P}'$ features a date-t deviation from $\mathcal{P}$ by authority $X \in \{F; M\}$ if i) the first policy action in $\mathcal{P}'$ that differs from its counterpart in $\mathcal{P}$ is taken by X at date t; ii) the continuation of $\mathcal{P}'$ that starts at the deviating action is feasible with default on already issued debt.*

For example, if F and M take the same actions across $\mathcal{P}$ and $\mathcal{P}'$ until date 5, at which M acts the same in the reserves market, F issues the same quantities of new bonds, but M bids differently in the secondary market for bonds ($b_5'^M \neq b_5^M$), then $\mathcal{P}'$ is a date-5 deviation from $\mathcal{P}$ by M if there exists a feasible policy from this date-5 secondary bond market on that may include default on B_4 at date 5 or B_5 at date 6. We are now equipped to define our equilibrium concept.

Definition 5. ***(Game of chicken)*** *A policy $((\sigma_t, \tau_t, \theta_t, B_t, X_t, b_t^F, b_t^M, R_t))_{t\in\mathbb{N}}$ is a game of chicken if it is feasible with default and if any date-t deviation by $X \in \{F; M\}$ that strictly increases U_t^X is not compatible with any liquidity demand (b', x') such that $x'_{t+1} \leq x_{t+1}$ and $b'_{t+1} \leq b_{t+1}$.*

A feasible policy is an equilibrium—"a game of chicken"—if neither F nor M can deviate at some date t and strictly increase its utility unless the continuation economy associated with this deviation features date-$t+1$ liquidity inflows strictly above (b_{t+1}, x_{t+1}).

[8] In Section 5 where we introduce a lower bound on interest rates, we relax this assumption so as to tackle situations in which the central bank is tempted to deflate the economy.

This equilibrium concept is a natural adaptation of subgame perfection in our environment in which i) the actions of the players F and M must be consistent with a competitive equilibrium, ii) there are many such competitive equilibria with varying bubble patterns. We impose that when either authority deviates and embarks on a different continuation path, this new path cannot involve a competitive equilibrium with higher future liquidity inflows than the initial ones. This way, the public sector cannot select a given path of bubbles, and must take its future mobilizable resources as given.

We will see that there exists a continuum of games of chicken as there exists a continuum of feasible bubbly paths (b, x) consistent with this definition. We are interested in studying how a given path (b, x) shapes M's independence measured as its ability to set the price level to target.

Remark. In our definition of equilibrium, the fiscal and the monetary authorities behave strategically, but the private sector remains non-strategic. In particular, this means that we assume that even off-equilibrium, a competitive outcome has to form for the private sector. This is consistent with a large share of the literature on games between an authority and a continuum of agents in macroeconomics as reviewed by Ljungqvist and Sargent (2018), but extended to a multiple-authority setting.

4.2 Equilibrium outcomes

We now characterize the games of chicken, starting with the following useful result:

Lemma 4. ***(Reserve dynamics in a game of chicken)*** *Any game of chicken is either such that* $x_t = 0$ *for all* $t \in \mathbb{N}$*, or such that* $x_{t+1} \geq x_t/\phi_t > 0$ *for all* $t \in \mathbb{N}$*.*

Proof. See Appendix D. □

As the only resources of M result from the rise of bubbles on reserves, one cannot rule out a "no-trade" equilibrium in which there is no bubble on reserves. The market for reserves is however active at all dates as soon as it is active at at least one date. The remainder of the paper focusses on these latter equilibria in which $x_t > 0$ for all t. [9]

[9] It is easy to eliminate "no-trade" equilibria ($x = 0$) without affecting the rest of the analysis by endowing M with (arbitrarily small) real resources at date 0 that it can store using the safe technology (with return $e^{-\eta}$).

We characterize games of chicken in two steps. We first tackle the case $L = 0$ in which all the liabilities of the public sector are endogenously issued. We then solve for the case $L > 0$. This distinction enables a clear identification of the important economic insights.

Endogenous liabilities ($L = 0$). We start by describing the equilibrium outcome in the absence of exogenous liability ($L = 0$). Let us introduce for all $t \in \mathbb{N}$,

$$\delta_t = \phi_t(b_{t+1} + x_{t+1}) - \min\{\lambda\phi_t(b_{t+1} + x_{t+1}); 1\}. \tag{19}$$

In words, δ_t is the date-t present value of the date-$t+1$ resources that the public sector cannot pledge at date t. Limited pledgeability $\delta_t > 0$ stems either from a fiscal requirement ($\lambda < 1$) or from the present value of these future resources exceeding (unit) aggregate savings. Equivalently, the public sector can pledge its entire future income ($\delta_t = 0$) if and only if there are no fiscal requirements and unit aggregate savings exceed the present value of its future resources. We adopt the convention that $\delta_{-1} > 0$ as there are no aggregate savings at date -1.

Proposition 5. ***(No monetary determination of the price level without fiscal requirements)*** *Suppose $L = 0$. Suppose a game of chicken is such that $x_t > 0$ for all t. Fix $t \in \mathbb{N}$. If $\delta_{t-1} = \delta_t = 0$, then $P_{t+1} = P^M_{t+1} + \alpha_M$. Otherwise $P_{t+1} = P^M_{t+1}$.*

Furthermore, the public sector issues reserves and bonds for the maximum amount $b_t + x_t = \min\{\lambda\phi_t(x_{t+1} + b_{t+1}); 1\}$ at date t, and any surplus after the reserve market clears and date-t-1 bonds are repaid is transferred to young entrepreneurs.

Proof. See Appendix E. □

Proposition 5 states that there is fiscal dominance as soon as i) M incurs costs from sovereign default ($\alpha_M > 0$); ii) F can borrow against the entire resources of the public sector net of the value of reserves. The only way to eliminate fiscal dominance across all games of chicken is to impose a fiscal requirement $\lambda < 1$, where λ can be arbitrarily close to one.

This result stands in sharp contrast with that in Proposition 2 suggesting that in the presence of a pleasant monetary arithmetic, M has a free hand at selecting price levels regardless of fiscal policy and without relying on any fiscal resources. In a strategic context, the monetary arithmetic is pleasant mostly to the fiscal authority.

We sketch here the important and most instructive part of the proof of Proposition 5. Suppose that $\delta_{t-1} = \delta_t = 0$ at some date t. We show by contradiction that $P_{t+1} = P^M_{t+1} + \alpha_M$. Suppose otherwise that M seeks to set $P_{t+1} = P^M_{t+1}$. In this case M announces $R_t = P^M_{t+1}/(\phi_t P_t)$ at the opening of date t and sets the date-t price level P_t, whichever it is, by issuing reserves $\Delta_t = (x_t - x_{t-1}/\phi_{t-1})P_t$. We show that F can in this case deviate and set a higher date-$t+1$ price level that inflates away reserves at date $t+1$. It does so by issuing in the date-t primary bond market a number of bonds equal to

$$B_t = \left(\phi_t(b_{t+1} + x_{t+1}) - \frac{x_t P^M_{t+1}}{\phi_t(P^M_{t+1} + \alpha_M)}\right)\left(P^M_{t+1} + \alpha_M\right). \tag{20}$$

Flooding the primary bond market with paper this way sets fiscal dominance by forcing M to set the date-$t+1$ price level at $P^M_{t+1} + \alpha_M$. To see why, notice first that M cannot buy any of these bonds back from the savers in the date-t secondary market because it has no resources to do so. Since $\delta_{t-1} = 0$, F has optimally borrowed a maximum real amount $\phi_{t-1}(x_t + b_t) - x_{t-1}$ at date $t-1$, thereby forcing M to use its entire date-t resources $x_t - x_{t-1}/\phi_{t-1}$ to contribute to the repayment of bonds issued at date $t-1$. Second, in the absence of default at date $t+1$, P_{t+1} is the smallest price level such that:

$$\left(\phi_t(b_{t+1} + x_{t+1}) - \frac{x_t {P^M}_{t+1}}{\phi_t(P^M_{t+1} + \alpha_M)}\right)\left(P^M_{t+1} + \alpha_M\right) + \frac{P^M_{t+1} x_t}{\phi_t} \leq P_{t+1}\left(x_{t+1} + b_{t+1}\right). \tag{21}$$

The left-hand side of (21) is the (nominal) value of the liabilities of the public sector at date $t+1$, and the right-hand-side that of its real resources. Thus, P_{t+1} must be equal to $P^M_{t+1} + \alpha_M$, the maximum price level that M is willing to implement rather than let F default.

