A State Theory of Price Levels*

Jean Barthélemy Eric Mengus Guillaume Plantin

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Abstract

This paper studies whether public financial policy, defined as the collection of transfers and trades of money between public and private sectors, can determine the price level. In an economy in which all agents are free to set the prices at which they privately trade money for goods with each other, we identify policies that elicit a single equilibrium price level. For policies that fail to do so, for example because different official and unofficial prices may coexist in equilibrium, we still offer tight restrictions on the set of predictable price levels.

*Barthélemy: Banque de France, 31 rue Croix des Petits Champs, 75001 Paris, France. Email: jean.barthelemy@banque-france.fr. Mengus: HEC Paris and CEPR, 1 rue de la Libération, 78350 Jouy-en-Josas, France. Email: mengus@hec.fr. Plantin: Sciences Po and CEPR, 28 rue des Saints-Pères, 75007 Paris, France. Email: guillaume.plantin@sciencespo.fr. We thank participants in various seminars and in particular Manuel Amador, Marco Bassetto, Michael Choi, Christopher Phelan, Xavier Ragot, Guillaume Rocheteau, and Gilles Saint-Paul for very helpful discussions. Errors are ours. The views expressed in this paper do not necessarily reflect the opinion of the Banque de France or the Eurosystem.

1 Introduction

This paper studies the ability of a state to control the value of its legal tender. We specifically seek to identify the circumstances under which public financial policy alone, defined as the transfers and trades of money between the state and the private sector, suffices to determine the price level. We are also interested in identifying the predictable outcomes in situations in which the state fails to determine the price level this way.

There are several obvious channels through which public financial policy affects the price level. First, public sectors use like other agents money as a medium of exchange for goods and assets. Central banks in particular trade the money that they issue for other stores of value. Some of the prices at which such trades settle serve as important nominal anchors, and may thus be explicit targets for monetary policy. Historical and contemporaneous examples abound, including metallic standards, currency pegs, or the targeting of some short-term interest rates. Second, fiscal policy creates private tax liabilities that may also affect the determination of the price level, be it only because money is legal tender for these liabilities (e.g., Lerner, 1947; Smith, 1776; Starr, 1974). Finally, that public sectors can issue money in order to make good on their other (explicit and implicit) liabilities, such as sovereign debt, creates another possible connection between the value of money and the financial decisions of the issuing state (Sargent and Wallace, 1981).

Does such a collection of trades, and negative and positive transfers of money suffice to determine the price level? This paper studies this old question in an economy that has a novel, and we believe essential, feature: All agents are free to privately trade money for goods with each other at whichever prices they agree upon. Namely, subject to a natural collateral constraint, all agents including the state are free to create trading posts in which they can trade with each other. This unfettered trade is not subject to a cash-in-advance restriction in the sense that agents can act as leveraged intermediaries across such endogenously created trading posts, bidding in one post the expected proceeds from another one. A public financial policy determines the price level in this economy if all equilibrium trades occur at the same price.

In other words, we study whether the state can determine the price level without imposing any form of explicit or implicit trading ban nor price control on society. We think that this is the appropriate environment to study price-level determination. This enables us to separate out the pure contribution of public financial policy to the determination of the price level from that of other features of social interactions, such as restrictions on trading. A pervasive restriction on trading protocols that we depart from in particular is the assumption of a Walrasian environment. It imposes *de facto* strong restrictions on trade, leaving agents, including the state, with no option but to submit demand schedules in a centralized official market. Enforcing such extreme restrictions to exchange seems in fact out of reach in even the most controlled societies. To be sure, Walrasian markets are a natural frictionless limit when agents with standard objectives trade standard economic goods. We will show that it no longer need be the case when one good, money, is desirable to private agents only insofar as the state trades it, and accepts it as legal tender for the tax liabilities of its own making.

We carry out our analysis in an environment that is fully strategic, and we use Nash equilibrium as our concept of predictable outcome. Unlike with Walrasian environments, this enables a distinction between on one hand the policies that are feasible, and on the other hand the policies that determine the price level among these feasible ones. As we will show, some feasible policies that fail to determine the price level have interesting and plausible properties such as the coexistence of official and unofficial prices. In this case the price level is not determined but all the prices at which money may trade are fully characterized by public financial policy.

We first study an elementary one-commodity one-date economy. In this economy, public financial policy has three central components: i) a maximum quantity of goods that the state is willing to trade for money, ii) a price at which the state is willing to trade goods for money, and iii) a vector of monetary transfers from the state to each private agent. We deem fiscal debtors the private agents who receive a negative transfer from the state, and fiscal creditors those who receive a positive transfer.

Fixed policies. We first study the natural situation in which both the price at which the state is willing to trade and the positive transfers are fixed, as opposed to contingent on the actions of the private sector. This is empirically relevant as a fixed official price resembles a currency peg, a metallic standard, or some of the allocation mechanisms currently used by central banks in their refinancing operations. Fixed transfers correspond to the repayment of nominally safe public liabilities issued in the (for now unmodelled) past such as central-bank reserves.

Our first key insight is that such a combination of fixed transfers and a fixed official trading price determines the price level if and only if it is impossible for any subgroup of private agents to place trades in the official market that would lead other agents to be rationed in it. A policy that does not satisfy this property opens up the possibility of situations in which some agents coordinate on squeezing the official market in order to induce the other agents to trade at unfavorable unofficial prices with them. In order to discourage such behavior, the state must commit to trade quantities of goods or money that are strictly larger than the ones it ends up trading in equilibrium.

More concretely, suppose for example that the large fiscal debtors in this economy cannot purchase enough money from the state to meet their tax liabilities if other agents coordinate on buying more money than they need in the official market. This might lead to a situation akin to debt deflation, whereby these distressed large fiscal debtors are willing to offload desirable commodities at a low unofficial price, and their counterparts use the cash that they obtain in the official market to snap up these cheap commodities. Symmetrically, in the presence of fiscal creditors, financial repression may create situations in which some agents sell goods at a high unofficial price level, and use the proceeds to increase their bids in the official market. The resulting crowding out of the other bidders in the official market justifies in turn their willingness to buy goods at a high price in the unofficial market.

Applications. We argue that these findings shed light on multiple historical but also more recent episodes. In particular, we show how our theory of financial repression offers a parsimonious framework to understand the various phases of the "assignats" crisis—paper money issued during the French Revolution. We also relate our setup to the emergence of parallel markets in response to exchange rate pegs or during periods of price controls, to the introduction of in-kind taxation in periods of financial repression, and to private monies such as stablecoins, banknotes during the US Free banking era, or money market funds. In particular, we discuss how the Walrasian approach makes it more difficult to account for important features of these episodes.

Contingent policies. Then we study policies that are contingent on the actions of the private sectors in ways that are empirically relevant. Our contingent official price is a market-clearing price à la Shapley and Shubik (1977)—the price that makes the in and out-of equilibrium private demand for goods equal to the maximum quantity that the government supplies. Our contingent transfers are defaultable payments, in the sense that the state never transfers more in aggregate than the amount of money that it collects from trading—that is, the state does not create money to honor its liabilities. We find that policies such that the price or/and the transfers are contingent this way never determine the price level, except possibly in the limit in which all private agents become negligible in the sense that the maximum price impact that each of them can have tends to zero.

What do we learn compared with the Walrasian approach? We show that our approach helps clarify—and offers rigorous foundations for—the ones based on the Walrasian equilibrium concept. In particular, our framework leads to the following insights. First, the fiscal theory of the price level usually described in Walrasian environments relies on full debt monetization out-of-equilibrium. Second, allowing for default breaks the connection between debt and the price level: defaultable nominal debt is akin to real debt for price determination. Third, the Walrasian approach is not necessarily inconsistent with fixed-price policies, but in this case it resembles a "hidden" peg, in which the state adjusts its traded quantities to peg the price at its objective. Fourth, and more important, the Walrasian approaches assume away situations of financial repression in which the state only partially controls the price level because they feature by assumption either a unique price or no equilibrium. Yet such situations are pervasive in practice.

Dynamics. Finally, we write a two-date extension of our model in order to endogenize all transfers as resulting from voluntary earlier private decisions to buy government claims. This also makes money intrinsically desirable at date 0 as a store of value. We obtain that the initial price level is determined if and only if the state promises a sufficiently high real return on money and makes good on this promise, for example, by limiting the initial real resources it seeks to obtain by issuing money.

Related literature. The title of this paper is an unsubtle reference to the state theory of money outlined in Knapp (1924). As epitomized by the opening sentence of the book—*"Money is a creature of law."*—the state theory of money contends that the state has a unique ability to impose something as money due to its legislative capacity. Our contribution is to formally study the extent to which this capacity may suffice to determine the price level. Here, the formalization of the distinctive capacity of the state is that it is the only agent which can print money, declare taxes, and expropriate bankrupt private agents.

Bassetto (2002) pioneers the strategic foundations of price-level determination by public financial policy. His goal is to offer an example of an economy in which the fiscal theory of the price level applies. We share with him a strategically closed environment that highlights the importance of credible out-of-equilibrium actions in shaping equilibrium outcomes. By lifting his restriction to centralized markets and market clearing as in Shapley and Shubik (1977), and by considering various types of nominal promises by the state—thus allowing for sovereign default, we also generate a number of additional and, we believe empirically relevant, insights.

Our approach also has points of contact with the literature that endogenizes trading frictions as coordination failures in economies that are not plagued by informational nor search frictions. Important contributions include Lagos (2000) and Burdett et al. (2001). One can view our results as identifying public financial policies that eliminate the possibility that the private sector coordinates on other prices than that targeted by the state. We emphasize in particular the central role of the trading protocol selected by the government. In this sense, our approach applies to a context of such endogenous frictions the approach pioneered by Hu et al. (2009), that endogenizes trading mechanisms in the presence of search frictions.

The search literature has like us emphasized that the willingness of the state to back its money by accepting to trade it for desirable goods is important (Aiyagari and Wallace, 1997; Li and Wright, 1998). In our model without exogenous frictions such backing is simply a necessary condition for price-level determination. In these papers, this source of value for money coexists with its role of mitigating search frictions, and they show that more backing makes it easier to sustain the Pareto-dominant monetary equilibria.

Bassetto and Phelan (2015) find like us that bounds on public interventions in money markets may generate mutlitple equilibria.

Finally, given the central role of strategic exchange in our framework, we revisit the old and large literature on the strategic foundations of Walrasian equilibrium. A review is beyond the scope of this paper, important contributions include Dubey (1982) and the references herein, Dubey and Shubik (1980), Schmeidler (1980), and Shapley and Shubik (1977). We show that when an economic good—money—is only desirable to optimizing agents as legal tender, the producer of this good still has the ability to coordinate private agents on a given nominal anchor.

The paper is organized as follows. Section 2 outlines our baseline model. Section 3 solves it in the case in which the economy features only fiscal creditors. Section 4 shows how our approach helps clarify—and offers rigorous foundations for—the ones based on the Walrasian equilibrium concept. Section 5 introduces fiscal debtors. Section 6 introduces policy uncertainty in order to endogenize our collateral constraint as risk management. Section 7 outlines and solves a two-date model. Section 8 discusses historical and current situations for which we believe our insights to be particularly relevant. Section 9 concludes. Proofs follow the propositions because we find most of them instructive, yet the paper is written so that they can be skipped in a first reading.

2 One-date model

This section outlines our simple one-date economy. It presents a baseline public financial policy that consists in fixed negative transfers (taxes), in fixed positive transfers that may be interpreted as extinguishments of nominal liabilities issued in an unmodelled past (e.g., reserves with the central bank), and in a commitment to trade a given maximum quantity of goods for money at a fixed official price.

2.1 Setup

The economy comprises a public sector—"the state"—and $n \geq 2$ private agents indexed over $\mathcal{I} \equiv \{1, ..., n\}$. There are two divisible economic goods, one deemed "the good" and the other "money" henceforth. The good is intrinsically desirable to private agents whereas money is not. Each private agent thus ranks any bundles of the good and money using the standard ordering of their respective quantities of the good only.

Each private agent is endowed with $e > 0$ units of the good. The state is endowed with $n\tau > 0$ units of it. The state can produce money. Private agents cannot. All private agents and the state can trade money for the good as described below.

Public financial policy. The state enforces a policy that features monetary transfers, money creation, and trade. We describe each component of a policy in turn.[1]

Negative transfers (taxes). The state requires that each private agent $i \in \mathcal{I}$ pay a tax equal to $T_i \geq 0$ units of money.

Positive transfers. The state makes a cash transfer $L_i \geq 0$ to each agent $i \in \mathcal{I}$.

Money creation. Policy also features the production of $nM \geq 0$ units of money.

Trade. The state posts an order to buy a quantity $n\delta_G \in \mathbb{R}$ of the good at the price level $P^* > 0$, with the convention that this is a sell order if $\delta_G \leq 0$.

In sum, a policy consists in a vector $\mathcal{P} = ((T_i)_{i\in\mathcal{I}}, (L_i)_{i\in\mathcal{I}}, M, P^*, \delta_G)$. We will also later introduce the possibility that, unlike in this baseline case of fixed policies, some policy components be contingent on the actions of the private sector.

The state also consumes $nc_{G,C} \in \mathbb{R}$ units of the good and $nc_{G,M} \in \mathbb{R}$ units of money.

Remark. We do not model this state consumption as a component of policy but rather as a payoff to the state determined by both policy and by the private sector's strategy profile as detailed below. Bassetto (2002), unlike us, models public spending as a decision that is not contingent on the private sector's strategy, but he posits that taxes, unlike here, are adjusting in response to (in and out of equilibrium) private strategies in order to maintain this fixed spending level. Both approaches are thus equivalent and merely reflect that since the state's surplus depends on voluntary trades by the private sector, then either taxes or expenditures (or both) must be modelled as contingent on actions by all agents—as payoffs rather than actions in a game-theoretic setting.

Net transfers. We will make intensive use of the following natural concepts of net transfers associated with a policy $\mathcal{P}$.

[1]Policy could also feature in-kind transfers. This would clutter notations without generating significant insights. Section 8 interprets τ as an in-kind tax levied by the government in order to analyze some historical examples of in-kind taxation under the lens of our model.

