

Backing Money*

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Abstract

Issuers of intrinsically worthless currencies typically commit to exchange them for other currencies, assets or commodities so as to pin down their relative values. We study the circumstances under which such a backing policy uniquely determines this relative value. We exhibit a very general condition for determinacy—All agents must earn the same marginal return when they redeem their monetary holdings. When this condition is not satisfied, for example because backing is fractional, rent seeking by some agents leads to the possibility of unofficial trade at various price levels. Among a wide array of standard backing policies, only a peg with excess backing of the outstanding money supply satisfies this condition in a robust sense.

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1 Introduction

An issuer of money backs it when it supports its value by committing to exchange it for assets, commodities, or currencies according to a rule governing the terms and quantities of exchange. Backing has been pervasive throughout monetary history. Metallic standards have historically been the purest form of backing, and currency pegs remain prevalent today. Money is ultimately a liability of the consolidated government, and thus fiscal resources can be deployed to back it. According to the fiscal theory of the price level, such fiscal backing alone can in fact be sufficient to determine the price level. Backing is not unique to state-issued monetary instruments. Stablecoins derive their value from commitments to redeem digital tokens for official currencies at fixed rates. Across these diverse settings, the value of intrinsically worthless monetary instruments is supported by their issuers' commitments to absorb them at specified terms.

This paper asks whether the issuer of an intrinsically worthless money can determine its value relative to other commodities, currencies, or assets by exchanging it against them. This simple question is important given the pervasiveness of backing. Perhaps surprisingly, it proves difficult and leads to subtle answers.

We first address this question in the simplest one-date one-good endowment economy. In this economy, the issuer of an intrinsically worthless money—the state—implements a policy comprised of transfers and trades of money. Each private agent i receives a positive transfer of L_i monetary units. The state also maintains an official exchange in which the private sector can redeem money for goods as follows. If agent i bids Y_i monetary units in the official exchange, he receives $X_i(\mathcal{Y})$ units of the desirable good, where $\mathcal{Y} = (Y_i)$ is the profile of all bids. This general specification only assumes that $X_i(\cdot)$ be increasing concave in Y_i .

In addition to participating in this official exchange, all private agents are free to unofficially trade money for goods with each other. Subject to a natural solvency constraint, each agent can submit orders to buy or sell a given quantity of goods at a given price. That is, an order features a trading direction (buy or sell), a quantity of goods, and a unit price, and each agent can submit any number of such orders. Orders submitted at the same price are executed using proportional rationing: If total buy and sell quantities at this price differ, orders on the larger side of the market are scaled down proportionally.

Given a policy of transfers and official exchange, we deem a price level predictable if there exists a Nash equilibrium with active trade at this price. We consider that a policy determines the price level if there is at most one such predictable price level associated with it. Thus, we propose a novel approach to price-level determination that is inspired from mechanism design: We impose an incentive-compatibility constraint on policies requiring that private agents do not trade in other terms than the official one.

Motivating the setup. We believe that our approach to price-level determination based on this strategic model of price formation is the appropriate one for two reasons. First, we do not want determinacy to be in part due to a form of trading ban or price control such as imposing a centralized market¹, or granting an exclusive price-setting privilege to a strict subset of (possibly unmodelled) agents. In our setup, all agents are price setters and are free to set any terms of trade with each other. Second, this strategically closed model enables us to study how out-of-equilibrium actions by the state shape equilibrium outcomes, a question that is beyond the reach of standard Walrasian environments. Yet some of the fiercest debates on price-level determination rely heavily on informal out-of-equilibrium arguments. Prominent examples, which we analyze in this paper, include the role of sovereign default in the fiscal theory of the price level, or the possibility of ruling out non-monetary equilibria through fractional backing (Obstfeld and Rogoff, 1983). One of our contributions is to formalize these arguments and assess them rigorously. The analysis of this conceptually simple model yields four main insights.

1. Price-level determinacy requires that all agents earn the same official marginal return. A simple and general necessary condition for determinacy is that when all agents bid their transfers L_i in the official exchange, they all earn the same marginal return— $\partial X_i / \partial Y_i(\mathcal{L})$ is constant over all agents, where $\mathcal{L} = (L_i)$. Otherwise, unofficial trading arises at multiple predictable prices. Agents with high official marginal returns bid more than the value of their transfers in the official exchange. They fund these bids with unofficial sales of goods to agents with low-official returns at prices in the convex hull of the official marginal returns. The low-return agents may be strictly worse off in the presence of this endogenous unofficial intermediation. Yet they still voluntarily participate conditionally on the trades of the high-return agents. In other words, in the presence of imperfect backing, rent extraction by some agents at the expense of others, as opposed to reaping universal gains from trade, may generate voluntary trade. We document in Section 8 that describes some empirical implications from our theory that rent seeking by agents with privileged official access is a pervasive source of unofficial prices and indeterminacy across time and contexts.

2. In particular, neither fractional backing nor letting the official market clear pin down the price level. We then focus on official exchange protocols with uniform pricing, that is, such that all agents pay the same unit price for each consumption unit that they receive from the state. The price is in general a function of the bids $\mathcal{Y}$. In the particular case of a peg whereby the state trades at a fixed price level, our condition for determinacy that marginal returns be equal implies that there is determinacy if and only if either all agents have the same transfer or there is strict excess backing. Strict excess

¹A centralized market may nonetheless endogenously arise here.

backing means that the state must commit to sell strictly more goods than the amount that actually trades in equilibrium. By contrast, fractional backing with rationing leads to indeterminacy.

If the official exchange is a market à la Shapley and Shubik (1977) whereby the state sells a fixed quantity τ at the price $(\sum Y_i)/\tau$ that clears the market given the bids $\mathcal{Y}$, then there is generic indeterminacy as well because of heterogeneous price impacts. Yet as agents become negligible in a sense made precise, all unofficial prices converge to this official endogenous market-clearing price. Put simply, the economy converges to a Walrasian one in which the backing τ of a money supply $\sum_i L_i$ determines a price level $(\sum_i L_i)/\tau$.

3. Sovereign default is an additional source of price-level indeterminacy. A central assumption behind this latter determinacy in the limit of negligible agents is that the transfers $\mathcal{L}$ be non-contingent on the actions of the private sector. This means in particular that the state commits to create as much money as needed to make good on them in the situation in which it fails to collect a sufficient amount of money in the official exchange to fund them. If by contrast the state uses only this collected money to fund transfers, and thus defaults on promised payments if need be, a market-clearing price leads to indeterminacy even with atomistic agents. Only a peg warrants determinacy in this case. In other words, we find that the fiscal theory of the price level, whereby debt and real surplus suffice to determine the price level, holds only if debt is a substitute for money, in the sense that the state stands ready to monetize all or part of it to avoid outright default.

4. Full in-equilibrium backing is still necessary for determinacy when the horizon is infinite and money is essential. Finally, we investigate whether the result that the state needs to fully back the entire money supply to ensure determinacy in case of a peg owes to the static nature of our model, in which official backing is the only reason money may be of any value. To do so, we embed our modeling of policy and private trades in an infinite-horizon model à la Wallace (1980), in which money is essential as a medium of exchange across overlapping generations of agents. We find that our results carry over. The state must back the entire supply of money of the private sector in the official exchange to ensure determinacy. The only difference is that this private supply is now the money supply of the old agents net of the money demand of the young agents estimated at the peg. This difference between net and gross private supply at the official exchange rationalizes why the gross supply is never fully backed in practice—think of the gold standard era. Yet, this confirms the insight from the elementary static model that backing needs to take place along the equilibrium path and to absorb the entire net supply in order to determine the price level. Out-of-equilibrium fractional backing along

the lines suggested by Obstfeld and Rogoff (1983) leads to indeterminacy in our setup.

In sum, a salient result is that, among standard backing arrangements, only an overcollateralized peg uniquely determines the value of money by ruling out unofficial valuations below the official target (an excessively high price level). When the issuing authority instead allows the official market to clear, this determination fails to be robust to either market power—a common feature of wholesale money markets, or to the issuance of nonmonetary liabilities such as defaultable debt. Monetary policy in most jurisdictions, where a well-capitalized central bank sets an interest-rate target, broadly resembles such a stylized overcollateralized peg. The model implies that reducing the central bank’s resources, for instance by canceling public debt or requiring a large remittance, may lead effective short-term rates to drop below the official target and to display endogenous volatility as captured by multiplicity of equilibria in our setup.

Related literature. Bassetto (2002) pioneers the strategic foundations of price-level determination by public financial policy. His goal is to offer an example of an economy in which the fiscal theory of the price level applies. We share with him a strategically closed environment that highlights the importance of credible out-of-equilibrium actions in shaping equilibrium outcomes. We extend this important paper in three directions. We lift the restriction to centralized markets and market clearing and let any agent be a price setter. We allow for a general specification of official exchange protocols. Finally, we consider various types of claims issued by the state—allowing in particular for sovereign default. This way we generate a number of novel and, we believe empirically relevant insights.

Our setup revisits three old literatures. First, given the central role of strategic exchange in our framework, we build on a large body of work on the strategic foundations of Walrasian equilibrium. The contributions that we borrow most directly from are Dubey (1982), Dubey and Shubik (1980), Schmeidler (1980), and Shapley and Shubik (1977). Second, we also offer a natural connection between the determination of the price level with pegs and an older literature that studies the aggregate implications of allocation mechanisms in non-clearing markets—see Bénassy (1993) for a brief exposition of some of its important insights. Flynn et al. (2026) also study an economy in which firms quote both prices and quantities in therefore non-clearing markets, with the different objective of revisiting New Keynesian insights. The third literature that we relate to is the one that studies the interaction of non-price allocations mechanisms such as queues with a Walrasian market, (e.g., Polterovich, 1993; Sah, 1987; Stahl and Alexeev, 1985). Unlike in this literature, the structure and liquidity of private markets results endogenously from the nature of the official allocation protocol here.