There are two ways this deviation by F can be eliminated when future public resources are not entirely pledgeable, that is, when $\delta_{t-1} + \delta_t > 0$. First, if $\delta_{t-1} > 0$, M has some resources to invest in the date-t secondary bond market. It can defeat any expectation in the date-t primary bond market other than the date-$t+1$ price-level being P^M_{t+1} by buying some arbitrarily small amount of debt from savers in the date-t secondary market. In this case, (21) is strict when $P_{t+1} = P^M_{t+1} + \alpha_M$, because only the debt that is in the hand of the private sector matters for solvency. The internal claim of M on F can be settled with a netting from the transfer that M would have made to F anyway with the repayment.

Second, if $\delta_t > 0$, then this means that F cannot borrow against the entire value of $\phi_t(b_{t+1} + x_{t+1}) - x_t$ in the date-t primary market, in which case (21) holds at strictly smaller price levels than $P^M_{t+1} + \alpha_M$, thereby defeating the deviation as well.

In sum, in the absence of fiscal requirements, there is generic fiscal dominance as F can raise the price level and benefit from it by flooding the market with paper. In contrast, with fiscal requirements, the central bank can impose its views on the price level by buying government debt on the secondary market and defeat any price level expectations except its targeted one.

Exogenous liabilities ($L > 0$). We now tackle the case $L > 0$. We posit that the public sector is subject to a fiscal requirement $\lambda < 1$ that eliminates the type of deviations described above. Games of chicken are as follows.

After date 2. After date 2, the economy behaves as in the case $L = 0$ with fiscal requirements. There is monetary determination of the price level. More precisely, the timing inside date $t \geq 2$ is as follows. First, M announces a nominal rate $R_t = P^M_{t+1}/(\phi_t P^M_t)$. Young savers buy for an amount x_t of reserves in the market for reserves. The central bank issues new reserves $\Delta_t = P^M_t\,(x_t - x_{t-1}/\phi_{t-1}) \geq 0$. This pins down the price level to P^M_t. Then F issues $B_t = \phi_t^{-1} b_t P^M_{t+1}$ bonds in the primary market, where

$$b_t = \phi_t\,(x_{t+1} + b_{t+1}) - \delta_t - x_t, \tag{22}$$

and thus collects b_t. The bonds are traded at $Q_t = 1/R_t$. F and M are inactive in the secondary bond market. M transfers its real resources $x_t - x_{t-1}/\phi_{t-1}$ to F who pays back its date-$t-1$ bonds, and transfers any residual to young entrepreneurs.

In sum, M reaches its price-level target, and young entrepreneurs receive $\phi_t(x_{t+1} + b_{t+1}) - \delta_t$. Notice that M is indifferent about the amount x_t that private agents invest in the market for reserves (provided $x_t \in (x_{t-1}/\phi_{t-1}, \phi_t x_{t+1})$) as whichever amount left on the table is then borrowed by F in the primary market and used to repay debt and subsidize entrepreneurs the same way M would have used it.

At dates 0 and 1. We proceed by backwards induction and start from date 1, right after the market for reserves has cleared and M has set the price level at P_1. F collects

$b_1 = \phi_1 (b_2 + x_2) - \delta_1 - x_1$ in the primary bond market. At the end of the date, M transfers its entire resources to F. F uses these resources and hers to subsidize entrepreneurs for an amount $-\sigma_1 \geq 0$ and, in equilibria without default, to repay all or part of its date-1 liabilities. The public sector holds internal claims from date-0 interventions in the secondary market. Denoting $\hat{P}_1$ the date-0 anticipation of the date-1 price, F is therefore solvent if and only if

$$\frac{1}{P_1}\left[L + \frac{b_0 \hat{P}_1}{\phi_0} - \left(\frac{b_0^F \hat{P}_1}{\phi_0} + \frac{b_0^M \hat{P}_1}{\phi_0}\right)\right] \leq b_1 + x_1 - \frac{x_0 \hat{P}_1}{\phi_0 P_1} + \sigma_1, \tag{23}$$

where the left-hand side features the residual date-1 government debt (assuming there is any) comprised of the legacy liability L and of the government bonds issued at date 0, $b_0 \hat{P}_1/\phi_0$, net of F and M's respective interventions in the secondary market, $b_0^F \hat{P}_1/\phi_0$ and $b_0^M \hat{P}_1/\phi_0$ respectively.

Going one step earlier when M determines the price level at date 1, it can set it at target if (23) holds at $P_1 = P_1^M$ when $\sigma_1 = 0$. Otherwise it must set P_1 at the lowest value such that this is the case. M however prefers to reach its target P_1^M and let the government default if this latter value is larger than $P_1^M + \alpha_M$. M implements P_1 with $R_1 = P_2^M/(\phi_1 P_1)$ and $\Delta_1 = P_1(x_1 - x_0/\phi_0)$.

Consider now the situation at date 0. From the date-1 analysis, in any equilibrium without default, both F and M (and the private sector) anticipate that the date-1 price is either equal to P_M, or solves

$$\frac{L}{P_1} + \frac{b_0}{\phi_0} - \frac{b_0^F + b_0^M}{\phi_0} = b_1 + x_1 - \frac{x_0}{\phi_0}. \tag{24}$$

Equation (24), which is (23) with correct price expectations (and $\sigma_1 = 0$), is key to the strategic analysis. Other things being equal the date-1 price level P_1 is decreasing in the amount of public debt bought back by the public sector in the date-0 secondary market, $b_0^F + b_0^M$. F would like this amount to be as small as possible so as to maximize the resources σ_0 that can be paid to young entrepreneurs at date 0, while avoiding date-1 default if possible. On the contrary M would like $b_0^F + b_0^M$ to be sufficient that P_1 can be as close as possible to P_1^M, but is also willing to avert default as long as this does not entail that $P_1 > P_1^M + \alpha_M$. Suppose that M maximizes its date-0 resources:

$$x_0 = \min \{\phi_0 x_1; \lambda\phi_0(b_1 + x_1); 1\}, \tag{25}$$

and invests them all in the date-0 secondary market, whereas F sets $b_0^F = 0$. In this case, (24) implies $P_1 = \phi_0 L/[x_0 + \delta_0]$. Thus, if $x_0 \geq \phi_0 L/P_1^M - \delta_0$, M can always set $P_1 = P_1^M$ without any help from F. If $x_0 \leq \phi_0 L/(P_1^M + \alpha_M) - \delta_0$, then F must invest in the date-0 secondary market if it wants to avoid default at date 1. It does so as to maintain the date-1 price at the maximum that M finds acceptable, $P_1^M + \alpha_M$. There is however default if the total resources of the public sector do not suffice to achieve this ($b_1 + x_1 < L/(P_1^M + \alpha_M)$).

The following proposition summarizes these results.