Definition 1. ***(Net transfers)*** *For all $i \in \mathcal{I}$, let $N_i = L_i - T_i$. Let*

$$N = \frac{1}{n}\sum_{i\in\mathcal{I}} N_i,\ \ N_+ = \frac{1}{n}\sum_{i\in\mathcal{I}} \max\{N_i, 0\},\ \ \textit{and}\ \ N_- = \frac{-1}{n}\sum_{i\in\mathcal{I}} \min\{N_i, 0\}. \tag{1}$$

Notice that $N = N_+ - N_-$. In words, N is the net nominal transfer per capita, N_+ is the private net fiscal credit per capita (counting a net debt as zero), and N_- the absolute value of fiscal net debt per capita (counting a net credit as zero). In the following, we will deem "fiscal creditors" the agents such that $N_i > 0$ and "fiscal debtors" that for whom $N_i < 0$.

Private actions. Taking policy $\mathcal{P}$ as given, private agents play a simultaneous game whereby they make decisions to trade and pay taxes. We describe these decisions in turn, and the resulting payoffs.

Taxes. Each private agent $i \in \mathcal{I}$ decides on the amount of cash taxes $\hat{T}_i \geq 0$ that she pays to the state.

Trades. Each agent can submit any number of orders to buy or sell a given quantity of goods at a given price. The only restriction is that the total size of her sell orders—the sum of the quantities of goods over all her sell orders—cannot exceed e. This is essentially a no short-sales constraint, as one cannot sell goods that one needs to buy. We will see below that money can by contrast be sold short.[2]

The trading strategy of agent $i \in \mathcal{I}$ is conveniently described by the functions describing her cumulative orders. The respective cumulative buy and sell orders at prices (weakly) lower than P, $D_i(P)$ and $S_i(P)$ respectively, are increasing step functions over $[0, +\infty)$ satisfying:

$$D_i(0) = S_i(0) = 0, \tag{2}$$

$$\lim_{+\infty} S_i \leq e. \tag{3}$$

Let us denote, for all $P > 0$, $d_i(P)$ and $s_i(P)$ the respective buy and sell orders of i at

[2]We rule out infinite prices at this stage but will address them in Section 3.

the price P:

$$d_i(P) \equiv \int \mathbb{1}_{\{p=P\}} dD_i(p),\ s_i(P) \equiv \int \mathbb{1}_{\{p=P\}} dS_i(p). \tag{4}$$

In sum, the strategy of agent $i \in \mathcal{I}$ is $\mathcal{S}_i = (\hat{T}_i, D_i(.), S_i(.))$. Let $\mathcal{S} = (\mathcal{S}_i)_{i\in\mathcal{I}}$ denote the strategy profile of the private sector.

Market clearing, bankruptcy mechanism, and payoffs. We now describe how market clearing and a bankruptcy mechanism shape the payoff of each agent given a policy $\mathcal{P}$ and a strategy profile $\mathcal{S}$.

Market clearing. For all $P > 0$, let $d(P)$ and $s(P)$ denote the aggregate buy and sell orders at the trading post P:

$$d(P) = \sum_{i\in\mathcal{I}} d_i(P) + \mathbb{1}_{\{P=P^*\}}\delta_G^+,\ s(P) = \sum_{i\in\mathcal{I}} s_i(P) + \mathbb{1}_{\{P=P^*\}}(-\delta_G)^+ \tag{5}$$

If $d(P)s(P) = 0$, then no trade takes place. Otherwise, the smallest side of the market is fully executed and the other side is rationed pro rata the size of each order. Formally, each private agent $i \in \mathcal{I}$ buys and sells effective quantities $\hat{d}_i(P)$ and $\hat{s}_i(P)$ such that:

$$\hat{d}_i(P) \equiv d_i(P) \min\left\{1, \frac{s(P)}{d(P)}\right\},\ \hat{s}_i(P) \equiv s_i(P) \min\left\{1, \frac{d(P)}{s(P)}\right\}, \tag{6}$$

and the same uniform rationing rule applies to the state at $P = P^*$. We respectively denote $\hat{D}_i(P)$ and $\hat{S}_i(P)$ the respective cumulative effective purchases and sales of agent $i \in \mathcal{I}$.

The following definition is natural and important. It states that a trading post is active if and only if at least one private agent strictly gains or loses goods in it.

Definition 2. ***(Active trading post, net buyer, net seller)*** *Agent $i \in \mathcal{I}$ is net buyer (respectively net seller) at the trading post P if and only if $\hat{d}_i(P) > \hat{s}_i(P)$ ($\hat{s}_i(P) > \hat{d}_i(P)$ respectively). The trading post P is active if and only if at least one agent is net buyer or net seller at P.*

Implicitly, a net buyer is a net buyer at price P of the good at price P and the net seller is a net seller of the good at price P.

An active trading post always features both at least one net buyer and one net seller by definition, but one of them can be the state.

Why uniform rationing? The only property of uniform rationing that is crucial for our results is that a larger bid generates other things being equal a larger allocation.[3] Many other trading games share this property, including for example the sequential clearing of bids at the same price in random order.[4]

Bankruptcy mechanism and payoffs. Given a policy $\mathcal{P}$ and a strategy profile $\mathcal{S}$, the payoff to agent $i \in \mathcal{I}$ depends on whether she is solvent or not, where we define solvency as follows:

Definition 3. ***(Solvent agent)*** *Agent $i \in \mathcal{I}$ is solvent if and only if*

$$\hat{T}_i \geq T_i, \tag{7}$$

$$\int P dD_i(P) \leq L_i - \hat{T}_i + \int P d\hat{S}_i(P). \tag{8}$$

In words, agent i is solvent if and only if she pays her taxes (condition (7)), and she satisfies a collateral constraint that consists in covering the value of her (gross) buy orders with cash (condition (8)). We elaborate on this latter constraint below. Notice that this is not a cash-in-advance constraint as "cash" includes the proceeds from effective sales.

If an agent is solvent, then her payoff is given by the respective quantities of goods and money $c_{i,C}$ and $c_{i,M}$ resulting from her transfers and trades:

$$c_{i,C} = e + \int d\hat{D}_i(P) - \int d\hat{S}_i(P), \tag{9}$$

$$c_{i,M} = L_i - \hat{T}_i + \int P d\hat{S}_i(P) - \int P d\hat{D}_i(P). \tag{10}$$

Notice that these consumptions are positive for any strategy profile from conditions (3), (8), and $D_i \geq \hat{D}_i$. If the agent is insolvent, then the state seizes all the goods and money of that agent ($c_{i,C} = c_{i,M} = 0$) and replaces her in the market. The state creates all the money that is needed to execute this agent's buy orders and/or to make up for $T_i - \hat{T}_i$.

[3] Unless, of course, the post is inactive or there are no competing bids.

[4] This example would require explicit assumptions on how private agents assess random consumption bundles.

The collateral constraint (8) plays two important roles. It eliminates default contagion and it creates a demand for money above and beyond that created by taxes. We discuss each role in turn.

Eliminating default contagion. Because $D_i \geq \hat{D}_i$ for all $i \in \mathcal{I}$, the collateral constraint implies that a solvent agent satisfies a fortiori the resource constraint

$$\int P d\hat{D}_i(P) \leq L_i - \hat{T}_i + \int P d\hat{S}_i(P). \tag{11}$$

In words she has enough money to settle her effective buy orders. If insolvent, the agent is replaced by the state which creates enough money to emulate her orders if needed. Thus, each agent can take other agents' orders as fixed when deciding on her trades because there is no default contagion. An agent does not have to worry that reducing a given buy order may trigger a chain of defaults ultimately affecting other counterparts.

Creating a demand for money. The collateral constraint creates a demand for money because cash serves to dilute other agents in a rationed market, as it enables placing larger buy orders by relaxing the constraint.

Execution uncertainty and endogenous liquidity constraint. If we defined solvency as consisting in the tax-compliance constraint (7) and in the resource constraint (11) in lieu of the collateral constraint (8), then agents could place arbitrarily large orders to dilute each other, facing the only restriction that they can execute their effective trades. It is important to stress that this owes to the perfect-foresight nature of the model. If agent $i \in \mathcal{I}$ faces uncertainty regarding the fraction of her buy orders that will be executed, for example due to policy uncertainty, then she may get bankrupt if her orders are not backed by sufficient cash. In this case self-imposed liquidity constraints may arise to ensure that the standard resource (11) is satisfied in sufficiently many states of the world. Section 6 offers a simple version of this economy with ambiguity-averse agents in which the equilibria that we obtain here under perfect foresight and a collateral constraint can be sustained with a mere resource constraint (11). The agents seek to ensure that they can execute their buy orders under all contingencies, and thus that their orders are fully backed by cash. This leads to self-inflicted constraints (8). In other words, whereas liquidity constraints play an important role in our results, they could endogenously arise from liquidity management in a richer model instead of being assumed, as is necessary in this perfect-foresight one.

2.2 Equilibrium concept

We model social interactions as a game between private agents given policy. It is thus natural to adopt Nash equilibrium as our concept of predictable outcome. Formally, given that private agents do not care about their consumption of money, an equilibrium associated with a policy $\mathcal{P}$ is a strategy profile $\mathcal{S}$ such that for every $i \in \mathcal{I}$, strategy $\mathcal{S}_i$ maximizes i's consumption of the good $c_{i,C}$ given other strategies $\mathcal{S}_{-i}$ and policy $\mathcal{P}$. This equilibrium concept yields a natural definition of predictable price levels:

Definition 4. ***(Predictable price levels)*** *A price $P > 0$ is predictable given policy $\mathcal{P}$ if and only if there exists an equilibrium associated with $\mathcal{P}$ with active trading at P. Let $\Pi(\mathcal{P})$ denote the set of predictable price levels associated with a policy $\mathcal{P}$.*

This enables us in turn to characterize whether a public financial policy determines the price level:

Definition 5. ***(Determination of the price level)*** *A policy $\mathcal{P}$ weakly determines the price level if and only if $\Pi(\mathcal{P})$ is a singleton. A policy strongly determines the price level if and only if it weakly determines the price level and every equilibrium features active trade.*

The price level may fail to be determined for three reasons. First, it may be that there exists no equilibrium with active trade. Second, it may be that every equilibrium features active trade at a given equilibrium price, but that this latter price varies across equilibria. Finally, an equilibrium may feature active trades at different prices. We will see that there exist policies leading to each of these three configurations, together with the ones that actually determine the price level.

2.3 Feasible policies

The consumption of goods and money by the state in the absence of private bankruptcy, $c_{G,C}$ and $c_{G,M}$, are by conservation of quantities:

$$nc_{G,C} = n(e+\tau) - \sum_{i\in\mathcal{I}} c_{i,C}, \tag{12}$$

$$nc_{G,M} = nM - \sum_{i\in\mathcal{I}} c_{i,M}. \tag{13}$$

These state consumptions of goods and money are not necessarily positive. The following proposition characterizes policies such that the state consumes positively no matter the private strategy profile. We will deem policies that satisfy these restrictions "feasible":

Definition 6. ***(Feasible policies)*** *A policy is feasible if and only if the state consumes positively goods and money for every private strategy profile.*

We have:

Proposition 1. ***(Characterization of feasible policies)*** *A policy is feasible if and only if*

$$\tau + \delta_G \geq 0, \tag{14}$$

$$M \geq N + P^* \min\left\{\delta_G^+, e\right\}. \tag{15}$$

Proof. We prove each inequality in turn. **Positive consumption of goods.** The state transfers goods to the private sector only through sales, and not more than $-\delta_G$ per capita, ensuring that condition (14) is sufficient. Suppose that agent $i \in \mathcal{I}$ buys an arbitrarily small quantity at an arbitrarily large price from agent $j \in \mathcal{I}$ whom in turn bids the money, supposed to be larger than $-P^* n\delta_G$, in the official market. Other agents do not trade in the official market. Agent i also sells e at an arbitrarily low price. Then the state must sell $n\delta_G$ units and receives arbitrarily few goods from the possible bankruptcy of i, establishing that (14) is also necessary.

Positive consumption of money. The right-hand side of condition (15) corresponds to the amount of money that the state must transfer to the private sector when the latter sells as many goods as possible and pays its taxes. This is the maximum amount M that the state needs to issue across all private profiles since the state issues additional money when agents are insolvent by assumption. □

Conditions (14) and (15) state that the state has enough real resources $n\tau$ and prints enough money nM to consume positively goods and money given policy $\mathcal{P}$, no matter the strategy profile $\mathcal{S}$ of the private sector. Condition (14) ensures that the state consumes a positive quantity of goods no matter the private strategy profile. Condition (15) ensures that the state consumes a positive quantity of money for all private strategies. It is worthwhile stressing that as soon as the states issues a nominally safe aggregate net promise $N > 0$, then it must stand ready to entirely monetize it—$M \geq N$, as there is no guarantee that (in and out-of-equilibrium) trades with the private sector generate any cash.

In the balance of the paper we will present our results for any policy, whether they are feasible according to the above definition or not. Conditions (14) and (15) make it easy to single out, among the set of policies that we consider, the ones that are feasible.

2.4 Preliminary results

Before starting the core of the analysis, we establish in this section two sets of results on agents' trading strategies that will be useful for our analysis in the next sections. First, we show that only their net position in a given trading post matters. Second, we show how rationing may affect their trading strategies.

Netting. The following lemma first shows that one can offset trades by the same agent at a given price in the following sense.

Lemma 2. ***(Netting)*** *Consider a strategy profile such that agent $i \in \mathcal{I}$ is a non-bankrupt net buyer at the trading post P. If she deviates and sets $s'_i(P) = 0$, $d'_i(P) = d_i(P) - d(P)s_i(P)/s(P)$ then she does not affect her allocation nor that of other agents. Symmetrically, suppose she is net seller at P. If she deviates and sets $d'_i(P) = 0$, $s'_i(P) = s_i(P) - s(P)d_i(P)/d(P)$ then she does not affect her allocation nor that of other agents.*

Proof. The results stem directly from the fact that these deviations do not affect the rationing coefficients as when $s(P) \neq s_i(P)$ and $d(P) \neq d_i(P)$,

$$\frac{d(P) - \frac{d(P)si(P)}{s(P)}}{s(P) - s_i(P)} = \frac{d(P) - d_i(P)}{s(P) - \frac{s(P)di(P)}{d(P)}} = \frac{d(P)}{s(P)}.$$

The effective trades of the other agents are thus unaffected by the deviation of i. Nor is that of i. To see this, suppose that i is net buyer. The reduction in her buy order $d(P)s_i(P)/s(P)$ is weakly larger than that of her effective sales $s_i(P)\min\{d(P)/s(P), 1\}$, and so the deviation leaves her solvent. Furthermore,

$$\hat{d}_i'(P) - \hat{s}_i'(P) = \left(d_i(P) - d(P)\frac{s_i(P)}{s(P)}\right) \min\left\{1, \frac{s(P)}{d(P)}\right\} = \hat{d}_i(P) - \hat{s}_i(P),$$

leaving her allocation unchanged. The same reasoning applies for a net seller. □

This result is useful because it implies that whenever an agent is net seller or net buyer at one post, we can assume that she nets her trades this way before entering into a deviation so that we do not have to worry about the impact of small deviations from her larger effective order on her potential order on the other side.