It is interesting to relate the possible rise of unofficial markets in our setup to that of equilibria with runs in the banking literature as in Bryant (1980) or Diamond and

Dybvig (1983). In Diamond and Dybvig (1983), the patient agents, who benefit the least from going early to the bank, may create rationing if they coordinate on running on it, thereby extracting resources meant for the impatient ones. In our setup, similarly, the agents with the smallest amounts of money to redeem may extract rents from the others by coordinating to congest the official market. A difference is that in Diamond and Dybvig (1983), the run equilibria destroy resources through inefficient liquidation and are thus Pareto inferior, whereas equilibria with various degrees of unofficial trading are generically not Pareto-ranked here.

Bassetto and Phelan (2015) find like us that bounds on quantities for public interventions at a fixed price in money markets may generate multiple equilibria. In their setup, crises may arise during which the private sector exhausts the public lending capacity, money grows fast, and the private interest rate is higher than the official one. Their contribution highlights that the exact mechanism used to implement monetary policy—e.g., standing ready to trade at a given fixed rate up to a limit versus conducting open-market operations to achieve a target rate—can have significant aggregate implications. One can view our contribution as a related general study of the impact of the mechanisms by which the state transfers and trades money on the determination of the price level.

Our approach also has points of contact with the literature that endogenizes trading frictions as pure coordination failures in economies that are not plagued by informational nor search frictions. Important contributions include Lagos (2000) and Burdett et al. (2001). One can view our results as identifying public financial policies that coordinate the private sector on trading at the official price target. We emphasize in particular the central role of the exchange protocol selected by the state. In this sense, our approach applies to this context of trading frictions as coordination failures the approach pioneered by Hu et al. (2009), that endogenizes trading mechanisms in the presence of search frictions.

Finally, our dynamic model that studies backing in an environment in which money is essential echoes similar attempts in the presence of search frictions. Aiyagari and Wallace (1997) and Li and Wright (1998) show that more backing by the state makes it easier to sustain Pareto-dominant monetary equilibria.

2 Setup

This section outlines our simple one-date one-good economy. It studies a policy that consists in fixed monetary transfers to the private sector, that may be interpreted as resulting from liabilities issued in an unmodelled past, and in a commitment to exchange money for the unique consumption good. Such a static setup, unusual in monetary economics, showcases our main insights in the simplest environment. Section 7 shows how they carry over to a dynamic environment.

Formally, the economy comprises a public sector—“the state”—and $I \geq 2$ private agents indexed over $\mathcal{I} \equiv \{1, \dots, I\}$. There are two divisible economic goods, one deemed “the good” and the other “money” henceforth. The good is intrinsically desirable to private agents whereas money is not. Each private agent thus ranks any bundles of the good and money using the standard ordering of their respective quantities of the good only.

Each private agent is endowed with $e > 0$ units of the good. The state is endowed with $I\tau > 0$ units of it. The state can produce money. Private agents cannot.

2.1 Policy

The state enforces a policy that features backing—a mechanism to exchange goods for money—monetary transfers, and money creation. We describe each of these components in turn.

Official exchange. If agent $i \in \mathcal{I}$ bids $Y_i \geq 0$ monetary units in the official exchange, she gets $X_i(\mathcal{Y}) \geq 0$ units of the consumption good and transfers $\hat{Y}_i(\mathcal{Y}) \in [0, Y_i]$ monetary units to the state, where $\mathcal{Y} \equiv (Y_i)_{i \in \mathcal{I}}$. For each $i \in \mathcal{I}$, the function X_i is increasing concave in Y_i . Notice that $\hat{Y}_i$ is payoff irrelevant for private agents given that money has no consumption value.

Uniform pricing. One class of official exchange protocols that is pervasive in practice and in models are the ones in which each unit of good is exchanged by the state for the same quantity of money.

Definition 1. (*Uniform pricing*) An exchange protocol prices uniformly if and only if there exists a function $\mathcal{P}$ such that for all $i \in \mathcal{I}, \mathcal{Y} \geq 0$, $\mathcal{P}(\mathcal{Y})X_i(\mathcal{Y}) = \hat{Y}_i(\mathcal{Y})$.

Simple examples that we will use repeatedly include the sale of a fixed quantity of goods at the market-clearing price, and sale at a fixed price—“a peg”.

Example. (Fixed-quantity trading) Here the state sells off its endowment $I\tau$ (or any smaller fixed quantity) in a market à la Shapley and Shubik (1977) in which the price is $(\sum_{i \in \mathcal{I}} Y_i)/(I\tau)$. In this case,

$$X_i = \frac{I\tau Y_i}{\sum_{j \in \mathcal{I}} Y_j}, \tag{1}$$

and $\hat{Y}_i = Y_i$.

Example. (Fixed-price trading or peg) The state could also sell up to τI units at a price $P^* > 0$, using a rationing scheme if $\sum_{j \in \mathcal{I}} Y_j > P^* I\tau$. In the case of proportional

rationing,

$$X_i = Y_i \min \left\{ \frac{1}{P^*}, \frac{I\tau}{\sum_{j \in \mathcal{I}} Y_j} \right\}, \quad (2)$$

and each agent transfers only part of her bid to the state when rationed: $\hat{Y}_i(\mathcal{Y}) = Y_i \min\{1, I\tau P^*/(\sum_{j \in \mathcal{I}} Y_j)\}$.

For brevity, we restrict the analysis to protocols such that private agents cannot sell goods in the official exchange. We will see that this is actually without loss of generality.

Transfers. Each agent $i \in \mathcal{I}$ receives a transfer of $L_i(\mathcal{Y})$ monetary units from the state, where $L_i(\cdot) \geq 0$.

We interpret these positive transfers as resulting from liabilities issued in an unmodelled previous date. Most of our analysis will focus on the case in which these transfers are constant, corresponding to nominally safe claims such as base money. Section 6 also tackles the case in which they actually depend on the money received by the state via a dependence on $\mathcal{Y}$, which encompasses defaultable securities such as sovereign debt.

Money creation. Policy also features the production of $IM \geq 0$ units of money.

In sum, a policy $\mathcal{P}$ is defined by $\mathcal{P} = \{(L_i(\cdot), X_i(\cdot), \hat{Y}_i(\cdot))_{i \in \mathcal{I}}, M\}$. The state consumes $I_{CG,C} \in \mathbb{R}$ units of the good and $I_{CG,M} \in \mathbb{R}$ units of money after it has interacted with the private sector as detailed below.

2.2 Private sector

Taking policy $\mathcal{P}$ as given, private agents play a simultaneous game whereby they make decisions to bid money in the official exchange and trade money for goods with each other.

Unofficial trading. Each agent $i \in \mathcal{I}$ can submit any number of orders to buy or sell a given quantity of goods at a given price. That is, an order features a trading direction (buy or sell), a quantity of goods, and a unit price, and each agent can submit any number of such orders. The only restriction is that the total size of his sell orders—the sum of the quantities of goods over all his sell orders—cannot exceed her endowment e . This is essentially a no short-sales constraint, as one cannot sell goods that one needs to buy.

The trading strategy of agent $i \in \mathcal{I}$ is conveniently described by the functions detailing his cumulative orders. The respective cumulative buy and sell orders at prices (weakly) lower than P , $D_i(P)$ and $S_i(P)$ respectively, are increasing step functions over $[0, +\infty)$

satisfying:

$$D_i(0) = S_i(0) = 0, \quad (3)$$

$$\lim_{+\infty} S_i \leq e. \quad (4)$$

Let us denote, for all $P > 0$, $d_i(P)$ and $s_i(P)$ the respective buy and sell orders of i at the price P :

$$d_i(P) \equiv \int \mathbb{1}_{\{p=P\}} dD_i(p), \quad s_i(P) \equiv \int \mathbb{1}_{\{p=P\}} dS_i(p). \quad (5)$$

In sum, the strategy of agent $i \in \mathcal{I}$ is $\mathcal{S}_i = (Y_i, D_i(\cdot), S_i(\cdot))$, where $Y_i \geq 0$ is his official bid and D_i, S_i describe his unofficial trades. Let $\mathcal{S} = (\mathcal{S}_i)_{i \in \mathcal{I}}$ denote the strategy profile of the private sector. All that is left is the description of how a policy $\mathcal{P}$ and such a profile $\mathcal{S}$ map into the payoff of each private agent. This requires defining both how private trades settle and how the state handles insolvent agents—the bankruptcy process.

Order execution, bankruptcy mechanism, and payoffs. We now describe how order execution and a bankruptcy mechanism shape the payoff of each agent given a policy $\mathcal{P}$ and a strategy profile $\mathcal{S}$.

Order execution. For all $P > 0$, let $d(P) = \sum_{i \in \mathcal{I}} d_i(P)$ and $s(P) = \sum_{i \in \mathcal{I}} s_i(P)$ denote the aggregate buy and sell orders at the unofficial trading post P . If $d(P)s(P) = 0$, then no trade takes place. Otherwise, there is proportional rationing: The smallest side of the market is fully executed and the other side is rationed pro rata the size of each order. Formally, each private agent $i \in \mathcal{I}$ buys and sells effective quantities $\hat{d}_i(P)$ and $\hat{s}_i(P)$ such that:

$$\hat{d}_i(P) \equiv d_i(P) \min \left\{ 1, \frac{s(P)}{d(P)} \right\}, \quad \hat{s}_i(P) \equiv s_i(P) \min \left\{ 1, \frac{d(P)}{s(P)} \right\}. \quad (6)$$

We respectively denote $\hat{D}_i(P)$ and $\hat{S}_i(P)$ the respective cumulative effective purchases and sales of agent $i \in \mathcal{I}$ at prices lower than P . The following definition is natural. It states that a trading post is active if and only if at least one private agent strictly gains or loses goods in it.

Definition 2. (Active trading post, net buyer, net seller) Agent $i \in \mathcal{I}$ is net buyer (respectively net seller) at the trading post P if and only if $\hat{d}_i(P) > \hat{s}_i(P)$ ($\hat{s}_i(P) > \hat{d}_i(P)$ respectively). The trading post P is active if and only if at least one agent is net buyer P .

Notice that an active trading post always features both at least one net buyer and

one net seller by definition.