Proposition 6. ***(Games of chicken)*** *Assume that the public sector is subject to a fiscal requirement ($\lambda < 1$). Take two scalars $x_1 > 0$ and $b_1 \geq 0$ such that $x_1 + b_1 \leq 1$. If the following condition is satisfied:*

$$b_1 + x_1 \geq \frac{L}{P_1^M + \alpha_M}, \tag{26}$$

then any game of chicken compatible with date-1 demand for public liquidity (b_1, x_1) features no default, and prices

$$P_1 = \min\left\{\max\left\{P_1^M; \frac{\phi_0 L}{x_0 + \delta_0}\right\}; P_1^M + \alpha_M\right\}, \tag{27}$$

$$\forall t \neq 1, P_t = P_t^M, \tag{28}$$

where x_0 is given by (25) and δ_0 by (19).

If (26) does not hold, there is full default on the legacy liability and $P_t = P_t^M$ for all $t \in \mathbb{N}$.

Proof. See above. □

Games of chicken can be indexed by the demand for public liquidity at date 1, (x_1, b_1). Given our definition of a game of chicken, these private demands for public liquidity cannot be affected by M nor F at date 0.

Condition (26) states that the game of chicken involves default on the legacy liability L if and only if the real resources $x_1 + b_1$ available at date 1 are too small to extinguish this liability deflated at $P_1^M + \alpha_M$, the highest price level that M is willing to tolerate to avert default. Thus, F and M reach a solvent equilibrium when it is their joint preferred option.

Expression (27) shows that M always imposes its price-level targets as soon as either $\alpha_M = 0$ or $L = 0$. The intuition is straightforward. In the former case $\alpha_M = 0$, M does not incur disutility from sovereign default and thus nor faces a tradeoff when setting the price level. In the latter situation $L = 0$, we have seen in Proposition 5 that fiscal requirements suffice to enforce monetary determination of the price level.

If $\alpha_M L > 0$, expression (27) shows that M may have to deviate from target even though i) F is infinitely averse to default, and ii) the public sector has sufficient date-1 resources $x_1 + b_1$ to sustain $P_1 = P_1^M$. This occurs when F can attract a sufficiently large quantity of liquidity at date 0. The share of date-1 resources that F cannot preempt at date 0 is comprised of the share x_0 that accrues to M in the date-0 market for reserves and (less interestingly) of the present value of the non-pledgeable date-1 resources δ_0. It is interesting to compare this result to that in Proposition 3 showing that in a non-strategic environment, the lowest date-1 price that can be reached in the absence of default, $L/(b_1 + x_1)$, depends on the entire resources $b_1 + x_1$ that the public sector can avail itself of at date 1. In the presence of strategic concerns, it is not these entire resources that matter, but only the share that F cannot preempt at date 0.

As sketched above, the reason is that the share b_0 preempted by F serves to finance a date-0 deficit from subsidizing date-0 entrepreneurs with the same amount, thereby increasing the level of government liabilities maturing at date 1. In other words, *strategic fiscal irresponsibility gives F the possibility to dictate the price level if it can preempt a sufficiently large share of total public resources. Such preempting prevents M from controlling the amount of government bonds in the hands of the private sector.*

It is important to stress that strategic fiscal irresponsibility is in essence a *dynamic strategy*. To see this, suppose that the game starts out at date 1 at which the public sector receives $x_1 + b_1$. In this case, M being first-mover, it can impose the lowest price

P_1 that is compatible with solvency and force F to then accommodate:

$$P_1 = \min \left\{ \max \left\{ P_1^M ; \frac{L}{b_1 + x_1} \right\} ; P_1^M + \alpha_M \right\} \tag{29}$$

Overall, the authority that puts its hands first on the total demand for public liquidity imposes its views.

Corollary 7. ***(The authority that preempts liquidity imposes its views)*** *Over the set of games of chicken that share the same level of public date-1 resources $b_1 + x_1$, the utility of M (given by (17)) is (weakly) increasing in x_1 whereas that of F (given by (16)) is (weakly) decreasing in it. In this sense, the authority that preempts liquidity imposes its views.*

A related insight is that if two equilibria are associated with distinct levels of date-1 total public resources $b_1 + x_1$, M may well be better off in the one with the smallest total resources if it attracts a larger fraction of them.

Proof. Straightforward from (27), as P_1 weakly decreases in x_1. □

There is monetary dominance when M has sufficient control over government bonds in the hands of the private sector. It is important to stress that the same forces that lead to fiscal dominance when $L = 0$ lead to possible date-1 inflation when $L > 0$. In both cases, M must chicken out at the next date as soon as it leaves a sufficiently large amount of bonds in the hands of the private sector relative to the future resources of the public sector. When $L = 0$, an arbitrarily small restriction to pledgeability in the form a fiscal requirement warrants monetary dominance because it suffices to eliminate any equilibrium in which the future price level is above M's future target. When the total liabilities of the public sector also feature an exogenous $L > 0$ and δ_0 is small, then M must have sufficient resources x_0 to intervene in the secondary market and take bonds off private hands. The reason these required resources x_0 are not arbitrarily small is that the nominal amount L need not be validated by the anticipations of savers as is the case with endogenously issued bonds. In both cases, F conversely seeks to maximize private holdings of bonds so as to force future inflation and thus maximize deficit spending, a behavior we deem strategic fiscal irresponsibility.

Remark. More generally, monetary dominance obtains when the central bank ensures that the net borrowing of the public sector, i.e. the gross stock of public liabilities in the hands of the private sector – reserves and government bonds – net of public savings, is sufficiently small. [10]

5 Lower bound on nominal rates

In this section, we explore the consequences of a lower bound on nominal rates on the policies conducted by the central bank.

More precisely, the potential convertibility of reserves into cash may lead to the existence of a lower bound on the interest rates on reserves (R_t). The presence of such a lower bound constrains what the central bank can achieve. To introduce such a bound on nominal rates, we make the following assumptions.

Further assumptions. First, assume that the the central bank interest rate has to satisfy:[11]

$$R_t \geq \bar{R}, \tag{30}$$

where $\bar{R} \geq 0$. Note that this constraint may bind when ϕ_t is large enough: with a constant price objective, $P_t^M = P^M$, the central bank has to set an interest rate $R_t = \phi_t^{-1}$ to reach its objective of price stability, which may violate the constraint (30) for large enough ϕ_t.

Second, assume the following restrictions on private returns: $\phi_0^{-1} \leq \bar{R}$ and $\phi_t^{-1} \geq \bar{R}$ for any $t \geq 1$. Note that this requires that the return on storage at date-0 $e^{-\eta_0}$ differs from the return at later dates. This assumption is made so that the lower bound may only bind initially. It plays the same role as preference shocks introduced in the literature on the ZLB (see Eggertsson and Woodford, 2003, among others).

Third, assume that the central bank can use the safe private storage. This allows the central bank to transfer resources across time, without relying on government bonds. Given our timing, it gives the option for the central bank to use its ressources to buy

[10] In a more general model, the central bank could also be able to save in private assets.

[11] This constraint may come from an arbitrage condition between reserves and currency. Consistently with our equilibrium definition, we require that a competitive outcome always forms, including out of equilibrium, so that (30) has to be satisfied in and out of equilibrium. See Bassetto (2004) for the explorations of situations where (30) does not hold out-of-equilibrium.

back reserves in the future (x_t^M) instead of transferring them to the government through dividends.[12]

Fourth, assume that $\alpha_M = 0$. This last assumption allows us to study the consequences of the lower bound on nominal interest rate when monetary policy is not constrained by fiscal moves.