Properties of equilibrium trades. When considering their trading strategy, agents do not only take into account prices, but also the extent to which their bids will be executed. The common denominator of the two following results is that rationing may push agents to adopt trading strategies in which they do not only compare prices—selling at the highest, buying at the lowest price.

To this purpose, let us define for any active trading post P and any $i \in \mathcal{I}$:

$$\Delta_i(P) = \begin{cases} \frac{s(P)(d(P)-d_i(P))}{d(P)^2} \text{ if } s(P) \leq d(P), \\ 1 \text{ otherwise.} \end{cases} \tag{16}$$

The coefficient $\Delta_i(P)$ measures the marginal return from increasing a buy order in a market in which buyers are (weakly) rationed ($s(P) \leq d(P)$). On one hand a marginal increase ϵ in i's order generates $\epsilon s(P)/d(P)$ additional marginal units. On the other hand it crowds out her own outstanding order $d_i(P)$, thereby costing a marginal reduction $\epsilon(d_i(P)/d(P)) \times (s(P)/d(P))$ in the return on this outstanding order.

To start with, we establish the following lemma on strategies of selling dear and buying cheap:

Lemma 3. ***(Selling high to buy low)*** *Suppose that in an equilibrium that features (at least) two active trading posts with price levels P and P', a non-bankrupt agent $i \in \mathcal{I}$ is net seller at P' and net buyer at P. Then $P' > P$, and if $s(P) < d(P)$,*

$$P'\Delta_i(P) \geq P. \tag{17}$$

Proof. We show that if $P' \leq P$ or if $s(P) < d(P)$ and condition (17) does not hold, i can strictly increase her utility by simultaneously reducing her sell and buy positions. Let us define:

$$\delta(P,\epsilon) = \min\left\{1, \frac{s(P)}{d(P)+\epsilon}\right\} \text{ and } \sigma(P,\epsilon) = \min\left\{1, \frac{d(P)}{s(P)+\epsilon}\right\}.$$

We let i modify her orders as follows. She first nets her positions as in Lemma 2. If she is the only seller at P' and is rationed, she also reduces her order up to the total buy order. For notational simplicity we maintain the notation $s_i(P')$, $d_i(P)$ for these new orders.

For $\epsilon > 0$ sufficiently small, define $\eta(\epsilon)$ as

$$-P\eta(\epsilon) = P'[(s_i(P') - \epsilon)\sigma(P', -\epsilon) - s_i(P')\sigma(P', 0)]. \tag{18}$$

In words $\eta(\epsilon)$ is the reduction in the buy order at P that has the same monetary value $P\eta(\epsilon)$ as that in the reduction in effective sales at P' when the sell order is reduced by ϵ. In particular, $\eta(\epsilon) = \epsilon P'/P$ when $\sigma(P', 0) = 1$. Suppose that i reduces her sell order at P' by ϵ and her buy order at P by $\eta(\epsilon)$. This leave her solvent by construction of $\eta(\epsilon)$, and brings a net change in consumption:

$$\begin{aligned}&(d_i(P) - \eta(\epsilon))\delta\left(P, -\eta(\epsilon)\right) - d_i(P)\delta\left(P, 0\right) - (s_i(P') - \epsilon)\sigma(P', -\epsilon) + s_i(P')\sigma(P', 0)\\ =&d_i(P)(\delta\left(P, -\eta(\epsilon)\right) - \delta(P, 0)) + \eta(\epsilon)\left(\frac{P}{P'} - \delta\left(P, -\eta(\epsilon)\right)\right).\end{aligned}$$

At first-order in $\eta(\epsilon)$, this is equal to

$$\eta(\epsilon)\left(\frac{P}{P'} - \delta(P) + \mathbb{1}_{\{\delta(P)<1\}}\delta(P)\frac{d_i(P)}{d(P)}\right),$$

Thus this deviation yields a strict benefit if $P' < P$ or if $\delta(P) < 1$ and (17) does not hold. $\square$

Intuitively, selling high to buy low is profitable only if the marginal redeployment of the sales proceeds to buy in the cheap trading post does not crowd out the outstanding order at this post.

Very much like rationing may limit how much agents want to sell at a high price, it may also lead some to be willing to buy at such a high price. The following lemma offers a necessary condition for a private agent being willing to buy at the highest of two prices.

Lemma 4. ***(Buying high instead of low)*** *Suppose that an equilibrium features (at least) two active trading posts with price levels P and $P' > P$. If a non-bankrupt agent $i \in \mathcal{I}$ is net buyer at P' then*

$$P'\Delta_i(P) \leq P\Delta_i(P'). \tag{19}$$

Proof. We show that if condition (19) does not hold, i can strictly increase her utility by moving some of her P'-order at P. Notice that Lemma 3 ensures that i cannot be a net seller at P as $P' > P$. We let i modify her orders as follows. She first nets her positions as in Lemma 2. Then she reduces her buy order at P' by ϵ and increases her buy order at P (possibly equal to 0) by $\epsilon P'/P$, where $\epsilon > 0$ is sufficiently small. Her solvency constraint still holds since the total cash value of her buy orders is unchanged. The net change in consumption units resulting from this deviation is

$$\begin{aligned}
&(d_i(P') - \epsilon)\delta(P', -\epsilon) - d_i(P')\delta(P') + \left(d_i(P) + \epsilon\frac{P'}{P}\right)\delta\left(P, \epsilon\frac{P'}{P}\right) - d_i(P)\delta(P) \\
=&\epsilon\left(\frac{P'}{P}\delta\left(P, \epsilon\frac{P'}{P}\right) - \delta(P', -\epsilon)\right) + d_i(P)\left(\delta\left(P, \epsilon\frac{P'}{P}\right) - \delta(P)\right) \\
&+ d_i(P')\left(\delta\left(P', -\epsilon\right) - \delta(P')\right).
\end{aligned}$$

At first-order w.r.t. ϵ this is equal to

$$\epsilon\frac{P'\delta(P)}{P}\left[1 - \mathbb{1}_{\{\Delta_i(P)<1\}}\frac{d_i(P)}{d(P)}\right] - \epsilon\delta(P')\left[1 - \mathbb{1}_{\{\Delta_i(P')<1\}}\frac{d_i(P')}{d(P')}\right],$$

strictly positive if condition (19) does not hold, which establishes the result. $\square$

Intuitively, an agent is willing to be net buyer at $P' > P$ if her order at P is sufficiently large that she would crowd herself out by rebalancing some of her expensive order P' towards P.

3 Price-level determination with fiscal creditors

This section first considers policies that feature only fiscal creditors. Section 3.1 studies price-level determination under our baseline fixed policies. Section 3.2 studies the limiting case of infinitesimal agents. Sections 3.3 and 3.4 study alternative policies in which positive transfers or/and the official price are contingent on the strategy profile of the private sector (outside bankruptcy).

3.1 Financial repression and unofficial markets

Consider a policy $\mathcal{P} = ((T_i)_{i\in\mathcal{I}}, (L_i)_{i\in\mathcal{I}}, M, P^*, \delta_G)$ in which there are only fiscal creditors—$N_- = 0$ and $N_+ = N > 0$. We have:

Proposition 5. ***(Financial repression and unoffical markets)*** *There are three types of predictable outcomes:*

1. ***No active trade.*** *If $\delta_G \geq 0$, then $\Pi(\mathcal{P}) = \emptyset$.*

2. ***Strong price-level determination.*** *If $N < -P^*\delta_G$, then $\Pi(\mathcal{P}) = \{P^*\}$ and the policy strongly determines the price level. Furthermore, the consumption of the government is given by*

$$c_{G,C} = \tau - \frac{N}{P^*}, \; c_{G,M} = M. \tag{20}$$

3. ***Financial repression.*** *If $N \geq -P^*\delta_G > 0$, then there is strong determination of the price level if and only if $N > -P^*\delta_G$ and $N_i = N$ for all $i \in \mathcal{I}$. Otherwise, there also exist equilibria with multiple active trading posts, with all unofficial prices strictly above $N/(-\delta_G)$. Whether the price level is determined or not, it is always*

the case that:

$$c_{G,C} = \tau + \delta_G \geq \tau - \frac{N}{P^*}, \tag{21}$$

$$c_{G,M} = M - N - P^*\delta_G \leq M. \tag{22}$$

Proof. Notice first that fiscal creditors can always avoid bankruptcy by not trading, and find it strictly preferable to going broke, so any equilibrium is without bankruptcy. The proof takes five steps.

Step 1: $\Pi(\mathcal{P}) = \emptyset$ when $\delta_G \geq 0$. Suppose otherwise that there is an active trading post. There has to be an active private net seller since the state buys. At the lowest price at which there is a private net seller, this net seller does not buy at a higher price from Lemma 3, and cannot by definition buy at a lower price. She would thus be strictly better off reducing her order, a contradiction.

Suppose for the rest of the proof that $\delta_G < 0$. There is no equilibrium with no trade in this case as one agent could deviate and buy goods from the state.

Step 2: All predictable prices are weakly larger than P^*. There exists a "P^*-equilibrium" in which each private agent $i \in \mathcal{I}$ places a buy order for N_i/P^* units at P^*. Suppose there exists an equilibrium with active trading at another price. Let us denote $\underline{P}$ the lowest unofficial price. There has to be an active net seller at this price. She must be net buyer too somewhere else otherwise she would be strictly better off cutting her order. She must buy below $\underline{P}$ from Lemma 3, and by definition cannot do so from a private seller, so she does so at $P^* < \underline{P}$.

Step 3: The P^*-equilibrium is unique when $N + P^*\delta_G < 0$. In this case buyers at P^* cannot be rationed since the private sector as a whole cannot bid more than N at P^* in an equilibrium without bankruptcy. Condition (19) then implies that there cannot be a private net buyer at $\underline{P} > P^*$ defined above.

Step 4: Equilibrium with unofficial trade when $N + P^*\delta_G = 0$ or $N + P^*\delta_G > 0$ and $N_i \neq N$ for some $i \in \mathcal{I}$. We leave it to the reader to check that one can construct verbatim the equilibrium that we construct in the proof of Proposition 10 in the presence of fiscal debtors. That is, the agent with the largest cash holdings buys at the official price and at an unofficial one $P > P^*$, whereas the others sell their entire endowments e at P and reinvest the proceeds plus their cash holdings in the official post. The only case which slightly differs is that in which $N + P^*\delta_G = 0$ and all agents are ex ante identical. In this case, one equilibrium can be such that one of them buys at an unofficial price defined as in the proof of Proposition 10. The others sell all their goods at this price and reinvest the proceeds in the official market. Unlike when $N + P^*\delta_G > 0$ and agents are ex ante identical tackled in Step 5 below, this is an equilibrium as condition (17) is not necessary in the case in which $N + P^*\delta_G = 0$, because this corresponds to the situation in which $s(P) = d(P)$ in Lemma 3. Finally, that unofficial prices are strictly above $N/(-\delta_G)$ follows directly from Lemma 3.

Step 5: The P^*-equilibrium is unique when $N_i = N$ for all $i \in \mathcal{I}$ and $N + P^*\delta_G > 0$. Suppose by contradiction that there is unofficial active trade, and let $\bar{P}$ and (again) $\underline{P} > P^*$ respectively denote the largest and smallest unofficial active prices. Let i denote a net seller at $\underline{P}$ and j a net buyer at $\bar{P}$. The official market is rationed on the buy side ($d(P^*) < s(P^*)$), so that condition (17) applies to i.[5] Conditions (17) and (19) together imply $\Delta_i(P^*) \geq P^*/\underline{P} \geq P^*/\bar{P} \geq \Delta_j(P^*)$, requiring that i is unofficial net buyer above $\underline{P}$ or/and j is unofficial net seller below $\bar{P}$, either way a contradiction given Lemma 3. □

Proposition 5 first states that there is no active trade if the state does not sell goods. Essentially, no private agent is interested in being net seller in this case, so nor can there be any net private buyer. Notice that we ruled out the posting of infinite prices when describing the action space of private agents. Yet they could arise as a variant of a no-trade equilibrium in which some agents transfer money without counterparts to others. By contrast, such "infinite price levels" are out of range when all equilibria feature active trade because money has strictly positive value in this case.

Proposition 5 then states that in the presence of heterogeneous net transfers, the state must supply strictly more goods $-P^*\delta_G$ than the equilibrium average net demand N ($N < -P^*\delta_g$) in order to determine the price level.

In the case of insufficient backing $N \geq -P^*\delta_G$ that we deem "financial repression", heterogeneity among net transfers creates room for unofficial trading among creditors.[6] Whereas there still exists an equilibrium without unofficial trades, there also exist equilibria with unofficial prices strictly above $N/(-\delta_G)$. In these equilibria, small creditors

[5] To see why, notice that if the official market is not rationed for buyers, then an unofficial one has to be, but then (19) implies that the active buyers must all have a strictly smaller footprint in this market than in the official one, which is impossible.

[6] As stated in the proposition, heterogeneity is not required in the particular case in which $N + P^*\delta_G = 0$.

are willing to acquire money at a low cost (at a high price level) in order to gain more dilution power in the official market. Conversely, agents with large claims accept to sell some money to them at such prices rather than further diluting their own positions in the official market. Thus, agents with low cash holdings arise as endogenous intermediaries between cash-rich agents and the state, selling goods high and buying low. In the absence of unofficial trading, these small fiscal creditors earn a higher marginal return on money in the official post than the large creditors. By using borrowed money in the unofficial market to crowd large creditors out, small creditors coordinate on extracting rents from them: Unofficial markets reduce consumption inequality.

Proposition 5 also shows that in the case of strict insufficient backing $N > -P^* \delta_G$, there is strong price-level determination if all the transfers N_i are identical because there is no room for unofficial trading. An inspection of the proof shows that this result is actually continuous in essence, in the sense that unofficial trading volume shrinks as private cash holdings become more homogeneous. The intuition is as follows. Applying Lemma 3 to any net seller j in an unofficial market and Lemma 4 to any net buyer k, it must be that $\Delta_j(P^*) \geq \Delta_K(P^*)$. This means that the unofficial transfer of cash for goods cannot be so large that unofficial sellers' positions exceed that of net unofficial buyers in the official market. But then, the unofficial trading volume has to tend to zero as their initial cash holdings become more similar. In other words, if all creditors earn the same marginal return on money in the official market, there is only limited room for rent extraction via unofficial markets across them.