Bankruptcy mechanism and payoffs. Given a policy $\mathcal{P}$ and a strategy profile $\mathcal{S}$, the payoff to agent $i \in \mathcal{I}$ depends on whether he is solvent or not, where we define solvency as follows:

Definition 3. (*Solvent agent*) Agent $i \in \mathcal{I}$ is solvent if and only if

$$\int PdD_i(P) + Y_i \leq L_i(\mathcal{Y}) + \int Pd\hat{S}_i(P). \quad (7)$$

In words, the monetary value of an agent's unofficial buy orders $\int PdD_i(P)$ and official bid Y_i cannot exceed the sum of his received transfer L_i plus his effective sales $\int Pd\hat{S}_i(P)$. Condition (7) therefore introduces a restriction on the sizes of the unofficial and official bids that agents can place. They need to post collateral, in the form of either money or effective sales, for each unit that they bid. This is stronger than a standard budget constraint in which the left-hand side of (7) would become $\int Pd\hat{D}_i(P) + \hat{Y}_i(\mathcal{Y})$, that is, bids would be replaced by money actually spent. This stronger constraint plays a role only when the official exchange protocol is manipulable. If the official exchange is not manipulable, all our results hold under a standard budget constraint.²

If agent $i \in \mathcal{I}$ is solvent, then his payoff is given by the respective quantities of goods and money $c_{i,C}$ and $c_{i,M}$ resulting from his transfers and trades:³

$$c_{i,C} = e + X_i + \int d\hat{D}_i(P) - \int d\hat{S}_i(P), \quad (8)$$

$$c_{i,M} = L_i + \int Pd\hat{S}_i(P) - \int Pd\hat{D}_i(P) - \hat{Y}_i. \quad (9)$$

If the agent is insolvent, then the state seizes all the goods and money of that agent ($c_{i,C} = c_{i,M} = 0$) and replaces him in the official and unofficial exchanges. The state creates all the money that is needed to execute this agent's buy orders.

Eliminating default contagion. As in Dubey (1982), this simple bankruptcy rule has the important implication that each agent can take other agents' orders as fixed when deciding on his trades because there is no default contagion. An agent does not have to worry that reducing a given buy order may trigger a chain of defaults ultimately affecting his other counterparts.

²A protocol is not manipulable if $X_i(\hat{Y}_i(\mathcal{Y}), \mathcal{Y}_{-i}) \geq X_i(Y_i, \mathcal{Y}_{-i})$. In words, an agent cannot strictly benefit from bidding more than the amount he actually pays. Under uniform pricing, fixed-quantity trading is not manipulable whereas so is a peg with proportional rationing.

³Goods consumption (8) is positive for any strategy profile from condition (4). So is money consumption (9) given condition (7), $\hat{D}_i \leq D_i$, and $\hat{Y}_i \leq Y_i$.

2.3 Equilibrium concept

The equilibrium concept is Nash. Formally, given that private agents do not care about their consumption of money, an equilibrium associated with a policy $\mathcal{P}$ is a strategy profile $\mathcal{S}$ such that for every $i \in \mathcal{I}$, strategy $\mathcal{S}_i$ maximizes i 's consumption of the good $c_{i,C}$ given other strategies $\mathcal{S}_{-i}$ and policy $\mathcal{P}$. This equilibrium concept yields a natural definition of predictable price levels:

Definition 4. (*Predictable price levels*) A price $P > 0$ is predictable given policy $\mathcal{P}$ if and only if there exists a Nash equilibrium associated with $\mathcal{P}$ with active unofficial trading at P , where active trading follows Definition 2. Let $\Pi(\mathcal{P})$ denote the set of predictable price levels associated with a policy $\mathcal{P}$.

This enables us in turn to characterize whether a public financial policy determines the price level:

Definition 5. (*Determination of the price level*) A policy $\mathcal{P}$ determines the price level if and only if $\Pi(\mathcal{P})$ has at most one element.

Remarks on the setup. We model trading as a Bertrand-Cournot game because we want to stack the deck against price-level determination. In this economy, every single agent is price setter, and as soon as at least two agents agree to trade at a given price given policy, this is a predictable price level. In other words, we deliberately adopt a setup such that no exogenous restriction to private trades contributes to price control. Such restrictions would include imposing a centralized market—which may still arise endogenously here, or granting the power to quote prices to only a strict subset of agents (the market makers). Also, whereas our results do not live or die on proportional rationing, it is an agnostic and tractable way of dealing with non-clearing orders at a given price.⁴ Notice that this delivers the same expected payoff to buyers and sellers as if buy and sell orders at the same price were placed in random queues and then sequentially matched. Proportional rationing basically eliminates execution uncertainty from such random matching.⁵ This simplifies the analysis, in particular because perfect foresight implies that our results are preference-free.⁶

2.4 Feasible policies

A natural restriction on policies is that they warrant that the state positively consumes goods and money for all the feasible actions of the private sector. We deem "feasible"

⁴Dubey (1982) and Dubey and Shubik (1980) also adopt proportional rationing.

⁵Michaillat and Saez (2015) also assume away the individual uncertainty associated with the random matching that they use to model non-clearing markets.

⁶Recall that we only assume strictly increasing preferences.

such policies. Formally, a policy $\mathcal{P}$ is feasible if and only if the consumption of goods and money by the state, $c_{G,C}$ and $c_{G,M}$, are positive for all private strategy profiles $\mathcal{S}$. We have:

Proposition 1. (*Sufficient conditions for feasibility.*) *A policy $\mathcal{P}$ is such that the state positively consumes money and goods for all strategy profiles if*

$$IM \geq \sup_{\mathcal{Y}} \sum_{i \in \mathcal{I}} (L_i(\mathcal{Y}) - \hat{Y}_i(\mathcal{Y})), \quad (10)$$

$$I\tau \geq \sup_{\mathcal{Y}} \sum_i X_i(\mathcal{Y}). \quad (11)$$

Proof. See Appendix A.2. □

Conditions (11) and (10) are sufficient because the right-hand sides are the maximum outflows of money and goods in the absence of bankruptcy, and we assumed that the state seizes goods and prints additional money if needed in case of bankruptcies. Condition (10) that ensures positive money consumption has important economic content. It highlights that as soon as the state has issued in the past an aggregate claim with a non-contingent component, that is, such that $\sum_{i \in \mathcal{I}} L_i(0) > 0$, then it must stand ready to honor it through money creation, as the private sector can always refrain from trading with the state ($\mathcal{Y} = 0$), implying that $\hat{\mathcal{Y}} = 0$ and the state does not get back through trading the money it transferred. In other words, a state liability that is nominally safe is a close substitute to money in the sense that the state must commit to monetize it as need be.

For clarity we postpone the analysis of contingent transfers until Section 6:

Assumption 1. (*Nominally safe liabilities*) *The transfers are constant functions of $\mathcal{Y}$ denoted $\mathcal{L} \equiv (L_i)_{i \in \mathcal{I}}$ such that $L \equiv \sum_i L_i / I > 0$.*

3 A simple necessary condition for price-level determinacy

This section offers a simple necessary condition for the determinacy of the price level for policies that are smooth in the sense that they are continuously differentiable at the transfers $\mathcal{L}$.

Proposition 2. (*A necessary condition for determinacy*) *Suppose a policy is such that for all $i \in \mathcal{I}$, X_i is continuously differentiable in a neighborhood of $\mathcal{L}$. If it determines the price level then*

$$\forall (i, j) \in \mathcal{I}^2, \frac{\partial X_i}{\partial Y_i}(\mathcal{L}) = \frac{\partial X_j}{\partial Y_j}(\mathcal{L}). \quad (12)$$

Proof. See Appendix A.3. □

Condition (12) simply requires that agents must earn the same marginal return on their own bids when each agent bids his transfer— $\mathcal{Y} = \mathcal{L}$. Notwithstanding the restriction to smooth policies, this condition is remarkably general in the sense that we do not impose anything on the nature of the dependence of X_i on $\mathcal{Y}_{-i}$. When condition (12) is not satisfied, there exist equilibria whereby agents unofficially trade money for goods at various prices in the convex hull of their marginal returns at $\mathcal{L}$. Agents with high official marginal returns unofficially sell goods to agents with low ones and reinvest the proceeds in the official market, thereby acting as intermediaries.

Unofficial trading in our economy is driven by rent extraction, not by the mutual gains typically associated with voluntary exchange. In standard microeconomic theory, voluntary trade is associated with the realization of mutual gains. A central insight is that this is not the case in our setup. The driver of trade is rent extraction by agents with high official returns. In equilibria with unofficial trading volume, these agents fund official bids larger than their transfers with money that they can unofficially acquire at a price level sufficiently high to make the trade profitable. This is in turn because the agents with low official returns accept to sell money at such unofficial terms. In general, low-return agents may or may not benefit from such unofficial trading, but this is a side show. In fact, for the policies that we will focus on in which the state has a fixed supply of goods up for sale, the low-return agents are necessarily always worse-off in the equilibria in which they unofficially trade than in the (always existing) equilibrium in which only the official exchange is active. Yet they still trade voluntarily conditionally on the trades and bids of the high-return agents. The following simple example illustrates this central point.

A simple example with a peg. Suppose $I = 2$, $L_1 = 0$ and $L_2 = 2$. The state sells up to $\tau = 1$ consumption unit at a fixed price $P^* = 1$. Agents' endowment satisfies $e > 0.25$. There is proportional rationing as in the unofficial markets. As a result, for $i \in \mathcal{I} = \{1, 2\}$, X_i is given by equation (2). Simple computations yield $\partial X_i(\mathcal{L})/\partial Y_i = L_{-i}/(L_i + L_{-i}) = \mathbb{1}_{\{i=1\}}$, and so condition (12) fails to hold. There are obviously no potential gains from trade between the two private agents because it is an endowment economy with one good and agent 2 alone can purchase the entire official supply ($L_2 = 2 > 1 = P^*\tau$).