Lower bound and a rationale for helicopter money. Under these assumptions, we obtain:

Proposition 8. *In any game of chicken with fiscal requirements, we obtain that:*

(i) *The lower bound binds at least at date 0: $R_0 = \bar{R}$;*

(ii) *$P_0 \leq P^M$ and $P_1 \geq P^M$;*

(iii) *The latter inequality is strict ($P_1 > P^M$) when $\beta < \bar{R}^{-1}\phi_0^{-1}$;*

(iv) *In this case, at date 0, the central bank does not fully use its storage technology and transfers at least part of its resources to the fiscal authority.*

Proof. See Appendix F. □

The low return for private sector investment opportunities pushes the economy to the lower bound on nominal rates on reserves (point (i)).

In the absence of such a lower bound, M would announce a sufficiently low nominal interest rate to make future reserves in line with the future price level objective. But the lower bound (30) prevents such a policy and hence introduces a potential trade-off between date-0 and date-1 price levels: as the monetary authority cannot adjust independently the amount of reserves at date 0 (X_0) and the amount of reserves held by the old generation at date 1 ($R_0 X_0$), issuing reserves at date 0 may also be inflationary at date 1. As a result, the lower bound on interest rates may lead to a price level below target at date 0 and potentially above target at date 1 (point (ii)).

Even though our model has important differences with the New-Keynesian model—we do not consider any nominal frictions among others, some of our conclusions regarding the handling of the ZLB are related. As in the New-Keynesian model, inflation between

[12] Such a use of private storage by M is immaterial in the absence of a lower bound on the nominal interest rate as the central bank never finds this option useful.

period 0 and 1 can mitigate the effects of the lower bound as, in equilibrium, the arbitrage condition requires that $P_{t+1}/(P_t R_t) = \phi_t$. This leads the central bank to desire a higher price level at date 1 relative to target (point (iii)). Given the preferences of M, this only occurs when its discount factor is sufficiently low: M only cares about the absolute deviations of the price level relative to its objective, which implies that it has no smoothing motive.

However, as for the New-Keynesian liquidity trap, this policy may not be time-consistent as the central bank may be tempted to decrease the price level to $P_1 = P_M$ ex-post. The ability of the central bank to decrease the price level depends on its resources at date 1. In the case where the central bank has some resources, it can buy back some of its reserves ($x_1^M > 0$). In our framework, a commitment device not to buy back reserves in the future and raise the price level at date 1 is to transfer the resources obtained by issuing reserves at date 0 (x_0) to the government by paying a dividend (point (iv)). In other words, in our framework, the time-consistent solution may feature helicopter money.

6 Policy implications

This section discusses the main policy implications from our analysis. Overall, our model allows to link in a common framework fiscal and monetary decisions regarding debt and reserves issuance and purchases.

Fiscal requirements for price stability are still necessary in an environment of low rates. Low rates may deliver a pleasant monetary arithmetic in which a given fiscal policy does not pin down the price level (Proposition 2). Even so, the central bank's ability to independently determine the price level is not granted. Our strategic analysis shows that the government can still force the central bank to chicken out and set future inflation above target under such pleasant arithmetic (Proposition 5). By flooding the market for debt, the government can force the central bank to increase seignorage and to transfer it to the government.

Importantly, this *strategic* motive does not disappear with low rates. It contrasts with the *dynamic* motive for fiscal requirements as put forward by Woodford (2001) among others: in this case, fiscal requirements are about avoiding the divergence of the real value of government's debt. Avoiding this divergence is also motivated by the fear of default but an important difference is that, with low rates, such a divergence is slowed down or even vanishes. As argued by Blanchard (2019), some debt may even come at no fiscal cost, thus limiting the need of fiscal requirements.

Low rates therefore do not obsolete fiscal requirements for central-bank independence such as debt or deficit caps, akin to the ones in the eurozone Maastricht Treaty.

Quantitative easing is useful for price stability, even when reserves and bonds are perfect substitutes. In our model, reserves and debt are always perfect substitutes from private agents' point of view.[13] In such a situation, central banks' interventions such as open market operations are commonly thought to be unable to affect macroeconomic outcomes (see Wallace, 1981; Chamley and Polemarchakis, 1984). For the same reason, quantitative easing is thought to have no effect, at least after interest rates on government debt have fallen to some lower bound (Woodford, 2012).

In contrast, we show that quantitative easing is still important, even when reserves and debt are always perfect substitutes, as, by changing portfolio composition, it modifies future strategic interactions between monetary and fiscal authorities. A key element behind this result is that, when government debt is held by the central bank, the central bank can still manipulate the price level, without triggering a default by the government: the central bank can always pay a dividend so as to satisfy the government's budget constraint using part or all the resources that the government owes to the central bank due to bonds held by the central bank.

In particular, in our framework, these interventions to ensure some control on future price levels are important off-equilibrium in Proposition 5 – to defeat undesired levels of borrowing – and in equilibrium in Proposition 6 in response to the exogenous liability L to limit the amount of nominal liabilities in circulation – or more generally to limit the net borrowing of public authorities – and ensure that the price level is in line with the objective (P_1^M).

Massive balance-sheet expansion is not *per se* a symptom of a central bank chickening out. Major central banks have responded to the 2008 and COVID-19 crises

[13] In the case where debt and reserves are not always substitutes, there is a clear role for quantitative easing, including in the case where this absence of substitutability comes from future differences between government's debt and reserves or money as in Auerbach and Obstfeld (2005) or Gali (2020) among others. See Bernanke (2000) among others for the policy proposal to use money financed deficits to alleviate the effects of a liquidity trap.

with large issuances of reserves invested in government bonds and private securities from the most distressed corners of the private sector. In our model, such date-0 balance-sheet expansions are not the consequence of the monetary authority chickening out. Quite the contrary, it is evidence that the central bank preempts a large fraction of private liquidity demand and uses it for reducing the amount of government debt in the hands of the private sector. This makes it in a stronger position, conditionally on a large shock to the economy, to impose a low price level and force fiscal consolidation at date 1.

In light of our model, an actual predictor of future inflation is not the size of the central bank's balance-sheet, but rather the net increase in the date-0 amount of government bonds held by the private sector—the amount issued in the primary market net of the purchases from the private sector by the central bank in the secondary market. This net increase is large if the fiscal authority preempts a large fraction of private liquidity demand (large b_0 relative to x_0), and this leads to date-1 inflation.

In sum, a central bank that massively issues reserves and keeps the amount of public debt and bailable private liabilities (L) in the hands of the private sector under control is not chickening out in our model. It preempts a lot of liquidity (x_0 large relative to b_0) in order to get ready to impose its views to the fiscal authority in the future.

Negative interest rates with large issuance of reserves are the first-best policy tool to address liquidity traps. Our model also has implications for liquidity traps. The central bank can always accommodate these traps by issuing important quantities of reserves and at the same time by setting potentially negative interest rates on these reserves. Overall, negative interest rates are a tool for the central bank to adjust the quantity of nominal public liabilities in the hands of the private sector.

More precisely, such a liquidity trap can be captured by a very low return on the private sector's technology (for example ϕ_0^{-1} very low). In such a situation, an equilibrium exists where the current demand for money x_0 can be large in comparison with future demand x_1 and the condition $x_0 \leq \phi_0 x_1$ still holds. In this view, date 0 is the period of the liquidity trap and date 1 is the post-crisis period.

The proof of Proposition 6 shows that, despite this pattern for the demand for reserves, the central bank is still able to implement its desired path for the price level. In the case where its target price level is constant ($P_t^M = P_M$), this requires to issue a large amount of reserves $X_0 = P_M x_0$ and to set an interest rate $R_t = \phi_0^{-1}$. In particular, when ϕ_0^{-1} is low enough, R_t can be below 1, which corresponds to negative interest rates.

In our benchmark model, negative interest rates are feasible as there are no arbitrage condition with no-interest bearing assets such as cash that would lead to a lower bound on what the monetary authority can set on its reserves. In line with Rogoff (2017), our model also suggests that avoiding the convertibility of reserves into no-interest bearing cash is desirable.