Remark on cash-in-advance. Had we imposed a cash-in-advance constraint instead of collateral constraint (8),[7] the state would strongly determine the price level at P^* no matter the amount of backing, that is, even when $N + P^* \delta_G > 0$. The reason is simply that unofficial sales only serve to improve one's position in the official market, and this motive vanishes under a cash-in-advance constraint. Our setup thus highlights that the assumption that the state delegates price setting to an umodelled Walrasian auctioneer which can impose centralized trading is essential in ruling out the possibility of financial repression in Walrasian environments with a Clouwer constraint. A price-setting state can just pick any price level regardless of its real resources given such a constraint.

[7]Formally, this means removing $\int P d\hat{S}_i(P)$ from the right-hand side of condition (8).

Remark on velocity. Our setup offers a direct observation of velocity properly defined as the average number of times a monetary unit is traded for goods. In the particular equilibria that we construct with a single unofficial market, the larger the unofficial trading volume, the larger the velocity as this means that more cash gets traded twice, unofficially and then officially. Thus we predict a positive comovement between the rise of unofficial inflation and velocity.

Empirical applications. In sum, Proposition 5 offers a minimalist model in which unofficial markets coexist with official ones if a state is unable to provide excess backing for its currency ($N + P^*\delta_G < 0$). We are not aware of other simple formalizations of such situations, even though they are pervasive in practice. Section 8 details applications of our settings that we briefly sketch here. First, there is a plethora of historical and current situations where price controls spur the creation of black markets in which price levels exceed the official prices to an extent that is commensurate to their economic irrelevance. A prediction of our very stylized model is that the agents with the lowest money holdings are the ones selling in the black market. Sargent and Velde (1995) mention anecdotal evidence consistent with this in their macroeconomic analysis of the French Revolution. They find that after the state has issued an excess of paper money ("assignats") and lifted trading bans, outsiders supplied gold in the unofficial market and used the proceeds in assignats to buy goods on the cheap in Paris. The other area in which our theory applies is that of currency pegs. In typical models of unsustainable pegs pioneered by Krugman (1979) or Flood and Garber (1984), the Walrasian environment implies that the peg either holds or not. We depict a more intermediate situation in which significant official and unofficial trading volumes may coexist. Reinhart and Rogoff (2004) show that, accordingly, this coexistence prevails for more than half of the pegs since WWII. Our results on who buys and sells unofficially also suggests that capital controls can be substitutes for backing when maintaining a peg. If foreign institutions with small structural positions have limited access to the official market, they have no reason to purchase the currency in unofficial markets as did the above-mentioned arbitrageurs during the French Revolution. Finally, one may also interpret money here as a stablecoin that the issuer seeks to peg to dollar ("the good" here). In their description of the collapse of the UST stablecoin, Liu et al. (2023) describe how heterogeneous players intervene differently in parallel markets that are reminiscent of the stylized ones that endogenously arise here.

3.2 Asymptotically atomistic economies

It is interesting to separate out, among the results in Proposition 5, the ones that survive in the limit in which each private agent becomes negligible. To be sure, the results that hinge on private agents' price impact are interesting in their own right, as there is ample evidence that the large institutions that participate in the primary markets for public liabilities have some price impact in practice. Yet assessing our results in the limiting case of negligible agents is also instructive. Here we show that when the economy converges to one in which each agent becomes negligible, the unofficial prices that may arise in the presence of financial repression all tend to $N/(-\delta_G) \geq P^*$.

Corollary 6. ***(Negligible agents)*** *Consider a sequence $(\mathcal{P}^n)_{n\in\mathbb{N}}$ of policies with financial repression each associated with an economy of size n, and each such that the net transfers are not all identical. Suppose that $\mathcal{P}^n \to \mathcal{P}$ such that $-P^*\delta_G > 0$ and that*

$$\max_{i\in\mathcal{I}} \left\{ \frac{N_i^n}{nN^n} \right\} \underset{n\to+\infty}{\to} 0. \tag{23}$$

For every $\epsilon > 0$, there exists $m \in \mathbb{N}$ such that for all $n \geq m$, the unofficial prices that are predictable given $\mathcal{P}^n$ belong to $(N/(-\delta_G) - \epsilon, N/(-\delta_G) + \epsilon)$.

Proof. Conditions (19) and (23) together imply that every unofficial price must become arbitrarily close $P^*/\delta(P^*) = N/(-\delta_G)$ as $n \to +\infty$. □

Condition (23) encodes that agents become negligible in the limit. Notice that this proposition implies in particular than when the state backs money with the exact real amount given P^*—$N + P^*\delta_G = 0$—then all equilibrium prices converge to the official target P^*. If $N + P^*\delta_G > 0$, there still are multiple equilibria with varying trading volume at unofficial prices, including possibly no unofficial trade. Yet all unofficial prices become arbitrarily close to $N/(-\delta_G)$.

The coefficient $\Delta_i(P^*)$ defined in (16) that drives agents' indifference between distinct prices illustrates the respective contributions of insufficient backing on one hand ($N + P^*\delta_G > 0$) and of the price impact of non-negligible agents on the other hand to the possible rise of unofficial trades. This coefficient is the product of the term

$(d(P^*) - d_i(P^*))/d(P^*)$ that reflects individual price impacts and vanishes as agents become negligible, and of $s(P^*)/d(P^*)$. That $-\delta_G = s(P^*) < d(P^*) = N/P^*$ in the case of financial repression is then the only remaining source of unofficial trade as agents become negligible.

The rest of this section studies policies such that transfers or/and official price-setting are contingent on the actions of the private sector in empirically relevant ways. We first introduce the possibility that the positive transfers of the state are defaultable. In this case, the state uses only the proceeds from its sales of goods to fund transfers. It does not use money creation to make them (nominally) safe as in the above fixed policies. Thus transfers resemble sovereign debt that cannot be repaid by money printing. We then also open up the possibility that the state commits to quote an official price that clears the official market in-and-out of equilibrium. This will help compare our economy to Walrasian environments in which price setting is delegated to an unmodelled "auctioneer".

3.3 Contingent policy: Defaultable security

The policy assumed thus far comprises a fixed nominal payment L_i from the state to private agent $i \in \mathcal{I}$. This section considers policies in which the payments to the private sector are subject to default, in the sense that the aggregate payment cannot exceed the aggregate amount of money that the state collects. In other words, the state does not use the money that it creates to honor its transfers, rather, it uses only the one that it receives from the private sector.

We modify the baseline policy with only fiscal creditors studied in Proposition 5 as follows. For simplicity, we assume away cash taxes: $T_i = 0$ for all $i \in \mathcal{I}$. More important, we suppose that the positive cash transfer from the state to agent $i \in \mathcal{I}$ depends on the amount of money that the state collects in the market. In game-theoretic language, this transfer is no longer an action of the state but rather a payoff vector $(L_i(\mathcal{S}))_{i\in\mathcal{I}}$ that depends on the private strategy profile $\mathcal{S}$. Formally, there exists $(B_i)_{i\in\mathcal{I}}$, a positive vector such that $B = \sum_{i\in\mathcal{I}} B_i/n > 0$, and such that the transfer to agent $i \in \mathcal{I}$ is

$$L_i(\mathcal{S}) = B_i \min\left\{1, \frac{\left(\sum_{i\in\mathcal{I}} P^*\hat{d}_i(P^*) - P^*\hat{s}_i(P^*)\right)^+}{nB}\right\}. \tag{24}$$

In case of default, each agent receives a share in the nominal value of the effective net demand at the official post. Definition (24) implies that creditors are treated pari passu. This is to fix ideas, all our insights carry over with alternative seniority rules.

The rest of the policy is unchanged and so are the trading and bankruptcy mechanisms. For brevity we consider only the case $\delta_G < 0$. We have:

Proposition 7. ***(Defaultable security and price-level determination)*** *The policy is feasible if and only if condition* (14) *holds. For all* $D \in [(B + P^*\delta_G)^+, B]$*, there exist equilibria in which the state pays* $B - D$ *per capita. Across such equilibria,*

$$c_{G,C} = \tau - \frac{B - D}{P^*}, \; c_{G,M} = M. \tag{25}$$

There is no price-level determination. However, all active price levels converge to P^* *as agents become negligible in the sense of Proposition 6.*

Proof. Notice first that agents can always avoid bankruptcy by not trading and are strictly better off this way, so that any equilibrium must be without bankruptcy. Notice also that there exists a no-trade equilibrium, so that determination of the price level is at best weak. We leave it to the reader to check that for every $D \in [(B + P^*\delta_G)^+, B)$, there exists a P^*-equilibrium in which only the official trading post is active. Agent $i \in \mathcal{I}$ bids $B_i(1 - D/B)$ at P^* and collects the same transfer from the state. The rest of the proof is in three steps.

Step 1: Feasibility. By construction the state transfers only what it receives, and so it does not need to create M to consume money positively (but of course can always do so), that is, $c_{G,M} = M$ in and out of equilibrium.

Step 2: Unofficial prices tend to P^* as agents become negligible. Suppose that an equilibrium features active unofficial trading. Suppose that there is a net buyer at $P^b > P^*$. Lemma 4 applies with $P' = P^b$ and $P = P^*$. To see this, notice that the deviation used to prove the lemma—shifting part of the P'-bid towards P—strictly improves solvency as it increases the transfer received by the deviating agent. Thus condition (19) holds and P^b becomes arbitrarily close to P^* as agents become negligible. Suppose then that there is a net buyer below P^*, and let P^b denote the smallest price at which there is one. There must be net sellers at this price. Lemma 3 applies between P^b and any other unofficial price $P \neq P^*$, implying that these net sellers, who must be buying somewhere, buy at P^*. Suppose that such a net seller $i \in \mathcal{I}$ reduces her effective sales at P^b by $\epsilon > 0$ sufficiently small. Her order at P^* must then shrink by x such that

$$P^* x = \epsilon P^b + \frac{B_i P^* x}{nB}, \tag{26}$$

where the second term on the right-hand side reflects that her smaller bid reduces her net transfer from the state, and thus tightens her solvency constraint. Equilibrium requires that $x \geq \epsilon$, or $P^b \geq P^*[1 - B_i/(nB)]$, and so, as $P^b < P^*$, P^b must become arbitrarily close to P^* as agents become negligible.

Step 3: There exist equilibria with unofficial trade. For D such that $B - D + P^* \delta_G < 0$, let $i \in \mathcal{I}$ an agent with a minimum value of B_i— i need not be unique. Let this agent buy a sufficiently small quantity at $P = P^*[1 - B_i/(nB)]$ and all the others sell their e at this post and invest the proceeds at P^*. It is an equilibrium as the unofficial buyer is indifferent between buying at P and P^* from the same reasoning as that used in step 2 and sellers may strictly prefer to sell more (unless their claim is equal to B_i in which case they are indifferent) but cannot. □

Within the limits of a static model, the equilibria are reminiscent of self-fulfilling debt crises. Equilibria with default here are gridlocks whereby the private sector does not bid much cash for goods because it expects sovereign default in the form of a small transfer, and these small bids in turn vindicate the small transfer. A key difference with the case of a nominally safe security is that there is no longer room for strict financial repression since the aggregate transfer of the state by construction never exceeds the amount of money that it is willing to purchase at P^*. As a result, the situation bears similarities with that in which $N + P^* \delta_G = 0$ with safe securities: Unofficial trades are made possible only because of individual price impacts. Thus unofficial prices all become arbitrarily close to the official one when agents become negligible. Yet there are multiple equilibria with varying default severity even in this negligible limit, but only the goods consumption of the state varies across them. The price level does not. A higher loss given default creates additional real resources for the state.

More on out-of-equilibrium monetization. We find this result that the absence of any monetization of the transfer creates self-justified default to be interesting. Still, it is a knife-edge one. By committing to arbitrarily small out-of-equilibrium financial repression, the state can eliminate all the equilibria but the one without default. To see this, suppose that for $\epsilon > 0$ arbitrarily small, one replaces (24) with

$$L_i(\mathcal{S}) = B_i \min \left\{ 1, \frac{\left(\sum_{i \in \mathcal{I}} P^* \hat{d}_i(P^*) - P^* \hat{s}_i(P^*) \right)^+}{nB} + \epsilon \right\}, \tag{27}$$

which requires a money creation of ϵ to be feasible in the sense of Proposition 1. Then it cannot be that an agent receives less than B_i in equilibrium as this would imply that the private sector does not bid its aggregate cash holdings in the official market, which is inconsistent with the rationality of at least one private agent.

3.4 Contingent policy: Market-clearing policy

The goal of this section is to compare the outcomes when the state's trading strategy consists in posting a fixed price as above with those when the state acts as an auctioneer à la Shapley and Shubik (1977), setting a price that absorbs all the private demand for goods. This will highlight the crucial role of the trading protocol on the set of predictable price levels.

We modify again the baseline policy with fiscal creditors only as follows. First, for brevity, we restrict again the analysis to policies such that $T_i = 0$ for all $i \in \mathcal{I}$ and $\delta_G < 0$.

Second, the official trading post no longer operates as the unofficial ones, but rather as a "sell-all" market à la Shapley and Shubik (1977). Private agent $i \in \mathcal{I}$ bids a positive quantity of money $C_i \geq 0$. The official price is then a function of the private strategy profile $\mathcal{S}$ defined as

$$P(\mathcal{S}) \equiv \frac{\sum_{i \in \mathcal{I}} C_i}{-n\delta_G}. \tag{28}$$

Finally, since our main goal is to highlight how defaultability and the trading protocol jointly determine the price level or fail to do so, we posit that the transfers to the private sector feature both a safe and a defaultable component. There exists a positive sequence

$(l_i, B_i)_{i\in\mathcal{I}}$ such that $l = \sum_{i\in\mathcal{I}} l_i/n > 0$ and that the net transfer to agent $i \in \mathcal{I}$ is

$$L_i(\mathcal{S}) = l_i + \frac{B_i}{B}\min\left\{B, \left(\frac{1}{n}\sum_{i\in\mathcal{I}} C_i - l\right)^+\right\}, \tag{29}$$

where $B = \sum_{i\in\mathcal{I}} B_i/n$.

Proposition 8. ***(Market-clearing policy and price-level determination)*** *The policy is feasible if and only if condition* (14) *holds and* $M \geq l$. *The set of predictable price levels includes* $[-l/\delta_G, -(l+B)/\delta_G]$. *For every* $D \in [0, B]$, *there exists an equilibrium with trades only in the official market at the price* $-(l + B - D)/\delta_G$, *and there exist equilibria in which such official trades coexist with active unofficial markets. Furthermore, across all equilibria* $c_{G,C} = \tau + \delta_G,\ c_{G,M} = M$.