There are equilibria in which only agent 2 bids at least one monetary unit in the official market and gets the consumption unit sold by the state. For every $B \in (0, 1)$, there also exists an equilibrium with an unofficial market with price $P = 4/B > 4$. Agent 2 bids B units of cash in the unofficial market and $2 - B$ in the official one. Agent 1 fulfills the unofficial buy order and uses the collected B to bid in the official market. To

see that it is an equilibrium, notice that if agent 2 shifts money $P\epsilon$ to the official from the unofficial market, he remains solvent and his net gain is

$$\underbrace{-\epsilon}_{\text{in the unofficial market}} + \underbrace{\frac{2 - B + \frac{4\epsilon}{B}}{2 + \frac{4\epsilon}{B}} - \frac{2 - B}{2}}_{\text{in the official market}}, \quad (13)$$

which is maximum at $\epsilon = 0$. Symmetrically, if agent 1 reduces his effective unofficial sale by $\epsilon > 0$ and his official bid by $P\epsilon$, he remains solvent and his net consumption gain is

$$\underbrace{\epsilon}_{\text{in the unofficial market}} + \underbrace{\frac{B - \frac{4\epsilon}{B}}{2 - \frac{4\epsilon}{B}} - \frac{B}{2}}_{\text{in the official market}}, \quad (14)$$

which is negative for any reduction in the official bid $P\epsilon < B$. In sum, agent 1 extracts a rent from agent 2 by diluting him in the official market with a bid that he funds with an unofficial sale at a sufficiently high price. The lower official return earned by agent 1 implies that he accepts such an unofficial purchase of expensive goods. The larger B , the larger the rent extracted by agent 1 from agent 2. $\square$

The equilibria that we construct to prove Proposition 2 have essentially the same structure as the ones in this elementary example. This showcases that the generic force driving unofficial trades and price-level indeterminacy when condition (12) is not satisfied is rent extraction by agents with high official returns. It is also possible to come up with policies such that all agents benefit from such unofficial trades. Here is an elementary example:

Example: discriminatory linear pricing. Suppose that for each $i \in \mathcal{I}$, $X_i(\mathcal{Y}) = Y_i/P_i$, where $(P_i)_{i \in \mathcal{I}}$ is strictly increasing. This corresponds to the situation in which the state has an arbitrarily large supply and discriminates agents along (linear) prices. Obviously all agents strictly benefit from having agent 1 unofficially selling goods to buy the transfers of all the other agents at any price in (P_1, P_2) (assuming his endowment of goods is such that he can afford it) and reinvesting them at P_1 . $\square$

Yet, when they happen to occur, such universal gains from unofficial trades remain only a side show rather than the essential driver of trade. Rent extraction by agents with high official marginal returns is this driver, whether other agents are better or worse off with unofficial trades. Interestingly, as detailed in Section 8, observers often mention the rent seeking motives of the agents with privileged official access as a rationale for of the coexistence of official and unofficial markets.

Empirical implications. We would like to emphasize that the indeterminacy made possible by the heterogeneity in returns in the official market is not a mere theoretical

curiosity. Quite the contrary, we believe that this phenomenon is a pervasive consequence of backing. Section 8 offers a wide array of past and current situations, both for public and private monies, in which imperfect backing generates trade away from the official price. In these examples, there are three types of empirical counterparts to the abstract heterogeneous official marginal returns leading to the failure of condition (12). First, it may just be that the issuer of money offers an explicit or implicit preferential treatment to some agents. It may also be that some agents have a superior trading ability. Finally, it may be that some agents value the commodities or currencies that the state sells for money more than others—think of heterogeneous needs for dollars among firms.⁷

Finally, it is worthwhile pointing out that given our fairly general specification of policy, condition (12) alone does not suffice to warrant price-level determinacy.⁸

4 Application to uniform pricing

Proposition 2 has strong implications for protocols that price uniformly.

Proposition 3. (*Determinacy under uniform pricing*)

- *Fixed-quantity trading determines the price level if and only if $L_i = L$ for all $i \in \mathcal{I}$. When it does not, all unofficial prices are above the official one L/τ .*
- *Fixed-price trading with proportional rationing determines the price level if and only if either $P^*\tau > L$ or $P^*\tau < L$ and $L_i = L$ for all $i \in \mathcal{I}$. When it does not, all unofficial prices are above the official one P^* .*
- *Suppose that an exchange protocol that prices uniformly does not ration and is continuously differentiable over an open set. If it determines the price level over this set, the elasticity of the official price is constant across bids for each $\mathcal{Y}$. Equivalently, the official price is only a function of the product $\prod_{\mathcal{I}} Y_i$.*

Proof. See Appendix A.4. □

Proposition 3 has three interesting results. First, both in the case of fixed-quantity trading and in that of fixed-price trading with fractional backing— $P^*\tau < L$, there is price-level determination if and only if all the transfers are equal— $L_i = L$ for all $i \in \mathcal{I}$. This is simply because heterogeneous transfers imply heterogeneous market impacts (price impact for fixed-quantity trading and quantity impact for fixed-price trading) and thus

⁷This corresponds to a straightforward extension of the model in which the backing good differs from the good in private endowments and agents have heterogeneous relative tastes.

⁸Suppose that there are two agents with $L_1 = L_2$ and $X_i(Y_i, Y_{-i}) = Y_i f(Y_{-i})$, where f is strictly positive decreasing. An interpretation is that an agent has more bargaining power with the state if he is bigger than the other. We leave it to the reader to check that one can construct unofficial trades despite equal marginal returns at $\mathcal{L}$.

heterogeneous official marginal returns. Conversely, uniform transfers warrant price-level determination, even with a peg with arbitrarily small backing. There is no room for rent extraction in this case as the unofficial net sellers of money would earn a larger marginal official return than the unofficial net sellers of goods and so they could not all accept the same unofficial price. This result is continuous in nature in the sense that the maximum feasible unofficial trading volume shrinks to 0 as transfers converge to the same value.

Second, fixed-price trading with strict excess backing— $P^*\tau > L$ determines the price level no matter the sizes of the agents. This may help understand the ubiquity of such fixed price-targets in practice. To reach such a target, the state must stand ready to sell strictly more than what it sells in equilibrium. In the case of exact backing $P^*\tau = L$, the price-level is not determined even when transfers are identical because the functions X_i are not differentiable—reducing a bid has no market impact whereas so does increasing one.⁹

Finally, the third bullet point shows that if we impose that the official exchange fully executes buy orders within some range at a uniform price, then price-level determination over this range requires that the official supply curve depend only on the product of the bids. In this case, condition (12) implies indeed constant price elasticity across bids, and this is equivalent to the supply of goods having this property. Suppose for example a price of the form $\alpha(\prod Y)^\beta$, where $\alpha > 0$, $\beta \in [0, 1)$. The case $\beta = 0$ is the overcollateralized peg. As β increases, the official supply of goods gets more inelastic.

5 Asymptotically atomistic economies

The wholesale markets for monetary instruments that we seek to model typically feature “large” agents, such as leading financial institutions. There are a plethora of examples of market impact and manipulation by a small number of players in the markets for assets or commodities that back money (currencies, precious metals, government bonds, short-term debt,...). In particular, the decentralized nature of many of these markets makes such impacts possible despite substantial market size. This motivates our focus on solving a model that does not assume any form of atomicity.

Still, this section tackles the limiting case in which each private agent becomes negligible. We do so for two reasons. First, our results on defaultable debt and infinite horizon in the following sections are simpler to spell out and analyze in this limit. Second, it is well-known that some equilibria in market games converge to the Walrasian outcome in the negligible limit in standard settings. We find it interesting to see what type of limit we obtain in our setup with an intrinsically worthless good (money) and an official exchange.

⁹Non-differentiability implies that this case is outside the scope of Proposition 2. We prove price-level indeterminacy even with uniform transfers in this case.

Proposition 4. (*Negligible agents*) Consider a sequence $(\mathcal{P}^I)_{I \in \mathbb{N}}$ of policies with fixed-price trading each associated with an economy of size I . Suppose that $(P^{*I}, \tau^I, L^I) \xrightarrow{I \rightarrow +\infty} (P^*, \tau, L)$ such that $0 < P^* \tau < L$, and that

$$\max_{i \in \mathcal{I}} \left\{ \frac{L_i^I}{IL^I} \right\} \xrightarrow{I \rightarrow +\infty} 0. \quad (15)$$

For every $\epsilon > 0$, there exists $n \in \mathbb{N}$ such that for all $I \geq n$, the unofficial prices that are predictable given $\mathcal{P}^I$ belong to $(L/\tau - \epsilon, L/\tau + \epsilon)$.

If the state adopts instead policies with fixed-quantity trading τ^I in these economies, all predictable unofficial prices tend to the official one which tends in turn to L/τ as $I \rightarrow +\infty$.

Proof. See Appendix A.5 □

In the case of a peg with fractional backing, all unofficial prices converge to the value L/τ that reflects the fundamentals of public finance—the equilibrium rationing coefficient. Insufficient backing still creates a form of indeterminacy—indeterminacy of the volume-weighted price level—because there are equilibria with varying levels of unofficial trading volume, including the one in which all trading is official.¹⁰

When the state sells a fixed quantity at the price that clears the official exchange, Proposition 4 states that, perhaps unsurprisingly, the economy converges to a Walrasian situation whereby money trades in the official market at the price L/τ in the negligible limit. In this limit, our model resembles a static version of the fiscal theory of the price level. We will now see that the assumption that $\mathcal{L}$ is not contingent—that is, that transfers are nominally safe in and out of equilibrium—plays a central role for price-level determination in this case.

6 Backing debt

The policies assumed thus far comprise fixed transfers $\mathcal{L}$. The feasibility condition (10) applied at $\hat{\mathcal{Y}} = 0$ (which is in particular the case when $\mathcal{Y} = 0$) implies that $M = L$. In words, the state must commit to monetize its entire transfers if need be. This subsection introduces the possibility that the transfers correspond to the extinction of debt-like liabilities that are subject to default. In their concluding remarks, Obstfeld and Rogoff (2021) conjecture that the possibility of sovereign default as an alternative to monetization should add indeterminacy to the fiscal theory of the price level. We validate

¹⁰For example, in our example of a peg above, suppose we “clone” each of the 2 agents in equal numbers so that the total population is I . For all $B \in (0, 1]$, the version of the equilibrium that we constructed for $I = 2$ features unofficial trading volume $BI/2$ and a price $P = 2I/(I - 2 + B)$ that tends to 2 as $I \rightarrow +\infty$.

this conjecture in our static environment by studying the polar case of a complete absence of monetization, in which the state uses only the money it collects in the market to fund its transfers.