A lower bound on the interest rate gives a motive for the central bank to inflate in the future. However, potential arbitrage with cash, even imperfect, may lead to a lower bound on nominal interest rates. We show that introducing such a lower bound on interest rate leads to a potential motive for the central bank to increase inflation at date 1 above its price-level objective.

A potential collateral effect is that the central bank may also inflate the government's debt, thus relaxing the government budget constraint at date 1. In a way, the lower bound aligns the interests of the central bank and that of the government which then both desire a higher price level at date 1, but for different reasons.

Helicopter money allows the central bank to inflate in the future while maintaining its independence. The time-consistent solution to inflate the economy by the central bank implies to issue reserves and to make sure that the central bank will not have the internal resources to buy back reserves in the future. A way to do this is helicopter money, which amounts to issuing reserves (X_t) followed by a transfer to the government (θ_t).

Instead of helicopter money, the central bank can also let the government issue debt at date 0 and chicken out at date 1, due to the large stock of nominal debt in the hands of the private sector. However, helicopter money allows the central bank to finely tailor its incentives to inflate in the future in accordance with the price stability objective —the central bank issues its desired amount of reserves and makes the corresponding transfer, while the issuance of debt by the government can force the central bank to inflate in the future in an uncontrolled way as in the game of chicken.

Central banks should not be allowed to intervene in primary government debt markets. Central banks are usually required not to buy government bonds in the primary market. They are constrained to buy them from banks in the secondary market. Such a requirement applies to the ECB and the Fed only trades US treasuries through primary dealers. Our model offers a rationale for this as a device that makes strategic fiscal irresponsibility more difficult. Governments may thus establish such a rule as a commitment device if they find ex-post strategic irresponsibility undesirable ex-ante because it undermines central banks' credibility at stabilizing the price level.

To see this, note that the assumed timing in our setup, according to which the monetary authority can intervene in the secondary bond market after the fiscal one has issued bonds in the primary market stacks the deck in its favor. Prohibiting primary-market bond transactions between governments and central banks is a way to make sure that the central bank can effectively control the amount of public liabilities in the hands of the private sector and avoid governments to get direct funding from the central bank seeking to invest its reserves this way.

7 Concluding remarks

This paper studies fiscal and monetary interactions in an environment in which a high demand for liquidity storages enables the public sector to issue unbacked liabilities. An analysis of feasible policies suggests that this environment is prima facie favorable to central-bank independence. First, the tight interdependence between fiscal surpluses and price level induced by an intertemporal budget constraint vanishes when such a constraint is no longer a necessary condition for equilibrium. In particular, the monetary authority can control the price level without relying on any resources from the fiscal authority nor even on government bonds as safe stores of value.

Our main contribution is to go beyond the mere studies of feasible policies and assess central-bank independence in a setup in which it matters because the fiscal authority has no commitment to nor interest in price stability. The diagnostic from this more relevant analysis is much more pessimistic. We find that the independence of the central bank crucially relies on its ability to keep the amount of government debt in the hands of the private sector under control. This ability is nonexistent in the absence of fiscal requirements. In the presence of a negative fiscal shock, the central bank expands its balance sheet right away so as to avoid inflating away privately held debt in the future. This is effective only insofar as it preempts a sufficiently large fraction of private demand for public liquidity.

Our setup is tractable and yet sufficiently rich to capture many aspects of fiscal and monetary interactions. There are several interesting avenues for future research. First, we could carry out a similar analysis when the monetary arithmetic is unpleasant. In this case, public liabilities must be backed by taxes and seigniorage accruing to each authority. We conjecture that the monetary authority can keep the price level under control in the game of chicken only if seigniorage is sufficiently high relative to government debt. Second, a stochastic version of the game of chicken could yield interesting insights into the effect of shocks to the economy on the prevalence of fiscal and monetary regimes.

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Appendix - For online publication

A Proof of Proposition 1

We proceed in three steps. We first show that the conditions for the feasibility of simple policies stated in the proposition are necessary and sufficient. We then prove that the allocation achieved by a feasible policy is also achieved by a simple policy. We finally show that a feasible policy is associated with a unique sequence $(P_t, Q_t)_{t\in\mathbb{N}}$.

Conditions {(8);(9);(10);(11);(12);(13)} characterize simple policies. We start by showing that these conditions are necessary. Suppose that the simple strategy

$$(\sigma_t, \tau_t, \theta_t, B_t, X_t, R_t)_{t\in\mathbb{N}} \cup \{b_0^M\}$$

is feasible. Condition (8) has to be satisfied because entrepreneurs can always claim a zero-return so as to avoid taxation, and each date-t saver can always claim that her endowment is $\bar{\tau}_t$. Feasibility implies that there exists a sequence of prices $(P_t, Q_t)_{t\in\mathbb{N}}$ so that private agents are indifferent between storage options, which leads to condition (9). Real private savings $X_t/P_t + Q_tB_t/P_t - \mathbb{1}_{\{t=0\}}b_0^M$ have to be lower than after-tax income $1-\tau_t$, which yields condition (10). Condition (11) is simply a feasibility condition for the buyback of date-1 debt at date 0. Conditions (12) and (13) are the fiscal and monetary authorities' budget constraints.

We now show that these conditions are sufficient. Consider a simple policy

$$(\sigma_t, \tau_t, \theta_t, B_t, X_t, R_t)_{t\in\mathbb{N}} \cup \{b_0^M\}$$

such that there exists a price vector $(P_t, Q_t)_{t\in\mathbb{N}}$ so that conditions (8), (9), (10), (11), (12), (13) are satisfied. We need to show that this implies that this policy is associated with a competitive equilibrium. Denoting $z_t = X_t/P_t$ and $w_t = Q_tB_t/P_t$, this policy is actually associated with a competitive equilibrium whereby savers invest w_t in the date-t primary market for bonds and z_t in that for reserves. By construction, such z_t and w_t ensure that the primary bond market and the market for reserves clear. Condition (10) ensures that such trades are feasible and (9) that private agents find them optimal.

Any feasible allocation can be implemented with simple policies. Consider a feasible policy $\mathcal{P} = (\sigma_t, \tau_t, \theta_t, B_t, X_t, x_t^F, b_t^M, b_t^F, R_t)_{t\in\mathbb{N}}$. Since the policy is feasible, there exists a sequence of prices $(P_t, Q_t)_{t\in\mathbb{N}}$ consistent with a competitive equilibrium. Consider now the simple policy $\mathcal{P}' = (\sigma_t, \tau_t, \hat{\theta}_t, \hat{B}_t, \hat{X}_t, R_t)_{t\in\mathbb{N}} \cup \{\hat{b}_0^M\}$ defined as follows:

$$\hat{\theta}_0 = \theta_0 - b_0^F - x_0^F, \tag{31}$$

$$\hat{\theta}_1 = \theta_1 + b_1^M + \frac{P_0}{Q_0 P_1} b_0^F - x_1^F + \frac{R_0 P_0}{P_1} x_0^F, \tag{32}$$

$$\forall t \geq 1,\ \hat{\theta}_t = \theta_t + b_t^M - \frac{P_{t-1}}{Q_{t-1} P_t} b_{t-1}^M - x_t^F + \frac{R_{t-1} P_{t-1}}{P_t} x_{t-1}^F, \tag{33}$$

$$\hat{B}_t = B_t - \mathbb{1}_{\{t>0\}} \frac{P_t}{Q_t}(b_t^F + b_t^M), \tag{34}$$

$$\hat{X}_t = X_t - x_t^F P_t, \tag{35}$$

$$\hat{b}_0^M = b_0^M + b_0^F. \tag{36}$$

In words, the simple transfers $\hat{\theta}$ add to the original ones θ the flows from investing and collecting repayments in bonds and reserves by M and F ((31),(32),(33)). The simple bond issuance after date 0 is the original one net of public interventions in the subsequent secondary bond market ((34)). The simple stock of reserves is the original one net of purchases by F ((35)), and the initial simple intervention of M in the secondary market is the aggregate one of M and F in the original policy ((36)).