The set $\Pi(\mathcal{P})\backslash[-l/\delta_G, -(l+B)/\delta_G]$ *becomes negligible as agents become negligible in the sense of Proposition 6.*

Proof. By construction the state transfers only what it receives beyond nl and positive consumption of money thus only requires $M \geq l$. Notice that there is always trade in equilibrium as $l > 0$ and $\delta_G < 0$. We leave it to the reader to check that for every $D \in [0, B]$, there exists an equilibrium whereby each agent $i \in \mathcal{I}$ bids $C_i = l_i + B_i(1 - D/B)$ at the official post and the price is $-(l + B - D)/\delta_G$. The proof that unofficial prices converge to the official one when agents become negligible is similar to that in Proposition 7, and so we omit it. All that is left to establish the proposition is the construction of an equilibrium with unofficial trade.

Equilibrium with unofficial trade. One can construct an equilibrium with a high unofficial price level similar to those when the official price is fixed and $N + P^*\delta_G = 0$. The reason is that an agent who buys dear has the same payoff from deviating towards the official post with a market-clearing price as with a fixed official price and uniform rationing. Formally, for $D \in [0, B]$, consider some agent $i \in \mathcal{I}$ and $0 < C_i < l_i + B_i(1 - D/B)$. Let us construct an equilibrium in which agent i posts C_i on the official market and sells a nominal value $l_i + B_i(1 - D/B) - C_i$ in an unofficial market with price P defined below. All the other agents post all their goods on the unofficial market and are uniformly rationed. They also bid a nominal amount $C_{-i} = \sum_{k\in\mathcal{I}} l_k + B_k(1 - D/B) - C_i$ in the official market. Let us denote $P(x) \equiv (C_{-i} + C_i + x)/(-n\delta_G)$ the price in the official market when agent i bids $x + C_i$ in it. The trading strategy for all agents except i

is optimal whenever $P > P(0)$ and posting C_i on the official market is optimal for agent i when $0 = \arg\max(C_i+x)/P(x)+(l_i+B_i(1+\min\{x;D\}/B-D/B)-C_i-x)/P$. The term $\min\{x;D\}B_i/B$ stems from the fact that bidding an additional x on the official market leads to an additional xB_i/B units of money for agent i as debt repayment. When $D > 0$, when $C_{-i}/(C_{-i}+C_i) = (1-B_i/B)P(0)/P$, the global optimum is $x = 0$. Otherwise, when $D = 0$, this condition is $C_{-i}/(C_{-i} + C_i) = P(0)/P$. As these two conditions may hold jointly, this proves the existence of an unofficial market with exchange $l_i+B_i(1-D/B)-C_i$ of money at the price $P = (C_{-i} + C_i)^2/(-n\delta_G C_{-i})$. □

An implication of Proposition 8 is that a market-clearing policy does not prevent the emergence of unofficial markets, at least away from the limit of negligible agents. When some agents bid more money than what they initially have on the official market by buying money on unofficial markets, they can force at least one agent to sell part of her money at a higher price level. This higher price level is the one that makes this latter agent indifferent between this unofficial trade and crowding herself out in the official market.

The change in trading protocol flips the relationship between default and price-level determination. The salient implication of Proposition 8 is that shifting the official trading protocol from fixed price to fixed quantity completely flips the relationship between the defaultable nature of public liabilities and price-level determination in the negligible limit. Proposition 5 shows that the fixed-price trading protocol may fail to determine the price level in the presence of a non-defaultable security because financial repression opens up the possibility of trade at multiple prices in equilibrium. Proposition 7 shows that by contrast, this fixed-price protocol determines the price level in the presence of a defaultable security in the negligible limit because only state consumption varies across equilibria with varying haircuts on public debt. The market-clearing trading protocol generates the exact opposite prediction. The price level is determined in the negligible limit if and only if the security is non-defaultable. In the presence of defaultable securities, it is the price level that absorbs all the fluctuations in the haircut on public debt across equilibria with varying default severity, whereas state consumption remains unaffected.

In sum, in the negligible limit, safe securities warrant price-level determination in the presence of a market-clearing official price whereas defaultable ones do so when the official price is fixed. This study of how the interplay between transfers and official trades shapes predictable price levels is novel to our knowledge. The following section shows how it can to some extent be captured in Walrasian models.

4 Getting the Walrasian approach right

We mentioned in the introduction the two reasons we study price-level determination in a strategically closed environment instead of using a more standard Walrasian one. First, Walrasian markets introduce *de facto* trading bans—via centralized market clearing—that contribute to the determination of the price level, and we did not want to impose them on private agents nor on the state. Second, as emphasized by Bassetto (2002), the Walrasian equilibrium concept makes it impossible to properly define feasible policies—policies that are physically possible no matter the actions of the private sector. This section applies the insights from our strategic model to analyze price determination in a Walrasian version of it. We perform a translation exercise: For each of the main policies studied in our strategic setup, we design a Walrasian counterpart that delivers outcomes that are similar to the ones we obtain in the limit of strategic (but negligible) agents. This exercise highlights the dimensions of price-level determination that are neglected under the Walrasian approach relative to the strategic approach in which the state may default, and markets do not necessarily clear. More specifically, we deliver the four following insights.

Insight #1: The fiscal theory of the price level is an extreme form of fiscal dominance. For notational simplicity, we study a version of our economy populated by $n = 1$ private agent—one needs only one representative agent in a Walrasian environment. Consider a policy comprised of a nominal transfer to that agent L, an in-kind tax τ levied on her, and government consumption of the good $\tau - \sigma$, all strictly positive real numbers. A Walrasian equilibrium associated with this policy (L, τ, σ) is comprised of consumption of money C_M and goods C_C by the private agent and of a price level $P > 0$ such that the price-taking private agent optimally consumes:

$$(C_M, C_C) = \arg\max C_C \tag{30}$$

$$s.t.\ C_M + PC_C + P\tau \leq Pe + L, \tag{31}$$

$$C_M,\ C_C \geq 0, \tag{32}$$

and, from Walras' law, such that the market for money clears:

$$L - C_M = P\sigma. \tag{33}$$

Condition (33) states that the private supply of money (endowment minus consumption of money) equals the state's demand of money (the nominal value of the goods it trades for money). Individual rationality requires that (31) binds and $C_M = 0$. Injecting this in the market-clearing condition (33) yields a unique equilibrium price level P associated with (L, τ, σ) that solves $L = P\sigma$: This is the so-called fiscal theory of the price level. In our setup, the case $B = 0$ in Proposition 8 corresponds (in the negligible limit) to such a policy with a fixed nominal liability and a fixed real quantity sold by the state for money. This policy with fixed L and σ is associated with two important assumptions. First, the state creates money and stands ready to use it to make good on its liability if it does not collect enough money in the market. We highlight that the state must stand ready to fully monetize the entire value of its liability this way in the (out-of-equilibrium) event that it does not collect any money in the official market. Second, the state adjusts the official price level in response to private demand so that its real surplus and in turn its consumption remain constant. In other words, our full-fledged model shows that an extreme form of fiscal dominance underlies the fiscal theory of the price level. The state prints money as needed to make sure that its liabilities are all perfect substitutes with money, standing ready to monetize all of it, and manipulates the price level so as to insure the government's consumption from the fluctuations of private demand. We now relax these assumptions and describe the Walrasian policy that generates the same outcomes as the strategic one in the absence of these assumptions.

Insight #2: Defaultable nominal debt is akin to real debt for price-level determination. We first relax the assumption that the transfer is nominally safe, while sticking to the one that the state trades at a market-clearing price. This corresponds to the other polar case in (the negligible limit in) Proposition 8 in which $l = 0$ and $B > 0$.[8] In this case, our setup predicts that there is price-level indetermination. All price levels within $[0, -B/\delta_G]$ can be sustained as equilibria with increasing levels of default and deflation. In the Walrasian setup, it is straightforward to see that these outcomes obtain when the policy (L, τ, σ) becomes $(\min\{L, P\sigma\}, \tau, \sigma)$. In words, the transfer is contingent on the equilibrium outcome as it depends on P. As in the strategic case, all price levels within $[0, L/\sigma]$ are then equilibrium outcomes. This basically shows that when debt is defaultable in our setup in the sense that it is not monetized, it is as if it was real in the Walrasian environment (up to an upper bound). As is well-known, the fiscal theory of the price level does not hold if debt is real.

Insight #3: An indexed surplus and defaultable debt make a hidden peg. The situation in which the two implicit assumptions in the fiscal theory of the price level—debt monetization and market-clearing price—are relaxed is covered in (the negligible limit of) Proposition 7, in which debt is defaultable and the state trades at a fixed official price. In this case, in the negligible limit, the price level is determined at P^* but there are a continuum of equilibria with varying default and surpluses. The more the state repays the less it consumes. This situation can be viewed as one of monetary dominance since there is no money creation to honor public liabilities, and policy sets a fixed price level no matter the consequences for state consumption. It is again possible to obtain these equilibria in the Walrasian environment by defining an appropriate contingent policy. The policy has to be contingent on two equilibrium outcomes, the real value of private money supply that we denote s, and the price level P. The policy that generates the outcomes in Proposition 7 is $(\min\{L, Ps\}, \tau, P^*s/P)$. In words, holding the real value of private money supply s fixed, the state makes its real surplus decreasing in the price level so that the only equilibrium price is P^*. Since the state cannot explicitly set an official price level in the Walrasian environment, it uses a contingent surplus that eliminates all possible equilibrium price levels but its target P^*. A potential connection with this strategy is the approach followed by Levy-Yeyati and Sturzenegger (2003) or Reinhart and Rogoff (2004) and the literature thereafter to identify "hidden pegs": these pegs correspond to

[8] Proposition 8 ruled out $l = 0$ only to avoid a price level equal to 0 without active trade, which we allow for here.

a low variability in the exchange rates and a large one in the amount of FX reserves. As in this Walrasian policy, there is no explicit price setting, but the trades amount to it.

Insight #4: The Walrasian auctioneer bans the state from partially controlling the price level with financial repression. A final situation that is interesting to bring to the Walrasian environment is that of financial repression—$N > -P^*\delta_G > 0$ in Proposition 5. In the negligible limit, there are multiple equilibria in which the trading volume is split between the official market at P^* and the unofficial one at $-N/\delta_G$. The unofficial volume has an upper bound that is decreasing when agents have more homogeneous money holdings. A situation with two rationed markets for the same good is of course out-of-reach of Walrasian environments. The Walrasian model prevents financial repression, and predicts given N and $-\delta_G$ a unique equilibrium price equal to the unofficial one under financial repression $N/(-\delta_G)$. Yet, the coexistence of official and unofficial prices is commonplace in practice.

5 Fiscal debtors

This section studies the baseline fixed policies defined in Section 2 that feature fiscal debtors—private agents such that $N_i < 0$. We first study policies such that all agents are fiscal debtors, and then that in which debtors and creditors coexist.

5.1 Only fiscal debtors: $N_+ = 0$ and $N_- = -N > 0$

Proposition 9. ***(Fiscal debtors and debt deflation)*** *Suppose that a policy $\mathcal{P}$ is such that $N_+ = 0$ and $N_- = -N > 0$. There exist equilibria without bankruptcy if and only if*

$$N_i + P^* e \geq 0 \; for \; all \; i \in \mathcal{I} \; and \; N + P^* \delta_G \geq 0. \tag{34}$$

In any equilibrium without bankruptcy, active prices are in $(0, P^*]$.

Condition (34) does not suffice to ensure price-level determination. A sufficient condition for strong price-level determination with $\Pi(\mathcal{P}) = \{P^*\}$ is

$$N_i + P^* \min\{\delta_G, e\} \geq 0 \; for \; all \; i \in \mathcal{I}. \tag{35}$$

Proof. We proceed in four steps.

Step 1: In any equilibrium without bankruptcy, active prices are in $(0, P^*]$**.** Suppose that an equilibrium is without bankruptcy. As agents are fiscal debtors, this implies that there must be active trading. Suppose that the highest active-trading price is strictly above P^*. Any net buyer at this price is a private agent and is not net seller anywhere from Lemma 3. But then she must be bankrupt, a contradiction.

Step 2: There exist equilibria without bankruptcy if and only if (34) **holds.** There always exists an equilibrium with a single trading post at P^* in which agent $i \in \mathcal{I}$ sells $\min\{e; -N_i/P^*\}$. This "P^*-equilibrium" features no bankruptcy if condition (34) holds because every agent can afford her taxes in this case. If (34) does not hold, any equilibrium without bankruptcy would require that the agents that are bankrupt in the P^*-equilibrium can sell goods at a strictly higher price than P^*, a contradiction from the above point.

Step 3: Condition (34) **does not suffice to ensure price-level determination.** We build a simple counter-example. Suppose that the number of agents satisfies $n > 2$, and, without loss of generality, that $(N_i)_{i \in \mathcal{I}}$ is increasing in i. Suppose that condition (34) holds, so that the P^*-equilibrium involves no bankruptcy. Suppose however that there exists $m \in [1, n-2]$ such that $(n-m+1)N_m + nP^*\delta_G < 0$ – such m exists when condition (34) holds but (35) does not. Notice that the existence of such an m given condition (34) implies that the fiscal debts of agents $i > m$ be sufficiently small in absolute values and $N + P^*\delta_G$ be sufficiently close to 0. If e is sufficiently large other things being equal, there also exists an equilibrium in which all agents $i \leq m$—the "large" fiscal debtors—are bankrupt and sell their entire endowments at an arbitrarily small unofficial price to agents $j > m$—the "small" fiscal debtors. These latter small debtors bid their entire endowment at the official post and reinvest the proceeds at this unofficial low price. To see why this is an equilibrium, notice first that for e sufficiently large, the small fiscal debtors squeeze the official market in this equilibrium. Thus a large fiscal debtor i would have a strict gain from deviating and buying cash on the official market to get out of bankruptcy if $nP^*\delta_G/(n-m+1) \geq -N_i$, which does not hold. Small fiscal debtors strictly benefit from this trade for e sufficiently large as they give up $\delta_G n/(n-m)$ consumption units in the official market and get $em/(n-m)$ in the unofficial one. Any of them would thus be strictly worse off just buying money in the official market to pay taxes and consuming strictly less than e.