The modified environment. Formally, denoting $(B_i)_{i \in \mathcal{I}}$ a vector with strictly positive components, we suppose that the transfer to agent $i \in \mathcal{I}$ given $\mathcal{Y}$ is

$$L_i(\mathcal{Y}) = B_i \min \left\{ 1, \frac{\sum_{i \in \mathcal{I}} \hat{Y}_i(\mathcal{Y})}{IB} \right\}, \quad (16)$$

where $B \equiv \sum_{i \in \mathcal{I}} B_i / I$. Thus, B_i admits a natural interpretation as the nominal promise to agent i . In case of default, creditors are treated *pari passu*—They receive a share in the available money proportional to the size of their claim B_i . This is to fix ideas, all our insights carry over with alternative seniority rules. For brevity, we focus on two exchange protocols, fixed-quantity trading τ and fixed-price trading P^* .

Proposition 5. (*Default and determinacy*) *Condition (11) alone is sufficient for feasibility.*

Fixed-quantity trading. *There is indeterminacy of the price level. There exists in particular a continuum of equilibria with only official trade indexed by their official prices $(B - D)/\tau$ for $D \in [0, B]$.*

Fixed-price trading. *There is indeterminacy of the price level. However, all unofficial prices converge to P^* as agents become negligible in the sense of Proposition 4, where we replace condition (15) with $\max_{i \in \mathcal{I}} \{B_i^I / (IB^I)\} \xrightarrow{I \rightarrow +\infty} 0$.*

Proof. See Appendix A.6. □

The salient implication of Proposition 5 is that making transfers contingent on the money that the state collects in the market as opposed to nominally safe completely flips the link between price-level determination and trading protocol in the negligible limit. In the case of nominally safe transfers, Proposition 4 shows indeed that the fixed-quantity trading protocol determines the price level in the negligible limit, whereas the fixed-price one may fail to do so in case of strictly insufficient backing. Proposition 5 shows that by contrast, the fixed-price protocol determines the price level in the presence of defaultable transfers in the negligible limit, whereas the fixed-quantity one fails to do so.

Essentially, the introduction of defaultable transfers creates a novel source of equilibrium multiplicity that is independent from the possibility of unofficial trades. There are a continuum of equilibria with varying default whereby agents cut their official bids all the more because they expect a large default. The resulting smaller collection of money by

the state in the official market vindicates agents' beliefs and results in smaller transfers. The trading protocol defines whether prices or quantities adjust in response to default. With fixed-price trading, a larger default translates into a larger real consumption of the state holding the official price P^* fixed. It is conversely the market-clearing price that adjusts to default under fixed-quantity trading.

Implication for the fiscal theory of the price level. In our static economy, a proponent of the fiscal theory of the price level would claim that if the state has a per capita surplus τ and maturing nominal liability L , the price level is L/τ . Propositions 4 and 5 clarify the in and out of equilibrium dimensions of the public financial policy that warrant such determination. The out-of-equilibrium dimension of policy is a blind spot of the Walrasian equilibrium concept. We find that the fiscal theory of the price level, corresponding to the situation of safe transfers and fixed-quantity trading in Proposition 4, is an extreme form of fiscal dominance. We have seen indeed that nominally safe transfers mean that the state commits to monetize its entire liabilities (but does not have to do it along the equilibrium path). In addition, fixed-quantity trading means that the state commits to clear the market in and out of equilibrium: It stands ready to let the price level adjust so that the real consumption of the state remains constant. At the polar opposite in Proposition 5, the price level is also determined in the negligible limit under fixed-price trading and defaultable transfers. This can be interpreted as monetary dominance, as the state commits to a fixed price level no matter the implications in terms of its solvency. Propositions 4 and 5 also show that the two remaining combinations of trading protocol and transfer safety fail to determine the price level.

7 Dynamic model

Our analysis, Proposition 3 in particular, suggests that price-level determination requires that the state back the entire money supply at the target price level. This is in contrast to Obstfeld and Rogoff (1983), where a commitment to an arbitrarily small amount of backing at a sufficiently high out-of-equilibrium price level implements the unique competitive monetary equilibrium by eliminating hyperinflationary dynamics. The reader may wonder if this discrepancy owes to our one-date setting in which backing is the only reason money has value. To address this, this section studies a simple infinite-horizon model in which money is essential as a store of value. More precisely, we embed our modeling of policy and private trades in an environment à la Wallace (1980), in which the overlapping-generations structure simply generates an endogenous demand for money. This generates two insights. First we show that an amended version of our static insights still holds in this environment. Second, our strategically-closed model allows us to conduct a formal analysis of the out-of-equilibrium arguments informally used in Walrasian

settings by Cochrane (2011) and Obstfeld and Rogoff (2021) in their debate on fractional backing.

Model. Time is discrete and indexed by $t \in \mathbb{N}$. There is a desirable consumption good at each date. Money is still intrinsically worthless and produced solely by the state. It is now a durable good that does not depreciate. At each date, a unit mass of agents are born who live for two dates. The cohort born at t is endowed at birth with $e > 0$ units of the date- t good and finds only the date- $t + 1$ good intrinsically desirable. There are also a unit mass of old agents around at date 0 who hold each $L > 0$ monetary units and value the date-0 good.

Policy. At each date t , the state collects a tax of $\theta \in [0, e)$ goods from each young agent, and stands ready to sell $\sigma \in [0, \theta]$ units at the unit price P^* . We interpret a strictly positive $\theta - \sigma$ as either state consumption or a deadweight loss from tax collection. The official trading post uses proportional rationing. To preserve time-homogeneity, we assume that the government seizes the money holdings of old agents if any and redistributes them lumpsum to young agents together with any money it might acquire by selling goods.

Unofficial trades and bankruptcy. At each date $t \in \mathbb{N}$, alive agents can submit any number of buy or sell orders for the date- t good. As in the static case, an order consists in a quantity and a price, and agents are subject to the restriction that they cannot place sell orders for a total quantity larger than their (after-tax) endowment at the outset of the date. Unofficial trading posts clear with proportional rationing as the official one. The strategy of an agent born at date t is $\mathcal{S}_t = (D_{t,Y}(\cdot), D_{t+1,O}(\cdot), S_t(\cdot))$, where $D_{t,Y}(\cdot)$ and $D_{t+1,O}(\cdot)$ are his buy orders when young and old, and $S_t(\cdot)$ his sell orders (agents have nothing to sell when old).¹¹ This includes his orders at the official price P^* .¹² While it does not show in notations because we will abstract from this, a strategy could be contingent on history. The respective solvency constraints at t and $t + 1$ of an agent born at t are:

$$\int PdD_{t,Y}(P) + \int Pd\hat{D}_{t+1,O}(P) \leq \int Pd\hat{S}_t(P) \quad (17)$$

$$\int Pd\hat{D}_{t,Y}(P) + \int PdD_{t+1,O}(P) \leq \int Pd\hat{S}_t(P). \quad (18)$$

For the date-0 old agents the solvency constraint reads $\int PdD_{0,O}(P) \leq L$.

¹¹Notice that we restrict the analysis to symmetric strategies which will turn out to be without loss of generality.

¹²We had to distinguish between unofficial trades and official bids in the static case because of the more general modelling of official exchange.

The equilibrium concept is subgame-perfect Nash. Notice that we directly work in the limiting case of negligible agents for simplicity. It is instructive to first describe some possible outcomes when the state is passive in the sense that $\sigma = 0$. We focus for parsimony on equilibria with a single trading post per date at a price that is constant over time. There clearly are a plethora of other equilibria.

Proposition 6. (*Laissez-faire*) *Suppose $\sigma = 0$. For every $P \geq 0$ there exists an equilibrium in which money trades for goods at the price P at each date. There also exists an equilibrium with no trade ($P = +\infty$).*

- *If $P \leq L/(e - \theta)$, old agents are (weakly) rationed buyers and consume the entire endowment of the young at each date.*
- *If $P > L/(e - \theta)$, young agents are rationed sellers and old agents consume only $L/P < (e - \theta)$ goods.*

Proof. See Appendix A.7. □

Proposition 6 first shows that the price level can take any value under laissez-faire, including the infinite one corresponding to a non-monetary equilibrium. For all values of P weakly smaller than $L/(e - \theta)$, agents reap all the possible gains from trade in this economy as the date- t goods are all purchased by the date- t old. If $P < L/(e - \theta)$, the economy is saturated with liquidity and the market does not clear. Yet all agents have the same utilities for all $P \in [0, L/(e - \theta)]$ no matter how awash with liquidity the economy is.

By contrast, every agent's utility strictly decreases in P over $[L/(e - \theta), +\infty)$. The decreasing purchasing power of money affects agents' ability to realize gains from trade. It forces the young to inefficiently hold on to the increasing fraction of their goods that the old cannot afford as money depreciates (P increases).

Comparison with Walrasian outcomes. If we were using Walras as our equilibrium concept in this very economy, we would obtain two perfect-foresight equilibria with constant price levels, a non-monetary one such that $P = +\infty$, and a unique monetary one in which market clearing yields $P = L/(e - \theta)$. Here instead, not imposing market clearing generates two categories of Nash equilibria. All the equilibria in which the economy is flush with liquidity— $P < L/(e - \theta)$ —generate the very same efficient outcome as the Walrasian one. To the contrary, when sellers are rationed at all dates— $P \in (L/(e - \theta), +\infty)$, the price level P can be viewed as a continuous measure of the "monetaryness" of the equilibria. As the equilibrium value of money continuously decreases— $P \rightarrow +\infty$, so do the realized gains from trade until they asymptotically vanish.

Suppose now that $\sigma > 0$. We have

Proposition 7. (*Price-level determination*) Suppose $\sigma > 0$.