We first show that this simple policy is feasible and can be associated with the same $(P_t, Q_t)_{t\in\mathbb{N}}$ as the original one. From the above characterization it suffices to show that

$$\frac{\hat{X}_t}{P_t} + \frac{Q_t \hat{B}_t}{P_t} \leq 1 - \tau_t + \mathbb{1}_{\{t=0\}} \hat{b}_0^M, \tag{37}$$

$$P_0 \hat{b}_0^M \leq Q_0(\hat{B}_0 + L), \tag{38}$$

$$\frac{Q_t \hat{B}_t - (\hat{B}_{t-1} + \mathbb{1}_{\{t=1\}} L)}{P_t} = -\sigma_t - \tau_t - \hat{\theta}_t, \tag{39}$$

$$\frac{\hat{X}_t - R_{t-1}\hat{X}_{t-1}}{P_t} + \frac{P_0}{Q_0 P_1} \mathbb{1}_{\{t=1\}} \hat{b}_0^M = \hat{\theta}_t + \mathbb{1}_{\{t=0\}} \hat{b}_0^M. \tag{40}$$

The respective budget constraints of F and M, (39) and (40), hold because using equations (31) to (36) to express all the simple policy variables in terms of the original policy variables yields the budget constraints of the original policy, (5) and (6). Equation (38) stems from the feasibility of the buybacks in the original policy ($P_0(b_0^M + b_0^F) \leq Q_0(B_0 + L)$)

and the definition of $\hat{b}_0^M$. That (37) holds stems again from $X_t/P_t + Q_t B_t/P_t \leq 1 - \tau_t + \mathbb{1}_{\{t=0\}}(b_t^M + b_t^F)$ because the original policy is feasible.

It remains to show that the private sector receives the same allocation under this simple policy as under the original one. Since σ_t and τ_t are unchanged, it suffices to show that savers invest the same quantities in bonds and reserves. The policy $\mathcal{P}'$ deduces the buybacks from the original policy $b_t^F + b_t^M$ from newly issued debt for any $t > 0$, and leaves the secondary market inactive, and so the net private bond holdings compatible with the policy are identical to the ones associated with the original policy at these dates. At $t = 0$, the primary issuance is unchanged ($\hat{B}_0 = B_0$) but the monetary authority buys back the equivalent $b_0^M + b_0^F$ of buybacks by the public sector in the original policy. Again this implies that private savers invest the amount b_0 of the original policy in bonds. Second, clearing the market for reserves imposes for the original policy $\mathcal{P}$:

$$R_{t-1}X_{t-1} + \Delta_t = P_t(x_t + x_t^F + x_t^M).$$

Letting

$$Y_t = \Delta_t + R_{t-1}P_{t-1}x_{t-1}^F - P_t(x_t^M + x_t^F),$$

we define

$$\begin{aligned}
\hat{\Delta}_t &= \mathbb{1}_{\{Y_t>0\}}Y_t, \\
\hat{x}_t^M &= -\mathbb{1}_{\{Y_t\leq 0\}}\frac{Y_t}{P_t}.
\end{aligned}$$

We have $\hat{X}_t = \hat{\Delta}_t - P_t\hat{x}_t^M$, and the market for reserves associated with the simple policy clears with the same private investment x_t as with the original policy.

A feasible policy is associated with a unique sequence $(P_t, Q_t)_{t\in\mathbb{N}}$. A policy includes all the nominal interest rates and hence inflation due to the equilibrium condition (9). Only the initial price level P_0 remains to be determined. Equation (5) leads to a unique price level P_0. Finally, equilibrium condition (9) leads to a unique sequence of bond prices $\{Q_t\}_{t\in\mathbb{N}}$.

B Proof of Proposition 2

We denote D_t the aggregate public liabilities held by the private sector: $D_t = R_t X_t + B_t - \mathbb{1}_{\{t=0\}} \frac{P_0}{Q_0} b_0^M$. Combining (12), (13), and (9), we obtain that:

$$\mathbb{1}_{\{t=1\}} \frac{L}{P_1} + \frac{D_{t-1}}{P_t} = \sigma_t + \tau_t + \phi_t \frac{D_t}{P_{t+1}}.$$

Case: $T > \eta/\rho$. In this case, at date 1:

$$\frac{L + D_0}{P_1} = \sigma_1 + \tau_1 + \sum_{t \geq 2} \left(\Pi_{t'=1}^{t-1} \phi_{t'}\right) (\sigma_t + \tau_t) \,. \tag{41}$$

as the sum is well defined. In addition, we have at date 0:

$$\frac{\phi_0 D_0}{P_1} = -(\sigma_0 + \tau_0)$$

Multiplying (41) by ϕ_0 and simplifying using the equation above, we obtain (14).

Finally, suppose $\theta_t \geq 0$. Condition (12) imposes:

$$\frac{X_t}{P_{t+1}} = -\phi_{t+1}\theta_{t+1} + \phi_{t+1}\frac{X_{t+1}}{P_{t+2}} = -\sum_{t'=t+1}^{\infty} \left(\Pi_{\tau=t'}^{t+1} \phi_\tau\right) \theta_{t'},$$

which shows that X_t cannot be strictly positive if transfers from M to F are always positive (that is, $\theta_t \geq 0$).

Case: $T \leq \eta/\rho$. Let us show that a simple feasible policy exists such that $X_t > 0$ and $\theta_t \geq 0$ with strict inequality at date 0. As $T \leq \eta/\rho$, there exists a sequence $\{x_t\}_{t\geq 0}$ such that $1 \geq x_t \geq \phi_{t-1}^{-1} x_{t-1}$ with $x_0 > 0$. Fix $P > 0$, a simple feasible policy is then:

$$\sigma_t = \tau_t = 0 \text{ and } X_t = x_t P$$
$$B_t = b_0^M = 0 \text{ and } R_t = \phi_t^{-1}.$$

As $x_0 > 0$, setting $X_0 > 0$ leads to $P_t = P < \infty$ for any date $t \geq 0$. At date 1, the government can repay $(1-h)L/P \leq \theta_1 = x_1 - \phi_0^{-1} x_0$.

Consider a simple feasible policy $(\sigma_t, \tau_t, \theta_t, B_t, X_t, R_t)_{t\in\mathbb{N}} \cup \{b_0^M\}$ such that $X_0 > 0$ and $\theta_0 > 0$. There exists a price vector $(P_t, Q_t)_{t\in\mathbb{N}}$ such that the conditions of Proposition 1 are satisfied. Consider the price vector $(P'_t, Q'_t) = (P_t(1+\bar{P}), Q_t)_{t\in\mathbb{N}}$, with $\bar{P} > 0$. This price vector still satisfies (9). Let us build a simple policy $(\sigma_t, \tau_t, \theta'_t, B'_t, X'_t, R'_t)_{t\in\mathbb{N}} \cup \{(b_0^M)'\}$ that is feasible with $(P'_t, Q'_t)_{t\in\mathbb{N}}$.

To this purpose, let us consider $B'_t = B_t(1+\bar{P})$ and $X'_t = X_t(1+\bar{P})$. For $t \geq 2$, we set $\theta'_t = \theta_t$, and $R'_t = R_t$ for $t \geq 1$ so that conditions (11), (12) and (13) are satisfied for any $t \geq 2$ and (10) is satisfied for any t.