Step 4: There is strong price-level determination if (35) **holds.** We show that the P^*-equilibrium is the only equilibrium if condition (35) holds. In this case, notice first that there is no equilibrium without active trading otherwise any agent such that $N_i < 0$ would be better off deviating and escaping bankruptcy by selling $-N_i/P^*$ at P^*. In any equilibrium in which there is trade at another price than P^*, there has to be a private net buyer and a private net seller. Let $\underline{P}$ denote the lowest price at which there is a private net seller i and $\bar{P}$ denote the highest price at which there is a private net buyer j. Net buyer j cannot sell at any lower price than $\bar{P}$ from Lemma 3 but must sell somewhere to avoid bankruptcy, which she could always achieve from condition (35). Thus she must sell at $P^* > \bar{P}$, which implies $\underline{P} \leq \bar{P} < P^*$. But then i, who is not net buyer at any post from condition (17) and $P^* > \underline{P}$, would be strictly better off selling only at P^* the amount required to pay her taxes, a contradiction. Condition (35) warrants that she is never too diluted by the other orders to achieve this. □

To grasp the intuition for the results behind Proposition 9, it is useful to start with the remark that there always exists an equilibrium with a single trading post at P^*, in which agent $i \in \mathcal{I}$ sells $\min\{e; -N_i/P^*\}$. We deem this equilibrium the "P^*-equilibrium".

If condition (34) holds, this equilibrium is without bankruptcy since i) each private agent has enough goods to sell to pay her net taxes ($-N_i > P^*e$), and ii) and their aggregate demand for money $-N$ is within the state's maximum supply $P^*\delta_G$. Proposition 9 shows that condition (34) is actually necessary for the existence of a bankruptcy-free equilibrium.

Debt-deflation equilibria. Interestingly, in terms of price-level determination, whereas condition (34) warrants that all predictable prices are smaller than the official target P^*, it does not suffice to rule out lower unofficial prices. As showcased by the example constructed in the proof, if (34) holds but i) fiscal debts are sufficiently heterogeneous, and ii) the maximum supply of money $P^*\delta_G$ is sufficiently close to the P^*-equilibrium demand N, then equilibria that we deem ones of "debt deflation" may arise. In these equilibria, the agents with low fiscal debt coordinate on squeezing the official market, purchasing more money than they need so as to ration the larger fiscal debtors. The small debtors can then redeploy this cash in an unofficial market in which they snap up goods sold by the distressed large fiscal debtors at a low price.

"Whatever it takes." These equilibria are reminiscent of debt-deflation dynamics in much less stylized macroeconomic models in which the financial distress of nominally indebted firms and deflation amplify each other because firms create balance-sheet or/and demand externalities for each other. Here the externality simply comes from the fact that a subset of agents can ration the others in the official market.

As a result, the state can eliminate such debt-deflation equilibria by eliminating this externality. It can do so by increasing δ_G, namely, by committing to buy more goods thereby injecting enough money in the economy so that condition (35) holds. The interpretation of condition (35) ensuring price-level determination is that the state commits to do whatever it takes to ensure that each single fiscal debtor can purchase money to honor her liabilities regardless of the (in or out of equilibrium) actions of the rest of the private sector. This implies standing ready to sell possibly much more money than the equilibrium quantity nN.

Symmetry with financial repression. Debt-deflation equilibria are such that agents with low fiscal debt corner the official market by flooding it with goods thereby forcing the more indebted ones to sell goods at a low unofficial price. These equilibria are thus symmetric to financial-repression ones in which agents with little cash coordinate on squeezing the official market by bidding acquired cash. This forces agents holding more cash to sell it at a high unofficial price level, thereby financing the squeezing strategy.

In sum, in both cases, agents with small trading needs squeeze agents with big ones out of the official market, and this forces the latter to accept less favorable unofficial trades. In both cases, the state ensures price-level determination by committing to trade larger volumes than the equilibrium one. The only difference is that this excess backing can be made arbitrarily small in the case of fiscal creditors, whereas it can be quite large in the case of fiscal debtors depending on the distribution of fiscal debts.

Finally, it is worthwhile noticing that in both cases, more unofficial trades yield more redistribution from the agents with the largest cash positions in absolute values towards the others, as the former force the latter to trade at unofficial prices that are less favorable than the official one.

5.2 Creditors and debtors: $N_+N_- > 0$

We now turn to the case in which there are both fiscal creditors and fiscal debtors.

Proposition 10. ***(Private gains from trade preclude the determination of the price level)***

(i) *If a policy is such that $N_+N_- > 0$—in words, it creates both fiscal creditors and debtors—then it does not determine the price level because $\{P^*\} \subsetneq \Pi(\mathcal{P})$.*

(ii) *If the state sells $-\bar{\delta}_G$ at $\bar{P}$ such that $N_+ + \bar{P}\bar{\delta}_G < 0$, and buys $\underline{\delta}_G$ at $\underline{P} < \bar{P}$ such that $N_i + \underline{P}\min\{\underline{\delta}_G, e\} \geq 0$ for all $i \in \mathcal{I}$, then the predictable prices must be within $[\underline{P}, \bar{P}]$, an interval that can be made arbitrarily small.*

Proof. **Point (i).** Without loss of generality, we suppose that $(N_i)_{i\in\mathcal{I}}$ is increasing. We denote n_- the fiscal debtor with the smallest debt (the smallest absolute value of $N_i < 0$). Notice first that there always exists an equilibrium with a single active trading post at P^* in which agent $i \in \mathcal{I}$ submits a buy order N_i/P^* if $i > n_-$ and sells $\min\{e, -N_i/P^*\}$ otherwise. This "P^*-equilibrium" features no bankruptcy if and only if i) $P^*e + N_i \geq 0$ for all $i \in \{1, ..., n_-\}$, and ii) $N + P^*\delta_G \geq 0$. Condition i) states that every fiscal debtor has enough goods to sell to acquire $-N_i$ of money. Condition ii) ensures that the demand of money by fiscal debtors is covered by public and creditors' supply at P^*.

We construct another equilibrium in which there is active trade at two prices, P^* and $P > P^*$. We construct the equilibrium supposing that $N_n > N_{n-1}$. We explain how to adapt the analysis to the case in which several agents share this same highest value of net transfers N_n in Step 3 below.

Step 1. Suppose first that the P^*-equilibrium features no bankruptcy. We construct an equilibrium in which agent n places a buy order with a sufficiently small (in a sense made precise below) nominal amount B in a trading post $P > P^*$. All the other agents are selling at P. The other fiscal creditors (if any) redeploy in the P^*-post the proceeds from selling their entire net endowment e at P. The fiscal debtors mix sales at P^* and at the rationed higher price P so as to meet their liabilities at the lowest cost.

We first define these fiscal debtors' strategies. For $B > 0$ sufficiently small, define

$S(B)$ the positive solution to

$$\frac{n_-e - S(B)}{(n-1)e - S(B)}B + P^*S(B) = nN_-. \tag{36}$$

For B sufficiently small, for every $i \in \{1, ..., n_-\}$, there exists a strictly positive solution $s_i(P^*)$ to

$$\frac{e - s_i(P^*)}{(n-1)e - S(B)}B + P^*s_i(P^*) = -N_i, \tag{37}$$

and by definition

$$\sum_{i=1}^{n_-} s_i(P^*) = S(B). \tag{38}$$

The equilibrium is then such that fiscal debtor $i \in \{1, ..., n_-\}$ sells $s_i(P^*)$ at the P^*-post and $e - s_i(P^*)$ at the P-post where P is defined below.

Let us now define fiscal creditors' strategies. Agent $j \in \{n_- + 1, ..., n-1\}$ (if any) sells e at P and invests a nominal amount equal to the proceeds plus N_j at P^*. Agent n invests a nominal amount $N_n - B$ at P^* and B at P. The supply at P^* is thus $s(P^*) = n(-\delta_G)^+ + S(B)$, the demand $d(P^*) = n\delta_G^+ + nN_+/P^* - B(n_-e - S(B))/[P^*[(n-1)e - S(B)]] = n\delta_G^+ + nN_+/P^* - nN_-/P^* + S(B)) \geq s(P^*)$. Let us define

$$P = \frac{P^*d(P^*)^2}{s(P^*)\left(d(P^*) - \frac{N_n - B}{P^*}\right)}. \tag{39}$$

Suppose B is sufficiently small that $N_n - B > N_{n-1} + Be/[(n-1)e - S(B)]$. Then n's trade is optimal from (39) and Lemma 4. So are the trades of the other fiscal creditors because Lemma 3 and (39) imply that they would like to sell more at P to reinvest at P^* but they hit their maximum supply e at P. Finally, fiscal debtors cannot meet their net liabilities at a lower cost as they sell as much as possible at $P > P^*$ subject to being solvent.

Step 2. Suppose now that the P^*-equilibrium features at least one bankrupt agent because there exists $i \in \{1, ..., n_-\}$ such that $P^*e < -N_i$ or because $N + P^*\delta_G < 0$. We re-create essentially the same equilibrium as in Step 1. First, for any fiscal debtor $i \leq n_-$ such that $P^*e < -N_i$, replace $-N_i$ with P^*e. Second, take one bankrupt agent, and make him add a buy order larger than N_n/P^* (which of course will be executed by the state) at P^* such that overall $N' + P^*\delta_G \leq 0$ where the new aggregate transfer per capita N' factors in the revised sell and buy orders of the bankrupt agents. It is easy to see that replacing this buy order of the bankrupt agent by another one split between P^* and P defined as in Step 1 for B sufficiently small is an equilibrium.

Step 3. In order to adapt the proof to the case in which $k > 1$ agents share the same maximum transfer N_n, we leave it to the reader to check that one only needs to let each of them invest a nominal amount B/k in a P-post defined as in Step 1.

Remark on other equilibria. Given that the goal of the proof is to offer one example of indetermination, we focused on a particular equilibrium with multiple active prices that has the advantage of being sustainable for any policy such that $N_-N_+ > 0$. It is also a particular equilibrium that we will repeatedly use in the balance of the paper. To be sure, there are in general plethora of other equilibria, including some with active trade at lower prices than P^*. Characterizing them further is not in the scope of this paper. As an illustration of this multiplicity, it is easy to see that in the case in which $\delta_G = 0$, there exists an equilibrium with a single active post with price P for any $P > 0$.

Point (ii). Point (ii) is a direct implication from Propositions 9 and 5. □

Proposition 10 states that if a policy opens up potential gains from trade between private agents because the transfers create both fiscal creditors and fiscal debtors, then it cannot determine the price level if the state is only on one side of the market (either buy or sell side). Price level determination obtains only if the policy features two official trading posts in opposite directions.

The essential reason private gains from trade make it impossible to peg the value of money with a single official trade is that money serves no other purpose than dodging bankruptcy in this economy.[9] Thus fiscal creditors are happy to trade money for goods at any price. Symmetrically, debtors are happy to trade goods for money at any price provided this makes them solvent. (They also are indifferent between any trade in the absence of any way out of bankruptcy.) The single trade of the state is thus not sufficient to coordinate the private sector on its price-level target P^*. In the presence of gains from trade between them, private agents can always simultaneously trade on this official

[9]Section 7 develops a two-date version of the model in which money may also be desirable as a store of value.

market and on unofficial ones at different price levels.

By using two trading posts, the state restricts the incentives for agents to engage in all these trades and then puts bounds on the price levels. The post on the buy side puts a floor on the price level as no agents will accept to trade at a lower price level. The post of the sell side puts a cap on the price level as no one has the incentive to trade at higher price.

The only situation that we have not covered so far is that in which policy features only a trade and no net transfers—$N_i = 0$ for all $i \in \mathcal{I}$.

Proposition 11. ***(No price-level determination without net transfers)*** *If a policy is such that $N_+ + N_- = 0$—in words, it features neither fiscal creditors nor debtors—there is no determination of the price level because there is no equilibrium with active trading:* $\Pi(\mathcal{P}) = \emptyset$.

Proof. By definition, $N_+ + N_- = 0$ implies that $N_i = 0$ for all $i \in \mathcal{I}$. As a result, agents can avoid bankruptcy by not trading and thus any agent actively trading in equilibrium cannot be bankrupt. Suppose by contradiction that an equilibrium is such that agent $i \in \mathcal{I}$ is active in at least one trading post. She cannot be net buyer in every post in which she is active since she would then be bankrupt from $\int PdD_i(P) \geq \int Pd\hat{D}_i(P) > \int Pd\hat{S}_i(P) + N_i$ with $N_i = 0$. This implies that if there exists at least one active trading post, at least one private agent is net seller somewhere. Let $\underline{P}$ denote the smallest price at which there is a private net seller. She must be net buyer somewhere else otherwise she would be strictly better off not trading. From Lemma 3, it has to be at a lower price, but since $\underline{P}$ is the smallest price at which there is a private net seller, the only possible net seller facing her is the government, and she buys at $P^* < \underline{P}$. But then this means there is a private net buyer at $\underline{P}$, as it cannot be the state which buys at this price. Let $\bar{P} \geq \underline{P}$ denote the largest price at which there is a private net buyer. A net buyer at this price cannot be net seller at any lower price from Lemma 3. But then she must be bankrupt, a contradiction. □

In this static model, the absence of net transfers implies indetermination of the price level because there exists no equilibrium with active trading in this case. Section 7 will show that by contrast, a policy with only trades may suffice to determine the price level in a dynamic environment in which money gains intrinsic value as storage.

6 Uncertainty and endogenous collateral constraint

This section constructs an equilibrium with unofficial trade in the presence of financial repression in a version of our economy in which the resource constraint (11) replaces the collateral constraint (8) in the definition of solvency. Policy uncertainty induces ambiguity-averse agents to self-impose such a collateral constraint, and thus some agents find it worthwhile selling goods in the unofficial market in order to fully cover their official orders with cash.

Formally, we modify here our baseline model along three dimensions. First, agents are ambiguity-averse and seek to maximize the minimum value of their consumptions across states of nature. Second, they define their strategies before policy is revealed, and policy can be of two types. Either it is a policy that leads to financial repression and possibly unofficial markets as in 3. in Proposition 5, or it is a policy with a market-clearing price as in proposition 8. In this latter case, the state sets the price that clears the market given its demand $\delta_G < 0$ and the nominal value of buy orders at P^*. The other dimensions of policy are unchanged. Finally, we substitute constraint (11) for constraint (8) in our definition of solvency.

The equilibrium with unofficial trading and binding collateral constraints that we construct in the proof of Proposition 10 is still an equilibrium in our environment with policy uncertainty. To see this, notice that whether the official market clears or whether there is uniform rationing does not affect the allocation of goods across bidders. The only difference is that they must pay up their entire orders in the former case and only the effective part in the latter. Thus the possibility of market clearing only implies that if agents do not impose on themselves a collateral constraint (8) in the official market, they go bankrupt in the market-clearing case. We leave it to the reader to check that private agents find it endogenously optimal to fully cover their bids in the official market, so that the construction of the equilibrium is verbatim that in the presence of an exogenous collateral constraint (8).