(i) *All equilibria feature trade.*

(ii) *There are equilibria with trades at prices strictly above P^* if and only if*

$$P^*\sigma < L - P^*(e - \theta). \quad (19)$$

(iii) *There are equilibria with trades at prices strictly below P^* if and only if*

$$P^*\sigma > L - P^*(e - \theta). \quad (20)$$

Proof. See Appendix A.8 □

The second point in Proposition 7 is the infinite-horizon version of our static result according to which in the absence of full backing, a peg may generate unofficial trade above the official price level. In our static framework, the gross private supply of money L must be backed. Here, the net private supply of money $L - P^*(e - \theta)$ computed at P^* —private supply L minus private demand $P^*(e - \theta)$ —must be fully backed.

The third point in Proposition 7 shows that unlike in the static case, unofficial trading can occur at prices below P^* . The reason is that all private agents were willing to sell money at any rate in the static case, whereas the young are willing to buy money at any rate here.

Determining the price level. A direct consequence from Proposition 7 is that the unique peg that determines the price level given fiscal policy (θ, σ) is $P^* = L/(e - \theta + \sigma)$, the only value such that neither (19) nor (20) hold. In the static case, all pegs above the cutoff L/τ determine the price level. There is by contrast one unique solution in this dynamic case because young agents' demand for money opens up the possibility of low unofficial prices in addition to that of high ones. Another important difference with the static case is that price-level determination does not require an arbitrarily large peg P^* as the backing resources become arbitrarily small. This is because the youngs' demand of money $e - \theta$ is real whereas the olds' supply L is nominal, and so the price level that makes the net private (real) supply of money small enough to meet the state's real demand σ , $P^* = L/(e - \theta + \sigma)$, is bounded above by $L/(e - \theta)$. It is only if the state has a price-level target significantly lower than $L/(e - \theta)$ that backing consumes substantial real resources.

An alternative policy to pin down the price level would be fixed-quantity trading, whereby the state posts σ on the official post and lets it clear. The market-clearing price would be P^* . As seen in the static case, this result hinges on the assumptions that agents are negligible and that L is nominally safe in and out of equilibrium.

Beyond these differences, both the static and dynamic analysis show that the determination of the price level requires that backing takes place in equilibrium. In both cases, the state does buy money and sell goods at the unique trading post that is active on the equilibrium path. To the contrary, a literature pioneered by Obstfeld and Rogoff (1983) argues that out-of-equilibrium backing can suffice to implement a unique monetary equilibrium in contexts related to ours. The view is that by credibly committing to buy a small amount of money at a price level above the equilibrium one, the state can eliminate all equilibria but the monetary one, and thus does not trade along the equilibrium path. This literature typically uses Walras as the equilibrium concept and so makes only informal out-of-equilibrium arguments. It is notoriously difficult to formalize concepts such as out-of-equilibrium feasible actions and price setting under Walras. We now use our strategically-closed model to rigorously assess the broad arguments in this literature.

Out-of-equilibrium backing and price-level determination. Recall that the application of the Walrasian equilibrium concept to our model generates two steady states under laissez-faire, the monetary equilibrium with price level $P^W = L/(e - \theta)$, and the non-monetary one without trade. In an environment essentially similar to ours, Obstfeld and Rogoff (2021) argue that by committing to sell an arbitrarily small real amount against money at a large price level, the state eliminates the non-monetary equilibrium. Cochrane (2011) however argues that this creates an additional equilibrium in which all agents trade with the state at this large price level. Obstfeld and Rogoff (2021) claim in turn that a deviation by the private sector undercutting the state eliminates this latter outcome. To assess these claims in our model, we suppose that the government intervenes by selling $\sigma > 0$ at $P^* > P^W$. We have:

1. There is an upper bound P^* on predictable price levels.
2. There always exist equilibria with trade at P^* .
3. If a "large" trader commits to sell at $P \in (P^W, P^*)$, there is an equilibrium in which all trade is at P .

The first point stems from the fact that $P^* > P^W$ implies that condition (19) is not satisfied. Thus we confirm the insight that the state can guarantee a level of "monetaryness" of all equilibria with such backing. The second point shows however that this backing has no reason to be an out-of-equilibrium one. There exist equilibria where all or part of the trading activity takes place at P^* . That all agents trading at P^* is always an equilibrium is obvious. The proof of Proposition 7 constructs equilibria with trading at P^* and below.

Our setting with simultaneous bids makes it difficult to assess the claim in Obstfeld and Rogoff (2021) that a private agent can eliminate such equilibria with trade at P^* with

a deviation that undercuts the state. By definition, no deviation can affect other agents' simultaneous behavior. To study this point in a more appropriate setting, the third point formalizes in our framework the idea that an agent may move first and undercut the state, thereby affecting other agents' strategies. Suppose that one agent has an arbitrarily small but strictly positive share of his cohort's endowment whereas the others remain atomistic. Suppose this agent can credibly commit to sell his endowment at $P \in (P^W, P^*)$ or, equivalently, places his publicly observed order before all the other agents. We show that there exists an equilibrium in which all trade is at P .¹³ In other words, by competing with the state to coordinate trading, this "large" agent may replace the state, but this does not suffice to eliminate trades with a high price level.

Overall, this vindicates our finding that in-equilibrium backing, as opposed to out-of-equilibrium, is necessary, and sufficient under some conditions, to determine the price level. To be sure, our models of money demand and market structure are special. We do not rule out that out-of-equilibrium backing works under other specifications in which the deviation suggested by Obstfeld and Rogoff (2021) applies. But the mere fact that the answer to this fundamental and very aggregate question of price-level determination hinges on the details of the microstructure of the money market would be interesting in its own right.

8 Empirical implications

This section offers a variety of contexts in which we believe that our abstract and general approach offers useful insights. We aim in particular at discussing the various forms that the failure of the equality in marginal returns (12) can take in practice. There are broadly three sources of heterogeneity in marginal returns: privileged access to the official market, heterogeneous trading abilities, and heterogeneous preferences for the backing commodities or currencies. This latter case corresponds to a straightforward extension of the model whereby the good sold by the state differs from the consumption good private agents are endowed with, and agents have different rates of substitution across them.

Currency pegs and parallel markets. Foreign exchange is perhaps the best-known instance of the pervasiveness of parallel markets. As noted by Gray (2021), "sometimes central banks maintain an [overvalued] official exchange rate at which supply does not meet demand and instead attempt to restrict demand administratively. They may do so by prioritizing certain transactions; rationing FX; allowing queues to develop; or setting

¹³See end of Appendix A.8. We do not characterize all the equilibria in this case as the existence of this one suffices to make our point.

ceilings for, or even prohibiting, certain current payments.” In response to such measures, Gray (2021) notes that: “In such cases, a parallel market for current transactions will develop as many economic agents will, if possible, move transactions to the parallel market, whether to obtain a better price if selling FX, or because they cannot obtain FX at the official price.” Gray (2021) also provides a list of 19 countries with official and parallel markets in the 2010-2020 period. Rationing may take different forms that generate heterogeneous official marginal returns. In Argentina for example, individuals were allowed to buy up to 200 dollars a month at the official exchange rate. We did not explicitly handle such uniform official rationing in our analysis, but this clearly opens up the possibility for agents who do not need as many dollars to sell their allotment dear in unofficial transactions. In order to account for such heterogeneous needs of dollars, specific firms and/or sectors may be privileged, such as medicine in Nigeria or PDVSA in Venezuela to pay for foreign equipments. Rationing may also take the form of queues as in Algeria, where the foreign currencies are allocated every week on a first-in-first-out principle.

Reinhart and Rogoff (2004) document that dual and parallel exchange-rate markets have been pervasive throughout the postwar period. In many fixed-exchange-rate regimes, roughly half of official pegs were accompanied by active parallel markets. They also find that parallel-market exchange rates tend to reflect monetary and fiscal fundamentals more accurately than official rates.. This resonates closely with our result that, when backing is insufficient, unofficial prices converge toward values implied by fundamentals as proxied by the backing ratio in our minimalist setup. They also document substantial dispersion in the economic importance of official and unofficial markets, in line with our prediction that equilibrium unofficial trading volumes can vary widely without fundamental reasons under imperfect backing, even when unofficial prices cluster around fundamental valuations. Taken together, the prevalence of parallel markets, their superior informational content about fundamentals, and the large variation in trading volumes between official and unofficial channels all support the paper’s core mechanism linking imperfect backing to unofficial valuations and market segmentation.

Rent extraction in currency markets. Gray (2021) actually argues that rent extraction may in some cases be a feature rather than a bug of rationed official markets:

The approach of using of one or more official rates may be supported by a belief that a strong currency will mean higher growth and lower inflation, while ignoring the underlying problems—such as excessive fiscal stimulus and/or monetary financing—that cause exchange rate weakness and inflation. It may also be promoted by those who can profit from privileged access to FX at the official exchange rate (rent seeking behavior).

Although rent-seeking behavior is by nature difficult to systematically identify, there is abundant anecdotal evidence consistent with it. An example is Venezuela. As noted by the Financial Crimes Enforcement network:¹⁴

Currency controls in Venezuela limit the supply of U.S. dollars for most economic activities, which encourages the demand for foreign currency and smuggled goods in the parallel market. Although illegal, Venezuela’s parallel market is a highly profitable business for those with regime connections that enable them to access inexpensive dollars and goods. This parallel market relies upon unregulated brokers, whose clients often include criminals who integrate illicit proceeds into the legal economy. Venezuelan officials who receive preferential access to U.S. dollars at the more favorable, official exchange rate, also exploit the multi-tier exchange rate system for profit.

The most famous case related to Venezuela is arguably that of Raül Gorrin, a businessman who bribed Venezuelan officials to buy dollars at the official rate, thereby showcasing the value of having a privileged official access.¹⁵

From determinacy to indeterminacy: Assignats during the French Revolution.

In 1790, during the Revolution, the French state issued paper money—“assignats”—to reimburse debts (L), and at the same time was selling the National Estates (τ) comprised of assets seized from the Church.¹⁶ Sargent and Velde (1995) describe the “rise and fall of the assignat” as a sequence of three periods: a “real-bills” period, a “legal restriction” period and a “hyperinflation” period. During the “real bills” period, the value of National Estates was about 2,400 millions livres and exceeded the value of the debt that the state had to honor, which was around 2,000 millions livres. In the terms of our model, $L < P^*\tau$: Assignats were then fully backed. Here, P^* is the face value of assignats.