So we only need to check that conditions (11), (12) and (13) can be satisfied for dates 0 and 1. To this purpose, we consider $\theta'_1 = \theta_1$, $(b_0^M)' = b_0^M + (\frac{Q_0}{P'_0} - \frac{Q_0}{P_0})L < b_0^M$ and $\theta'_0 = \theta_0 + (\frac{Q_0}{P'_0} - \frac{Q_0}{P_0})L < \theta_0$. Finally $R'_0 = R_0$.

By construction, $(b_0^M)'$ satisfies (11) and its value also makes sure that (12) is satisfied at dates 0 and 1. The only difference with the initial policy is then that $X'_0/P'_0 = \theta'_0 < \theta_0$, so this equilibrium corresponds to a lower $x'_0 < x_0$. This is possible: if there exists a sequence $\{x_t\}_{t\geq 0}$ such that

$$x_t - \phi_{t-1}^{-1} x_{t-1} \geq 0,$$

the sequence $\{x'_t\}_{t\geq 0}$ exists as well where $x'_t \leq x_t$.

C Proof of Proposition 3

Let us consider a sequence $(x_t, b_t)_{t\in\mathbb{N}}$ that admits compatible policies. This means that there exists at least one policy $((\sigma_t, \tau_t, \theta_t, B_t, X_t, R_t))_{t\in\mathbb{N}} \cup \{b_0^M\}$ that is feasible with a sequence of prices $(P_t, Q_t)_{t\in\mathbb{N}}$ and such that $X_t/P_t = x_t$ and $Q_t B_t/P_t = b_t$.

Using the conditions of Proposition 1, this is equivalent to:

$$\begin{aligned}
&\sigma_t \leq 0, \tau_t \leq \bar{\tau}, \\
&x_t + b_t \leq 1 - \tau_t + \mathbb{1}_{\{t=0\}} b_0^M \leq 1, \\
&\frac{P_{t+1}}{R_t P_t} = \frac{Q_t P_{t+1}}{P_t} = \phi_t, \\
&b_t - \phi_{t-1}^{-1} b_{t-1} - \mathbb{1}_{\{t=1\}} \frac{L}{P_t} = -\sigma_t - \tau_t - \theta_t, \\
&b_0^M \leq b_0 + \mathbb{1}_{t=0} \frac{\phi_0}{P_1} L, \\
&x_t - \phi_{t-1}^{-1} x_{t-1} + \mathbb{1}_{\{t=1\}} \phi_0^{-1} b_0^M = \theta_t + \mathbb{1}_{\{t=0\}} b_0^M.
\end{aligned}$$

Necessity. Plugging the last equation into the previous one, we obtain that:

$$(b_t + x_t) - \phi_{t-1}^{-1}(b_{t-1} + x_{t-1}) = -\sigma_t - \tau_t + \mathbb{1}_{\{t=1\}} \frac{L}{P_t} + \mathbb{1}_{\{t=0\}} b_0^M - \mathbb{1}_{\{t=1\}} \phi_0^{-1} b_0^M.$$

For any $t \geq 2$ with $\bar{\tau} = 0$, the right hand side of this equality is positive, which implies that $\phi_{t-1}^{-1}(b_{t-1} + x_{t-1}) \leq (b_t + x_t)$. We also have that $x_t + b_t \leq 1$.

At date 1 and date 0, we obtain:

$$b_1 + x_1 - \phi_0^{-1}(b_0 + x_0) = -\sigma_1 - \tau_1 + \frac{L}{P_1} - \phi_0^{-1} b_0^M. \text{ and } b_0 + x_0 = -\sigma_0 - \tau_0 + b_0^M.$$

As $\frac{\phi_0 L}{P_1} + b_0 \geq b_0^M$, hence $b_1 + x_1 - \phi_0^{-1} x_0 \geq 0$ and $b_0 + x_0 \geq 0$.

Sufficiency. Now, let us show that Lemma 3 provides a sufficient condition for admitting compatible policies. Let us assume that there exists a sequence $(x_t, b_t)_{t \in \mathbb{N}}$ such that $\phi_{t-1}^{-1}(b_{t-1} + x_{t-1}) \leq (b_t + x_t)$, $x_t + b_t \leq 1$, $x_t \geq 0$, and $b_t \geq 0$ at any date $t \geq 0$.

We can build $\theta_t = x_t - \phi_{t-1}^{-1} x_{t-1}$ and σ_t for any $t \geq 2$:

$$\phi_{t-1}^{-1}(b_{t-1} + x_{t-1}) - (b_t + x_t) = \sigma_t.$$

As $\bar{\tau} = 0$, we select $\tau_t = 0$. As $\phi_{t-1}^{-1}(b_{t-1} + x_{t-1}) \leq (b_t + x_t)$, we have that $\sigma_t \leq 0$.

Take $b_0^M = 0$. At date 1, there exists a price P_1 and a default rate h so that $\sigma_1 \leq 0$ and the following conditions are satisfied:

$$\phi_0^{-1}(b_0 + x_0 - b_0^M) - (b_1 + x_1) + \frac{(1-h)L}{P_1} = \sigma_1$$
$$- b_0 - x_0 = \sigma_0.$$

In particular, with $h = 1$, $\hat{P}_1$ can take any positive value and $\sigma_0, \sigma_1 \leq 0$.

If $h = 0$, we need to have:

$$\frac{L}{\hat{P}_1} \leq (b_1 + x_1) - \phi_0^{-1}(b_0 + x_0 - b_0^M) = (b_1 + x_1) + \phi_0^{-1}\sigma_0 \leq b_1 + x_1$$

So that $\hat{P}_1 \geq L/(b_1 + x_1)$. Notice that the minimum price level at date 1 is implicitly a function of b_0^M.

Select an arbitrary sequence for nominal rate $(R_t)_{t \in \mathbb{N}}$. We can then infer a sequence of prices $(P_t, Q_t)_{t\in\mathbb{N}}$ such that the price level at date-1 satisfies $P_1 = \hat{P}_1$ and Condition (9) is satisfied.

D Proof of Lemma 4

Suppose that a game of chicken is such that $x_{t_0} > 0$ for some t_0. Since the only inflows in the market for reserves are from the private sector, it must be that

$$x_{t+1} \geq \frac{x_t}{\phi_t} > 0 \tag{42}$$

for all $t \geq t_0$. Let $T = \inf\{t \in \mathbb{N} \mid x_t > 0\}$. Suppose that $T > 0$. M is better off announcing $R_{T-1} = P_T/(\phi_{T-1}P_{T-1})$ and issuing reserves $\Delta_{T-1} = P_{T-1}\min\{\phi_{T-1}x_T; \lambda\phi_{T-1}(b_T + x_T); 1\}$ at date $T-1$, thereby leading to $x_{T-1} > 0$, a contradiction. The reason is M can at least transfer these resources x_{T-1} to F, and this does not affect the continuation game since F would have raised them by issuing more debt anyway.

E Proof of Proposition 5

Lemma 9. *Suppose a game of chicken is such that $x > 0$. For a given $t \in \mathbb{N}$, suppose that the respective price levels at dates t and $t+1$ are P_t and P_{t+1}, respectively. Then M announces at date t a rate on reserves $R_t = P_{t+1}/(\phi_t P_t)$ and issues reserves $\Delta_{t+1} = (x_{t+1} - x_t/\phi_t)P_{t+1} \geq 0$ at date $t+1$. At date 0, M issues $\Delta_0 = P_0 x_0$.*

Proof. The rate R_t is imposed by savers' indifference condition. For all $t \in \mathbb{N}$ the date-t+1 newly issued reserves lead to the market-clearing price P_{t+1} from

$$R_t X_t + \Delta_{t+1} = P_{t+1}x_{t+1}. \tag{43}$$

□

Lemma 9 merely shows how M behaves in equilibrium for a particular sequence of price levels using the issuance of reserves and the interest rate on it.