The assumption of uncertain price-setting admits a natural interpretation as a situation in which currency-markets participants are unsure about the willingness of a country to maintain a peg. We leave it to the reader to check that we would obtain the same results if uncertainty was about the quantity $-\delta_G$ instead—holding the other ingredients of policy including the fixed official price P^* unchanged—equal either to its value under financial repression or arbitrarily large so that there is no rationing in the official market.

7 Two-date model

This section studies a simple extension of our model with two dates $\{0, 1\}$. The economy is still populated by a state and by $n \geq 2$ private agents. These agents value only a date-1 consumption good that is obtained out of the storage of a date-0 consumption good at a linear rate $\rho > 0$ between 0 and 1. Each agent $i \in \mathcal{I}$ is endowed with $e_i > 0$ units of the date-0 consumption good, where $(e_i)_{i\in\mathcal{I}}$ is increasing without loss of generality, strictly so for brevity. We denote $e = 1/n \sum_{i\in\mathcal{I}} e_i$. We focus for brevity on the following simple policies.

Policy. A policy $\mathcal{P} = (\delta_{G,0}, P_0^*, R, \delta_{G,1}, P_1^*)$ consists in two trades and one contingent transfer:

- **Trades.** The state stands ready to buy up to $n\delta_{G,0} > 0$ units of the date-0 good at a price P_0^*, and to sell up to $-n\delta_{G,1} > 0$ units of the date-1 good at a price P_1^*.
- **Transfers.** The state multiplies any outstanding net position in money by a private agent at the end of date 0 by $R > 0$, and this defines her net position at the outset of date 1.

Notice that, except for the fact that the interest rate applies to quantities that have been decided at date 0, we study only policies that are not contingent at date 1 on the date-0 actions of the private sector.

Private trades and bankruptcy. At each date $t \in \{0, 1\}$, private agents can submit any number of buy or sell orders of the date-t good, with the restriction that they cannot place sell orders for a total quantity larger than their endowment at the outset of each date. Trading posts clear with uniform rationing as in the one-date model.

With a straightforward extension of the one-date notations, the strategy of agent $i \in \mathcal{I}$ is $\mathcal{S}_i = (D_{0,i}(.), S_{0,i}(.), D_{1,i}(.), S_{1,i}(.))$. While it does not show in notations for parsimony, the date-1 orders are conditional on history, that is, on date-0 actions. Agent $i \in \mathcal{I}$ is bankrupt at date 0 if and only if

$$\int PdD_{0,i}(P) > \frac{1}{R}\left(\int Pd\hat{S}_{1,i}(P) - \int Pd\hat{D}_{1,i}(P)\right) + \int Pd\hat{S}_{0,i}(P), \tag{40}$$

and at date 1 if and only if

$$\int PdD_{1,i}(P) > R\left(\int Pd\hat{S}_{0,i}(P) - \int Pd\hat{D}_{0,i}(P)\right) + \int Pd\hat{S}_{1,i}(P). \tag{41}$$

Equilibrium concept. A profile $\mathcal{S} = (\mathcal{S}_i)_{i\in\mathcal{I}}$ is a predictable outcome given $\mathcal{P}$ if and only if it is a subgame-perfect Nash equilibrium.

This setup departs from the one-date model in three interesting dimensions. First, apart from interest payments, there are no transfers of any sign at any date imposed on private agents. Their date-1 net cash positions result only from their voluntary date-0 trades. Proposition 11 shows that this precludes any trade in the one-date model, we will see that it is no longer the case here. Second, money may deliver consumption at date 1, and thus serves as a store of value. In this sense, date 0 is an extension of the one-date model in which money may be intrisically desirable. Finally, the absence of cash-in-advance constraint opens up the possibility of inside-money creation, whereby an agent can pay for goods at date 0 with money backed by anticipated date-1 sales of goods. The following proposition characterizes price-level determination.

Proposition 12. ***(Price-level determination with two dates.)*** *Let $r^* \equiv RP_0^*/P_1^*$. The date-0 price level is weakly determined if and ony if $r^* = \rho$. It is strongly determined if and only if*

$$r^* > \rho \text{ and } \delta_{G,1} + r^* \min\{e, \delta_{G,0}\} \leq 0, \tag{42}$$

which holds if $\delta_{G,0}$ is sufficiently small or/and $-\delta_{G,1}$ sufficiently large other things being equal. The date-1 price level is determined if and only if the date-0 one is.

Proof. Notice that if $r^* \leq \rho$, no trade is an equilibrium. It is not if $r^* > \rho$, as one agent could deviate and strictly benefiting from selling in the date-0 official post and buying in the date-1 one with the saved proceeds. Notice also that agents can avoid bankruptcy by simply not trading. The proof is in five steps.

Step 1: There is no date-0 price-level determination if $r^* < \rho$. Suppose that $r^* < \rho$. Let $P_0 \in (P_0^*, \rho P_0^*/r^*)$. An agent $i \in \mathcal{I}$ buying some goods at P_0 from an agent $j \in \mathcal{I}$ at date 0, storing them and then selling them back to j at $P_1 = RP_0/\rho = (r^*/\rho)(P_0/P_0^*)P_1^* < P_1^*$ is an equilibrium because j cannot gain from selling in the official post at date 0, i cannot benefit from buying in the official post at date 1, and no other agent can strictly increase her date-1 consumption by intervening in the official or unofficial posts.

Step 2: There is strong date-0 price-level determination if condition (42) holds. There cannot be an equilibrium in which the date-1 official post is rationed on the buy side. Otherwise this would mean that some agents have sold goods at date 0 at a higher price than P_0^* and invested some proceeds at P_1^*. The counterparts of this inside money creation must finance their date-0 bids at P_0 by selling at $P_1 > P_1^*$, but they cannot receive enough cash given that some of their buyers bid at P_1^*, a contradiction. This no-rationing implies that if there exists an active unofficial trading post with price P_0 at date 0, sellers must earn r^* on their investment. Buyers can only generate that by investing their acquired goods in the date-0 official market. Thus it must be that $P_0 < P_0^*$. But then sellers should pivot to the official post, a contradiction. The absence of unofficial trading at date 0 implies that at date 1 from Proposition 9 case 2) applied at date 1.

Step 3: There is weak price-level determination if $r^* = \rho$. Notice that the date-1 official market cannot be rationed on the buy side. The rationed buyers would have to have sold at a price above P_0^* at date 0 to earn at least ρ. But then their counterparts could not fund their date-1 bids with sales above P_1^*. Suppose the lowest unofficial price is $P_0 < P_0^*$. Sellers must earn at least ρ and thus must invest the cash below $RP_0/\rho < P_1^*$ at date 1. But it cannot be that date-0 buyers sell them enough goods at this date-1 price. This would mean they break even at ρ. They would then be strictly better off selling some of their goods at P_0^* to fund their acquisitions, possibly reducing their order, thereby earning a strictly positive NPV. Suppose the largest unofficial price is $P_0 > P_0^*$. Buyers could not fund their date-1 bids with sales above P_1^* since sellers must invest part of their proceeds at P_1^*. The only case in which they could potentially not is if the P_1^*-buy side was exactly clearing from bids funded by P_0^*-investments. But in this case one agent would be strictly better off cutting her zero-NPV official trade and earning positive NPV by selling a bit at P_0 and buying at P_1^*.

Step 4: There is no date-0 price-level determination if $r^* > \rho$ but condition(42) does not hold. For brevity, we present the proof in the case in wich $n = 2$. We start with the (most involved) case in which $e \leq \delta_{G,0}$ and $\rho < -\delta_{G,1}(2e - e_2)/(2e^2) < r^*$. In this case, in the equilibrium without unofficial trade, all agents are all-in in the official markets. Let r_2 and $r_1 > r_2$ denote the respective marginal return that agents 2 and 1 respectively earn on their last unit. Let $r' \in (r_2, r_1)$. Suppose agents 1 and 2 agree on a trade whereby 2 sells an arbitrarily small quantity to 1 at $P_0 = r'P_1/R < P_0^*$ instead of selling to the state. 1 bids her cash in the official market at date 1, and sells what it takes to agent 2 at P_1 to finance her date-0 purchase. For $P_1 > P_1^*$ but sufficiently small, 2 is happy to buy at P_1 from Lemma 4, and 1 must sell to finance her date-0 acquisition. The case in which $e \leq \delta_{G,0}$ and $\rho \geq -\delta_{G,1}(2e - e_2)/(2e^2)$ is similar. The only difference is that 1 uses the storage technology ρ instead of the official market in this unofficial trade. The cases in which $\delta_{G,0} < e$ are also similar, the only difference is that all agents earn the same marginal return in the official market. Finally, in the case of n agents, the unofficial trade has the same structure. The agent with the largest marginal return in the official market is the unofficial date-0 buyer and sells to all the others at date 1 to fund her acquisition. The construction of the equilibrium is just more cumbersome because all these other agents must be happy to buy at P_1, and a system of equations determines the respective sizes of their bids that achieves this.

Step 5: Date-0 and date-1 price level determinations are equivalent. This follows from the steps above: Date-0 determination holds when the date-1 official post is not rationed and so the date-1 price level is determined from Proposition 5. All the equilibria with multiple date-0 prices that we constructed feature also multiple date-1 prices. □

8 Applications

This section discusses several applications of our framework.

Fiscal backing: Assignats during the French Revolution. During the French Revolution, the state issued paper money in 1790, "assignats", to reimburse debts (N) and at the same time was selling the National Estates ($-\delta_G > 0$) constituted by assets seized from the church. Sargent and Velde (1995) describe the "rise and fall of the assignat" as a sequence of three periods: a "real-bills" period, a "legal restriction" period and an "hyperinflation" period. We can connect each of these periods to specific aspects of our model.

In the "real bills" period, the value of National Estates was about 2,400 millions livres and exceeded the value of the debt that the state had to honor, which was around 2,000 millions livres. In the terms of our model, in nominal terms, $N + P^*\delta_G > 0$: assignats were then backed. Here, P^* is the face value of assignats.

During the next two periods, a large quantity of assignats were issued to compensate for limited fiscal resources due to war and internal chaos. We interpret these two periods as situations in which $N + P^*\delta_G < 0$: the state does not back its currency. However, we note important differences between the two periods.

During the "legal restriction" period, under the Terror, the state implemented very harsh restrictions on hoarding assets, closed markets, and imposed wage and price controls. As Sargent and Velde (1995) note, these restrictions led to a "guillotine-backed currency":

> Under the Terror, any citizen accused of violating these laws could expect swift and arbitrary proceedings. The law on parity of the assignat called for arraignment and trial within 48 hours of the offense. The law encouraged denunciations from informants and gave extravagant powers to local authorities to enforce the restrictions. In a few dozen instances, the death penalty was imposed for crimes against the assignat or for hoarding.

The outcome is that the price level was determined and no inflation arose, consistently with our model in which the absence of private trades leads to price determination with financial repression in the case $N + P^*\delta_G < 0$.

With military successes, the Terror was overthrown and the legal restrictions were alleviated and markets reopened. In this "hyperinflation" period, private agents tried to sell all their holdings of assignats for goods and specie. This period opened up arbitrage opportunities. As reported by Sargent and Velde (1995):

> In terms of gold, prices were lower than in 1790, creating trading opportunities for the savvy. A Swiss visiting Paris hastened to change his gold for

> paper; bought hundreds of shoes, stockings, and hats; shipped them off to Switzerland; and lived in Paris like a king for a month.

We find interesting that this anecdote would not hold with Walrasian markets but requires some arbitrage opportunity. The interpretation consistent with our model is that these arbitrage opportunities stem from the rationing on the demand side of the market between gold (or specie) and currency.[10] This kind of intermediation looks very much like the one we obtain in our model, where at least one agent may find optimal to sell its goods against money because of rationing on the post of the state.

Exchange rate pegs and parallel market exchange rates. Parallel foreign exchange markets may exist for multiple reasons (e.g., in order to evade capital controls or for illegal transactions). They may also emerge from countries maintaining an overvalued official exchange rate and rationing the supply of foreign currency. In particular, Gray (2021) notes that, among the motives for such a policy decision:

> It may also be promoted by those who can profit from privileged access to FX at the official exchange rate (rent seeking behavior).

This echoes the motive for the emergence of parallel markets in our model: they emerge from the heterogeneity of agents and their price impacts on the official market. Gray (2021) also provides a list of 19 countries with official and parallel markets in the 2010-2020 period. More generally, by using estimates of export misinvoicing practices,[11] Reinhart and Rogoff (2004) show that parallel markets are widespread in fixed exchange rate regimes and concern more than half the pegs, since at least World War II. As they show, parallel market exchange rates are better indicator of monetary policy stance and they even predict realignments in the official exchange rate. This aspect of parallel markets connects with our results that the price on parallel markets may reflect the actual backing of currency $N/(-\delta_G)$.

Price controls. Another interpretation of our model is price controls. With price controls, the government imposes the prices at which private agents trade goods against

[10] To be precise, this market should be more rationed than the one between currency and shoes, stockings, and hats, at least so for the "Swiss", who can export the goods to Switzerland, while locals have perhaps already more than satiated their demands for these goods.

[11] According to IMF (1991), the ways to circumvent official markets are "smuggling, over-invoicing of imports and under-invoicing of exports, workers' remittances from abroad, and tourism".

money. Imposing a price below the equilibrium one may lead to rationing. Such a rationing may be reinforced by lower production by firms facing lower prices. In this case, the post of the government can be interpreted as the supply of goods $-\delta_G > 0$ supplied by firms at the controlled price P^*. Notice that, with this interpretation, the feasibility conditions described in Section 2.3 do not apply, as the supply of goods $-\delta_G$ does not stem from the taxation power of the government.[12]

The connection between price controls and parallel markets, or, for instance, black markets, is well documented, and examples of black markets associated with price controls abound: the US during World War II (e.g., Rockoff, 2004), Argentina in 1973-1975 (e.g., Chu and Feltenstein, 1978), Chile in 1973 (e.g., Edwards, 2023) are famous examples. This connection is so well established that the Wikipedia page on black markets even mentions "Common motives for operating in black markets are to trade contraband, avoid taxes and regulations, or evade price controls or rationing." Notice that, in our model, black markets take place between net buyers of goods (net sellers of money). With price controls, another important dimension of black markets is that they take place between net sellers of goods (net purchasers of money) and net buyers of goods (net sellers of money): firms may participate themselves in black markets to sell their production at better prices compared with the controlled one.