During the following two periods, a large quantity of assignats were issued to compensate for limited fiscal resources due to war and internal chaos. We interpret these two periods as situations in which $L > P^*\tau$: The state does only partially backs its currency. However, we note important differences between the two periods.

During the “legal restriction” period, under the Terror, the state implemented very harsh restrictions on hoarding assets, closed markets, and imposed wage and price controls. As Sargent and Velde (1995) note, these restrictions led to a “guillotine-backed currency”:

¹⁴See FinCEN Advisory FIN-2017-A006 accessible through <https://www.fincen.gov/resources/advisories/fincen-advisory-fin-2017-a006>.

¹⁵See <https://www.justice.gov/archives/opa/pr/venezuelan-television-news-network-owner-charged-alleged-12b-money-laundering-scheme> for details of his indictment by the US DoJ.

¹⁶Interestingly, Bolt et al. (2024) comment on an episode of insufficient backing by the Bank of Amsterdam that is contemporaneous to the Assignats debacle discussed here.

Under the Terror, any citizen accused of violating these laws could expect swift and arbitrary proceedings. The law on parity of the assignat called for arraignment and trial within 48 hours of the offense. The law encouraged denunciations from informants and gave extravagant powers to local authorities to enforce the restrictions. In a few dozen instances, the death penalty was imposed for crimes against the assignat or for hoarding.

The outcome is that the price level was determined and no inflation arose, consistently with our model in equilibria without private trades. Arguably such equilibria were made stable by such large costs of isolated deviations. With military successes, the Terror was overthrown, legal restrictions were alleviated, and markets reopened. In this “hyperinflation” period, private agents tried to sell all their holdings of assignats for goods and species. This period opened up arbitrage opportunities. As reported by Sargent and Velde (1995):

In terms of gold, prices were lower than in 1790, creating trading opportunities for the savvy. A Swiss visiting Paris hastened to change his gold for paper; bought hundreds of shoes, stockings, and hats; shipped them off to Switzerland; and lived in Paris like a king for a month.

This anecdote corresponds to a situation in which the goods τ officially available are luxury Parisian goods that are marginally more valuable to visitors than to the already well-stocked locals. The “high-return” agents in our models are therefore these visitors valuing local goods relatively to gold more than the “low-return” Parisians.

Private monies. While our main set of implications pertains to official currencies, our framework may also be used to think about privately-issued monies and the question of singleness—whether these private monies trade at par with the official currency. Among these private monies, some combine key aspects of our model: They are bearer instruments and are redeemable by the issuer. These two properties are satisfied by, among others, stablecoins, bank notes during the Free banking era or money market mutual funds.¹⁷ For this type of private monies, deviations from par are likely as noted by Group of Thirty (2025). In what follows, we connect these deviations to the way such private monies are redeemed.

Stablecoins. Major stablecoins such as Tether and Circule—both account for more than 90% of the stablecoin market capitalization—aim to maintain a peg to the US dollar by backing tokens with USD-denominated assets, such as US Treasury bonds. Typically,

¹⁷This contrasts with private monies that are account-based and for which transactions are settled with central bank reserves (see Garratt and Shin, 2023). As noted by Gorton (1985), deposits are “double claims” that is ‘a claim on a specific agent’s account at a specific bank.

issuers exchange tokens for dollars in a primary market and promise to redeem them at par value. Stablecoins then trade in secondary markets.¹⁸ There are usually barriers to redemption (Group of Thirty, 2025). In the case of Tether, only amounts larger than \$100,000 are possible and Tether charges fees (0,1 percent of the amount and the minimum is \$1,000). In periods of illiquidity, redemption may be delayed. In the case of Circle, only institutional investors can redeem their stablecoins. As a result, retail investors are usually excluded from the primary market and the stablecoin value is pegged in the secondary market insofar as large investors arbitrage between the primary and secondary markets (see Harris and Rogoff, 2026).

While fiat-backed stablecoins have been the most successful, they are not the only type. Before the Terra Luna collapse in May 2022, algorithmic stablecoins were gaining popularity.¹⁹ In this ecosystem, the stablecoin UST was circulating and differed from fiat-backed stablecoins in its pegging mechanism. A smart contract allowed the exchange of one UST for \$1 worth of LUNA—the native token circulating on Terra and a claim on Terra’s transaction fees and block rewards, but also a token giving access to Terra applications. This setup resembles the primary market described earlier, except that τ was not fixed in USD but varied with LUNA’s dollar value, which depended on the overall value of the Terra system. The collapse of UST was triggered by a sharp decline in LUNA’s price, stemming from a sudden loss of confidence in the system’s sustainability (see Liu et al., 2023, for further details). Liu et al. (2023) highlight heterogeneity in the use of the primary versus secondary markets during the May 2022 crisis:

Interestingly, we find that Alameda Research, a cryptocurrency trading firm closely affiliated with the FTX exchange, conducted the largest amount of UST-LUNA swaps among Anchor depositors. It seems that the swap fees and uncertainty about the execution price of LUNA on exchanges discouraged most other Anchor depositors from utilizing the native swap contract as an exit strategy. But Alameda Research, with its advantageous access to the FTX exchange, had a competitive advantage over other market participants.

This observation suggests again that some agents, such as Alameda Research, had a competitive edge in trading on the primary market due to their superior access and resources to resell LUNA, while other participants opted for the secondary market, even at significantly discounted prices.

Free banking era. Under Free Banking, as experienced by the US between 1837 and 1863, banks’ liabilities were in form of banknotes redeemable in species. These

¹⁸Numerous studies have explored stablecoin stability: Lyons and Viswanath-Natraj (2023), d’Avernas et al. (2022), and Routledge and Zetlin-Jones (2022), among others.

¹⁹The Terra Luna ecosystem was the third largest in the market, with a capitalization of \$50 billion, which collapsed to nearly zero in just three days.

banknotes, in contrast with deposits, were massively exchanged on secondary markets. In particular, brokers specialized in trading these banknotes. As Gorton (1999) showed (see also Jaremski, 2011), the prices at which these banknotes traded relate to the option to redeem the banknote, that is banks' default risk and the transportation cost to travel to the bank to redeem it. "Wildcat" banking, i.e., banks which intentionally overissue money compared with their ability to redeem it, is considered to have been at most marginal. To be more precise, Jaremski (2011) notes that banknotes were traded at par locally, unless the bank was closed or suspended.

From the perspective of our model, money is banknotes while the good stands for species. The commitment of the bank to redeem banknotes at par is akin to a peg. Private agents have, however, an heterogeneous access to the bank, as they are spatially dispersed and potentially far from the bank. The functions $X_i(\cdot)$ are heterogeneous and take into account transportation costs. Within a same location, private agents may face heterogeneous transportation costs to the issuing bank with the ones with the lowest transportation cost specializing in the intermediation.

9 Conclusion

Many attempts to control the value of a currency through public financial policy ultimately boil down to the stylized backing problem studied in this paper. We establish a general and intuitive necessary condition for determinacy. When this condition fails, we show that rent seeking is the key driver of price dispersion. We then assess the ability of several standard policies to determine the price level and characterize, to the extent possible, the set of predictable prices when they fail to do so. We believe that this novel approach provides a useful way to uncover blind spots created by the Walrasian framework in the analysis of important dimensions of public financial policy. We provide two examples. First, outright default generates indeterminacy in the fiscal theory of the price level. Second, in a simple Wallace (1980) environment, out-of-equilibrium backing fails to implement the monetary equilibrium.

Our approach to price-level determination is innovative because it is inspired from mechanism design. We impose an incentive-compatibility constraint on policy requiring that private agents do not trade in other terms than the official one. Accordingly, a natural route for future research is to add to this approach the type of informational asymmetries that mechanism-design techniques typically tackle. For example, agents's valuations of the backing commodities could be private information to them. We conjecture that this could create an interesting interplay between price determination and price discovery.

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A Appendix

A.1 A netting result

Lemma 8. (*Netting*) Consider a strategy profile such that agent $i \in \mathcal{I}$ is a non-bankrupt net buyer at the trading post P . If he deviates and sets $s'_i(P) = 0$, $d'_i(P) = d_i(P) - d(P)s_i(P)/s(P)$ then he does not affect his allocation nor that of other agents. Symmetrically, suppose he is net seller at P . If he deviates and sets $d'_i(P) = 0$, $s'_i(P) = s_i(P) - s(P)d_i(P)/d(P)$ then he does not affect his allocation nor that of other agents.

Proof. First, such deviations by i do not affect the payoffs of the other agents. This stems from the fact that these deviations do not affect the rationing coefficients since when $s(P) \neq s_i(P)$ and $d(P) \neq d_i(P)$,

$$\frac{d(P) - \frac{d(P)s_i(P)}{s(P)}}{s(P) - s_i(P)} = \frac{d(P) - d_i(P)}{s(P) - \frac{s(P)d_i(P)}{d(P)}} = \frac{d(P)}{s(P)}.$$

Second, these deviations are payoff-irrelevant for i as well. To see this, suppose that i is net buyer. The reduction in his buy order $d(P)s_i(P)/s(P)$ is weakly larger than that of his effective sales $s_i(P) \min\{d(P)/s(P), 1\}$, and so the deviation leaves him solvent. Furthermore,

$$\hat{d}'_i(P) - \hat{s}'_i(P) = \left(d_i(P) - d(P) \frac{s_i(P)}{s(P)} \right) \min \left\{ 1, \frac{s(P)}{d(P)} \right\} = \hat{d}_i(P) - \hat{s}_i(P),$$

leaving his allocation unchanged. The same reasoning applies to a net seller. $\square$

Based on this result, we will implicitly assume in the following proofs that whenever an agent is net seller or net buyer at one post, he always nets his trades this way before entering into any deviation. These restricted sets of actions will suffice to establish all our results and simplify the exposition.