Lemma 10. *Suppose a game of chicken is such that $x > 0$. At every $t \in \mathbb{N}$, F borrows the maximum amount $b_t = \phi_t(b_{t+1} + x_{t+1}) - \delta_t - x_t$ in the primary bond market.*

Proof. F has no reason to leave money on the table as it can at least frontload subsidies to young entrepreneurs by borrowing as much as possible, because $\beta < e^{-\rho}$. $\square$

Suppose now that for some $t \in \mathbb{N}, \delta_{t-1} = \delta_t = 0$. We show by contradiction that $P_{t+1} = P_{t+1}^M + \alpha_M$. Suppose otherwise that the game of chicken is such that M announces $R_t = \hat{P}_{t+1}/(\phi_t P_t)$, where $\hat{P}_{t+1} < P_{t+1}^M + \alpha_M$, and issues $\Delta_t = (x_t - x_{t-1}/\phi_{t-1})P_t$ whatever P_t is. In this case, F can inflate away reserves at date $t+1$ by issuing in the date-t primary bond market a number of bonds equal to

$$\left(\phi_t(b_{t+1} + x_{t+1}) - \frac{x_t \hat{P}_{t+1}}{\phi_t(P_{t+1}^M + \alpha_M)}\right)(P_{t+1}^M + \alpha_M) \tag{44}$$

that will force M to set the date-$t+1$ price level at $P_{t+1}^M + \alpha_M$. Notice that M cannot buy any of these bonds back from savers in the date-t secondary market because $\delta_{t-1} = 0$ and Lemma 10 implies that F has optimally borrowed a maximum real amount $\phi_{t-1}(x_t + b_t) - x_{t-1}$ at date $t-1$, thereby forcing M to use its entire date-t resources to repay reserves and bonds issued at date $t-1$. Thus it is rational for savers to invest $b_t = \phi_t(b_{t+1} + x_{t+1}) - x_t\hat{P}_{t+1}/[\phi_t(P_{t+1}^M + \alpha_M)]$ in the primary bond market because M will have to set the price level at $P_{t+1}^M + \alpha_M$ in the date-$t+1$ market for reserves. To see this, notice that in the absence of default, P_{t+1} is the smallest price level such that:

$$\left(\phi_t(b_{t+1} + x_{t+1}) - \frac{x_t \hat{P}_{t+1}}{\phi_t(P_{t+1}^M + \alpha_M)}\right)(P_{t+1}^M + \alpha_M) + \frac{\hat{P}_{t+1} x_t}{\phi_t} \leq P_{t+1}(x_{t+1} + b_{t+1}). \tag{45}$$

The left-hand side of (45) is the (nominal) value of the liabilities of the public sector at date $t+1$, and the right-hand-side that of its real resources. Thus, P_{t+1} must be equal to $P_{t+1}^M + \alpha_M$, the maximum price level that M is willing to implement rather than let F default. F has benefitted from imposing such a maximum date-t+1 price level because it inflated away the value of X_t, and so the equilibrium must be such that $P_{t+1} = P_{t+1}^M + \alpha_M$: F dictates the price level.

When $\delta_{t-1} + \delta_t > 0$, there are two ways this deviation by F can go wrong and be eliminated. First if M has some resources to invest in the date-t secondary bond market, then it can defeat any expectation about a date-$t+1$ price-level above $\hat{P}_{t+1}$ in the date-t primary bond market by buying some arbitrarily small amount of debt from savers in the secondary market so that (45) is strict when $P_{t+1} = P_{t+1}^M + \alpha_M$, because only the debt that is in the hands of the private sector matters for solvency. The internal claim of M on F can be settled with a netting from the transfer that M would have made to F anyway with the repayment. M has resources to invest in the date-t secondary market this way if $\delta_{t-1} > 0$. In this case F could not borrow against the entire value $\phi_{t-1}(b_t + x_t) - x_{t-1}$ in the date-$t-1$ primary market and so M has free resources to invest in the secondary bond market provided $x_t > x_{t-1}/\phi_{t-1}$, which M can ensure at date $t-1$ at no cost.

Second, if $\delta_t > 0$, then this means that F cannot borrow against the entire value of $\phi_t(b_{t+1} + x_{t+1}) - x_t$ in the date-t primary market, in which case (45) holds at strictly smaller price levels than $P^M_{t+1}+\alpha_M$, thereby defeating the deviation, or any other deviation whereby $P_{t+1} > \hat{P}_t$.

This shows that the game of chicken must be such that $P_{t+1} = P^M_{t+1}$ whenever $\delta_{t-1} + \delta_t > 0$, which holds across all games of chicken at all dates provided $\lambda < 1$, and only occasionally when $\lambda = 1$.

A final remark is that a source of equilibrium multiplicity stems from the fact that M is indifferent across all values of $x_t \in (x_{t-1}/\phi_{t-1}, \phi_t x_{t+1})$. The reason is that whichever amount M leaves on the table in the market for reserves is picked up by F in the subsequent primary bond market, and F and M agree on how to use it given the price level.

F Proof of Proposition 8

Let us consider an equilibrium in which $\phi_t^{-1} x_t \leq x_{t+1}$ for all t and such that $\phi_0^{-1} \leq \bar{R}$. First, as $\alpha_M = 0$, the monetary authority only cares about its price-level objective. At date 0, the objective of the central bank is:

$$U_0^M = -\sum_{t' \geq 0} \beta^{t'} (\mid P_{t'} - P^M \mid + \alpha_M \Delta_{t'})$$

At date 1, P_1 cannot be lower than P^M. Otherwise the monetary authority issues new reserves so as to raise the price level to P^M. At date 0, the price level cannot be larger than P^M. Otherwise, the monetary authority issues less reserves at date 0. This comes at no cost at date 1, as the monetary authority can issue reserves then as well. Moreover, $P_0 = P^M$ and $P_1 = P^M$ is not a possible outcome either as it is not consistent with any interest rate $R_t \geq \bar{R}$ and the arbitrage relation (9).

For a given $P_1 \geq P^M$, the best response of the monetary authority to optimize the price level is:

$$P_0 = \min \left\{ P_1 \bar{R}^{-1} \phi_0^{-1} ; P^M \right\}$$

If $P_0 = P^M$, $R_t \geq \bar{R}$. To reach this price level, the central bank issues $X_0 = P_0 x_0$.

Let $\epsilon > 0$ be a positive real number. With $P_1 = P^M + \epsilon$, $P_0 = (P_1 + \epsilon)\bar{R}^{-1}\phi_0^{-1}$ and the date 0 objective function becomes

$$\beta\epsilon + \mid (P^M + \epsilon)\bar{R}^{-1}\phi_0^{-1} - P_M \mid$$

When $\bar{R}^{-1}\phi_0^{-1} > \beta$, the monetary authority is strictly worse off when $\epsilon = 0$. Otherwise, $\epsilon = 0$ is optimal. Importantly, if the central bank inflates at date 1 above P^M, it has to adjust R_1 so that $P^M/((P^M + \epsilon)R_1) = \phi_1$. As long as ϵ is sufficiently small, $R_1 \geq \bar{R}$ and the central bank can still achieve $P_2 = P^M$.

Suppose, finally, that the central bank fully uses its storage technology to transfer the resources collected at date 0 (x_0) to date 1. This allows the central bank to have at least $e^{-\eta} = \phi_0^{-1} x_0$ at date 1, which is sufficient to buy the whole stock of reserves $R_0 X_0$ and then to implement a price level P^M, thus contradicting that the central bank at 0 optimally prefers to have a price level P_1 strictly above P^M.