In-kind taxation in periods of financial repressions. Here we interpret the real resources of the state $n\tau$ as in-kind taxes. We show that in-kind taxation boosts the return on financial repression for the state. Under full backing, condition (20) relates the real value of state liabilities to the real surplus in the absence of financial repression as follows:

$$\frac{1}{P^*}\sum_{i\in\mathcal{I}} L_i = f - nc_{G,C}, \tag{43}$$

where $f \equiv n\tau + (\sum_{i\in\mathcal{I}} T_i)/P^*$ are the real fiscal resources of the state. Thus, among the policies that determine the price level, those that differ only along the modalities of tax payment—in-kind versus in cash—but not along the real value of taxes f lead to the same real allocations. By contrast, the modalities of tax payment are no longer

[12]A full model of price controls would include the endogenous supply of goods by firms $-\delta_G(P) > 0$ as a function of the controlled price P, with all the potential tools that the government may have to force firm production.

irrelevant under financial repression. As is transparent from expressions (21) and (22), a reduction in $\sum_{i\in\mathcal{I}} T_i$ and increase in τ holding f fixed shifts real resources from the private sector towards the state since this reduces the private demand for money for tax-payment motives, and thus increases households' forced money holdings. This is reminiscent of historical situations in which a financially distressed public authority imposed in-kind payments for some taxes, such as the Confederacy during the US civil war or the USSR in the 20s.

More precisely, the Confederacy faced important difficulties to levy taxes and had to rely on money printing to finance its war efforts. Through the lens of our model, the fraction of new money used to pay for the wages of the army, for example, can be understood as nominal transfers N. The large issuance of money led to hyperinflation. In particular, no fiscal backing was supporting the value of money: As noted by Nielsen (2005), "The Treasury bills issued during the war had a peculiar feature: They were redeemable for gold two years after the war ended, which meant that the value of the bills was partially tied to expectations of victory for the Confederacy." This lack of backing can be understood as a situation of financial repression in which $N + \delta_G P^* > 0$.

Consistently with this situation of financial repression, the mix between nominal and in-kind taxes did matter for the Confederacy: As documented by Burdekin and K. (1993), in-kind taxes contributed more than 50% of the Confederate revenue for example in the ten first months of 1863.

Private monies. While our main set of implications concerns government-issued liabilities, our framework may also be used to think about privately-issued monies.

Stablecoins. Our model provides a useful framework for analyzing the determination of stablecoin prices. Major stablecoins aim to maintain a peg to the US dollar by backing tokens with USD-denominated assets, such as US Treasury bonds. Typically, issuers exchange tokens for dollars in a primary market and promise to redeem them at par value. Stablecoins are then traded in secondary markets.[13] In the context of our model, the stablecoin represents "money", while the US dollar is the "good". On the primary market, the issuer (analogous to the state in our model) posts a sell order $-\bar{\delta}_G$

[13]Numerous studies have explored stablecoin stability: Lyons and Viswanath-Natraj (2023), d'Avernas et al. (2022), and Routledge and Zetlin-Jones (2022), among others.

at price $\bar{P}$ and a buy order $\underline{\delta}_G$ at price $\underline{P}$, with the spread $\bar{P} - \underline{P}$ representing transaction fees. The feasibility conditions involve the dollar value of reserve assets held by the stablecoin issuer. According to our model, stablecoin prices on the secondary market should fluctuate within this band as long as token issuance $\underline{\delta}_G$ exceeds all the potential demand ($\underline{\delta}_G \geq \max_{i \in \mathcal{I}}(-N_i)^+/\underline{P}$) and the backing remains sufficient, i.e., $-\bar{\delta}_G > N_+/\bar{P}$. If the backing is perceived as insufficient, prices may breach the upper bound $\bar{P}$, as seen during the brief depeg of USDC following the SVB failure in March 2023 (Aldasoro et al., 2023).

While fiat-backed stablecoins have been the most successful, they are not the only type. Before the Terra Luna collapse in May 2022, algorithmic stablecoins were gaining popularity.[14] In this ecosystem, the stablecoin UST was circulating and differed from fiat-backed stablecoins in its peg mechanism. A smart contract allowed the exchange of one UST for \$1 worth of LUNA —the native token circulating on Terra and a claim on Terra's transaction fees and block rewards, but also a token giving access to Terra applications. This setup resembles the primary market described earlier, except that δ_G was not fixed in USD but varied with LUNA's dollar value, which depended on the overall value of the Terra system. The collapse of UST was triggered by a sharp decline in LUNA's price, stemming from a sudden loss of confidence in the system's sustainability (see Liu et al., 2023, for further details). Liu et al. (2023) highlight heterogeneity in the use of the primary versus secondary markets during the May 2022 crisis:

> Interestingly, we find that Alameda Research, a cryptocurrency trading firm closely affiliated with the FTX exchange, conducted the largest amount of UST-LUNA swaps among Anchor depositors. It seems that the swap fees and uncertainty about the execution price of LUNA on exchanges discouraged most other Anchor depositors from utilizing the native swap contract as an exit strategy. But Alameda Research, with its advantageous access to the FTX exchange, had a competitive advantage over other market participants.

This observation suggests that certain sophisticated agents, such as Alameda Research, had a competitive edge in trading on the primary market due to their superior access and resources to resell LUNA, while other participants opted for the secondary market, even at significantly discounted prices. In this scenario, market segmentation

[14] The Terra Luna ecosystem was the third largest in the market, with a capitalization of \$50 billion, which collapsed to nearly zero in just three days.

arose not from differences in price impacts due to rationing and order sizes (as modelled in our setting), but from the varying capabilities of investors.

Another noteworthy aspect of the crash was the role of Anchor, a lending platform for UST. TerraForm Labs, the creator of the Terra network, subsidized the interest rate on UST deposits in Anchor to stimulate demand. Until May 2022, the interest rate stood at 19.5%.

Free banking era. Under Free Banking, as experienced by the US between 1837 and 1863, banks' liabilities were in form of banknotes redeemable in specie. These banknotes, in contrast with deposits,[15] were massively exchanged on secondary markets. In particular, brokers specialized in trading these banknotes. As Gorton (1999) showed (see also Jaremski, 2011), the prices at which these banknotes traded reflected banks' default risk, which relates to the option to redeem the banknote (and the transportation cost to travel to the bank to redeem it) – in particular, "wildcat" banking, i.e., banks which intentionally overissue money compared with their ability to redeem it, is considered to be at most marginal. To be more precise, Jaremski (2011) notes that banknotes were traded at par locally, unless the bank was closed or suspended. From the perspective of our model, money is banknotes while the good stands for species. The commitment of the bank to redeem banknotes at par is consistent with a fixed-price order. The situation in which there is no redemption risk is akin to one in which backing is sufficient ($\delta_G P^* + N < 0$), while the one in which banknotes are traded at a discount, even in their place of issuance, is one in which backing is insufficient, ($\delta_G P^* + N > 0$).

Money market funds and other asset-backed funds. Another application of our model could be Money Market Mutual Funds. These funds implicitly guarantee that the value of their shares is 1$. Interpreting the government in our model as the MMF and the private sector as the MMF's shareholders willing to withdraw their funds from the MMF, this guarantee can be modeled by a fixed-price order and $-\delta_G$ is the measure of resources that the MMF can use to redeem its shares. When this backing is sufficient ($N + P^*\delta_G < 0$), the price of shares is pegged at P^*. When this backing is insufficient ($N + P^*\delta_G > 0$), the price of shares may fall on secondary markets. Such situations of

[15] As noted by Gorton (1985), deposits are "double claims" that is 'a claim on a specific agent's account at a specific bank.

insufficient backing can be connected to the runs that MMF experienced in the aftermath of the 2008 financial crisis (see Gorton and Metrick, 2010, among many others).

In the case where the fund redeems a function of the value of its resources $-\delta_G$, the situation is then akin to the market-clearing price case that we discuss in Section 3.4.

9 Conclusion

This paper studies the extent to which distinctive capacities of the state—issuing money, declaring taxes, and implementing bankruptcy, together with its trades of money for other goods, imply that public financial policy determines the price level. Our concept of price-level determination is robust in the sense that we set all agents free to trade whichever quantities at whichever prices they want. In addition to characterizing policies that do determine the price level, we also offer a description of the set of predictable price levels even when this is not a singleton. We obtain realistic predictable outcomes such as the rise of unofficial prices and that of endogenous intermediaries in this case of price-level indetermination. Our strategically closed framework also unveils the important out-of-equilibrium dimensions of policies that shape the equilibrium outcomes behind the curtains in Walrasian environments.

Our focus has been on economies in which neither money nor other public liabilities play an important role at overcoming frictions. A natural route for future research is to incorporate such a role in the analysis. This would in particular allow us to develop a normative analysis, assessing for example the welfare costs of price-level indetermination. Other situations that our strategically closed model is well-suited to study are that of the coexistence of multiple (private or/and public) monies. We also leave this for future work.

References

Aiyagari, S. R. and N. Wallace (1997): "Government Transaction Policy, the Medium of Exchange, and Welfare," *Journal of Economic Theory*, 74, 1–18.

Aldasoro, I., P. Mehrling, and D. H. Neilson (2023): "On par: A Money View of stablecoins," BIS Working Papers 1146, Bank for International Settlements.

Bassetto, M. (2002): "A Game-Theoretic View of the Fiscal Theory of the Price Level," *Econometrica*, 70, 2167–2195.

Bassetto, M. and C. Phelan (2015): "Speculative runs on interest rate pegs," *Journal of Monetary Economics*, 73, 99–114.

Burdekin, R. and L. F. K. (1993): "War Finance in the Southern Confederacy, 1861-1865," *Explorations in Economic History*, 30, 352–376.

Burdett, K., S. Shi, and R. Wright (2001): "Pricing and Matching with Frictions," *Journal of Political Economy*, 109, 1060–1085.

Chu, K.-Y. and A. Feltenstein (1978): "Relative Price Distortions and Inflation: The Case of Argentina, 1963-76," *Staff Papers-International Monetary Fund*, 452–493.

d'Avernas, A., V. Maurin, and Q. Vandeweyer (2022): "Can Stablecoins Be Stable?" Working Paper 2022-131, SSRN.

Dubey, P. (1982): "Price-Quantity Strategic Market Games," *Econometrica*, 50, 111–126.

Dubey, P. and M. Shubik (1980): "A Strategic Market Game with Price and Quantity Strategies," *Zeitschrift fur Nationalokonomie / Journal of Economics*, 40, 25–34.

Edwards, S. (2023): "The Debauchery of Currency and Inflation: Chile, 1970-1973," Working Paper 31890, National Bureau of Economic Research.

Flood, R. P. and P. M. Garber (1984): "Collapsing exchange-rate regimes: Some linear examples," *Journal of International Economics*, 17, 1–13.

Gorton, G. (1985): "Clearinghouses and the Origin of Central Banking in the United States," *The Journal of Economic History*, 45, 277–283.

——— (1999): "Pricing free bank notes," *Journal of Monetary Economics*, 44, 33–64.

Gorton, G. and A. Metrick (2010): "Regulating the Shadow Banking System," *Brookings Papers on Economic Activity*, 41, 261–312.

Gray, S. T. (2021): "Recognizing Reality–Unification of Official and Parallel Market Exchange Rates. International Monetary Fund (IMF) Working Papers," Tech. rep., WP/21/25.

Hu, T., J. Kennan, and N. Wallace (2009): "Coalition-Proof Trade and the Friedman Rule in the Lagos-Wright Model," *Journal of Political Economy*, 117, 116–137.

IMF (1991): *Macroeconomic Models for Adjustment in Developing Countries*, USA: International Monetary Fund.

Jaremski, M. (2011): "Bank-Specific Default Risk in the Pricing of Bank Note Discounts," *Journal of Economic History*, 71, 950–975.

Knapp, G. F. (1924): *The State Theory of Money*, History of Economic Thought Books, McMaster University Archive for the History of Economic Thought.

Krugman, P. (1979): "A Model of Balance-of-Payments Crises," *Journal of Money, Credit and Banking*, 11, 311–325.

Lagos, R. (2000): "An Alternative Approach to Search Frictions," *Journal of Political Economy*, 108, 851–873.

Lerner, A. P. (1947): "Money as a Creature of the State," *American Economic Review*, 37, 312–317.

Levy-Yeyati, E. and F. Sturzenegger (2003): "To Float or to Fix: Evidence on the Impact of Exchange Rate Regimes on Growth," *American Economic Review*, 93, 1173–1193.

Li, Y. and R. Wright (1998): "Government transaction policy, media of exchange, and prices," *Journal of Economic Theory*, 81, 290–313.

Liu, J., I. Makarov, and A. Schoar (2023): "Anatomy of a Run: The Terra Luna Crash," NBER Working Papers 31160, National Bureau of Economic Research, Inc.

Lyons, R. K. and G. Viswanath-Natraj (2023): "What keeps stablecoins stable?" *Journal of International Money and Finance*, 131, 102777.

Nielsen, E. (2005): "Monetary Policy in the Confederacy," *Region Focus – Richmond Fed*, 9, 40–43.

Reinhart, C. M. and K. S. Rogoff (2004): "The Modern History of Exchange Rate Arrangements: A Reinterpretation," *Quarterly Journal of Economics*, 119, 1–48.

Rockoff, H. (2004): *Drastic measures: A history of wage and price controls in the United States*, Cambridge University Press.

Routledge, B. and A. Zetlin-Jones (2022): "Currency stability using blockchain technology," *Journal of Economic Dynamics and Control*, 142, 104155, the Economics of Digital Currencies.

Sargent, T. J. and F. R. Velde (1995): "Macroeconomic Features of the French Revolution," *Journal of Political Economy*, 103, 474–518.

Sargent, T. J. and N. Wallace (1981): "Some unpleasant monetarist arithmetic," *Quarterly Review*.

Schmeidler, D. (1980): "Walrasian Analysis via Strategic Outcome Functions," *Econometrica*, 48, 1585–1593.

Shapley, L. and M. Shubik (1977): "Trade Using One Commodity as a Means of Payment," *Journal of Political Economy*, 85, 937–968.

Smith, A. (1776): *An Inquiry into the Nature and Causes of the Wealth of Nations*, London: W. Strahan and T. Cadell.

Starr, R. M. (1974): "The Price of Money in a Pure Exchange Monetary Economy with Taxation," *Econometrica*, 42, 45–54.