A.2 Proof of Proposition 1

The right-hand sides of conditions (10) and (11) are the respective suprema of the aggregate net monetary transfer and goods sold assuming no bankruptcy: The state never has to transfer more money nor goods conditionally on no bankruptcy. These sufficient conditions need not be necessary because either supremum may come with bankruptcies, in which case the state seizes goods and creates any money needed in addition to IM by assumption.

A.3 Proof of Proposition 2

Suppose that condition (12) fails to hold. Suppose without loss of generality that the marginal returns $(\partial X_i/\partial Y_i(\mathcal{L}))$ are decreasing in i . Suppose that $\partial X_I/\partial Y_I$ is not constant in a vicinity of $\mathcal{L}$ and that $\partial X_I/\partial Y_I(\mathcal{L}) < \partial X_{I-1}/\partial Y_{I-1}(\mathcal{L})$. For $B > 0$ sufficiently small, define $P(B)$ such that

$$P(B) \frac{\partial X_I}{\partial Y_I} \left(L_I - B, \mathcal{L}_{-I} + \frac{B}{I-1} \right) = 1, \quad (21)$$

and take B sufficiently small that

$$eP(B) > \frac{B}{I-1}, \quad (22)$$

and for all $i < I$

$$\frac{\partial X_I}{\partial Y_I} \left(L_I - B, \mathcal{L}_{-I} + \frac{B}{I-1} \right) < \frac{\partial X_i}{\partial Y_i} \left(L_i + \frac{B}{I-1}, \mathcal{L}_{\mathcal{I} \setminus \{I, i\}} + \frac{B}{I-1}, L_I - B \right). \quad (23)$$

Here the notation $\mathcal{L} + B/(I-1)$ is a shorthand for the addition of $B/(I-1)$ to each component of $\mathcal{L}$. Local continuous differentiability warrants the existence of a continuum of such B . Then the situation in which I bids B monetary units at $P(B)$ and the other agents bid e each and reinvest their proceeds $B/(I-1)$ in the official exchange is an equilibrium with unofficial price $P(B)$.

Notice that there would be a unique price $P(B)$ across values of B defined this way in case $X_I(\mathcal{Y}) = kY_I$ for a constant k in the vicinity of $\mathcal{L}$. Yet we need several such prices to prove price-level indetermination. We omit for brevity the proof that it is still possible to have multiple unofficial prices in this case as this involves a lengthy discussion depending on how localized this linear behavior would be. The body of the paper offers a simple example.

Common marginal returns. Suppose that several agents, but not all of them given that (12) does not hold, share the same marginal return at $\mathcal{L}$. The above equilibrium construction can then be simply amended as follows. Denoting $\mathcal{J}$ the subset of agents with the same marginal return as I , suppose first that for all $\epsilon > 0$ there exists $B < \epsilon$ such that an agent $j \in \mathcal{J}$ has a strictly smaller $\partial X_j/\partial Y_j (L_j - B, \mathcal{L}_{-j} + \frac{B}{I-1})$ than the others. Then one can construct the same equilibrium as above with this agent. If this is not the case then the marginal return functions coincide over $\mathcal{J}$ in a neighborhood of $\mathcal{L}$. In this case one can replicate the equilibrium constructed above with the only difference that the members of $\mathcal{J}$ are all unofficial buyers, investing the same amount of money.

A.4 Proof of Proposition 3

No unofficial price below the official one. Suppose fixed-price trading. Suppose an equilibrium features unofficial trading. Let P the smallest active unofficial price. A net seller at this price must be net buyer somewhere else otherwise he would be strictly better off not selling. He must be net buyer at a strictly lower price than P otherwise he would be strictly better off cutting both sell and buy orders. By construction this can only be at the official post so $P > P^*$. The proof is verbatim with fixed-quantity trading.

Heterogeneous transfers and failure of (12). If the transfers are heterogeneous, expressions (1) and (2) show that condition (12) does not hold neither under fixed-quantity trading nor under a peg with $P^*\tau < L$.

Homogeneous transfers and price-level determination. Suppose the transfers are identical and the official exchange is either fixed-quantity or a peg with $P^*\tau < L$. If there is unofficial trade, net unofficial sellers have more money to officially invest than net unofficial buyers and thus earn lower official returns, which contradicts that they can trade at unofficial prices within the range of their official returns.

Peg with excess backing $P^*\tau > L$. Official net buyers cannot be rationed because unofficial trading cannot create net money—at least one agent would be bankrupt, which is strictly dominated. Thus there cannot be any net buyer at a higher price than the official one.

Case $P^*\tau = L$. At least one agent, agent i , say, has a strictly positive transfer. We let him bid $B < L_i$ at the unofficial price P such that $P[1 - (L_i - B)/(IL)] = P^*$, with B sufficiently small that $Pe > B$. Other agents bid their endowment at P and invest the proceeds $B/(I - 1)$ at P^* on top of their transfers. This is an equilibrium as the unofficial buyer is indifferent and the unofficial sellers strictly prefer their maximum bids, and there are a continuum of such P .

Third bullet point. Over the interval over which the properties hold, uniform pricing and no rationing imply that for all $i \in \mathcal{I}$

$$P(\mathcal{Y})X_i(\mathcal{Y}) = Y_i, \tag{24}$$

differentiating with respect to Y_i yields

$$\frac{\partial P}{\partial Y_i} X_i + \frac{\partial X_i}{\partial Y_i} P = 1, \quad (25)$$

$$\frac{\partial P}{\partial Y_i} \frac{Y_i}{P} + \frac{\partial X_i}{\partial Y_i} P = 1. \quad (26)$$

Since $\partial X_i / \partial Y_i$ does not depend on i from condition (12), nor does the elasticity. Standard methods (e.g., characteristics) then imply that P must be a function of the product of the Y_i .

A.5 Proof of Proposition 4

We first prove the result in the case of a peg. The proof is in three steps.

Step 1. Suppose that there is active trading at the unofficial price $P > P^{*I}$. Denote s a net seller and b a net buyer. We have

$$P \frac{s(P^{*I})(d(P^{*I}) - d_s(P^{*I}))}{d^2(P^{*I})} \geq P^{*I}, \quad (27)$$

$$P \frac{s(P^{*I})(d(P^{*I}) - d_b(P^{*I}))}{d^2(P^{*I})} \leq P^{*I} \mathbb{1}_{\{d(P) > s(P)\}} \frac{s(P)(d(P) - d_b(P))}{d^2(P)}. \quad (28)$$

The first condition states that s does not find it optimal to cut his long and short positions. The second one states that b does not find it optimal to shift some money from P to P^{*I} .

Step 2: All official buy orders become negligible in the negligible limit. Formally, denoting $Y^I = \sum_{i \in \mathcal{I}} Y_i^I$ the official buy orders of the size- I economy in a generic equilibrium, we have

$$\max_{i \in \mathcal{I}} \left\{ \frac{Y_i^I}{Y^I} \right\} \xrightarrow{I \rightarrow +\infty} 0.$$

To see why, notice first that any net buyer $i \in \mathcal{I}$ in unofficial markets invests less than L_i^I in the official exchange. That the official exchange is rationed and condition (15) together imply that net buyers in unofficial markets place negligible official buy orders in the negligible limit. Comparing conditions (27) and (28) at a given unofficial price P shows that any net buyer in an unofficial post has a larger official position than any net seller in an unofficial post. Thus all agents must have negligible buy orders in the negligible limit.

Step 3. Step 2 implies that as $I \rightarrow +\infty$, conditions (27) and (28) can only be jointly satisfied if $s(P)/d(P)$ becomes sufficiently close to 1: Buyers' unofficial rationing if any

becomes negligible. This implies that $s(P^{*I})/d(P^{*I}) \rightarrow P^*\tau/L$ as $I \rightarrow +\infty$, and conditions (27) and (28) then yield the result.

We leave it to the reader to check that the proof readily applies to the case of fixed-quantity trading. The key is that the conditions (27) and (28) still hold replacing P^{*I} with 1, $s(P^{*I})$ with τ , and the official orders expressed in quantities of goods $d_s(P^*)$, $d_b(P^*)$, $d(P^*)$ by the monetary bids Y_s , Y_b , and Y .

A.6 Proof of Proposition 5

Feasibility. By construction the state transfers only the money that it receives, and so it does not need to create M to consume money positively (but of course can always do so), that is, $c_{G,M} = M$ in and out of equilibrium. Condition (11) remains sufficient for $c_{G,C} \geq 0$ for all strategy profiles for the same reasons as that in Proposition 1.

Price-level indetermination under fixed-quantity trading. Let $D \in [0, B]$. There exists an equilibrium in which agent $i \in \mathcal{I}$ receives a transfer $B_i(1 - D/B)$ and invests all of it in the official exchange, which clears at $(B - D)/\tau$. This is independent of the number of agents and thus holds for I arbitrarily large.

Price-level indetermination under a peg. Take $D \in (0, B)$ such that $B - D < P^*\tau$. If agent $j \in \mathcal{I}$ increases his effective sales or reduces his buy order by a small ϵ at the unofficial post P , he can increase his official bid by x such that

$$P^*x = P\epsilon + \frac{B_j P^* x}{IB}. \quad (29)$$

We build an equilibrium with unofficial trade below P^* based on this. Let $i \in \mathcal{I}$ an agent with a minimum value of B_i — i need not be unique. Let this agent buy a sufficiently small quantity at $P = P^*[1 - B_i/(IB)]$ and all the others sell their endowment e at this unofficial post and invest the proceeds at P^* . It is an equilibrium as the unofficial buyer is indifferent between buying at P and P^* whereas unofficial sellers weakly prefer to sell as much as possible from condition (29).

Peg and negligible agents. Suppose there is unofficial trade at prices below P^* , and denote P the smallest such price. The net sellers at P must be buying at P^* . Suppose that such a net seller $j \in \mathcal{I}$ reduces his effective sales at P by $\epsilon > 0$. His order at P^* must then shrink by x defined by (29), and $x \geq \epsilon$ implies $P \geq P^*[1 - B_i/(IB)]$, and so P must become arbitrarily close to P^* as agents become negligible. Suppose there is unofficial trade at prices above P^* . It must be that $B - D = P^*\tau$ otherwise non-rationed official buyers would not buy above P^* . We leave it to the reader to check that the proof of Proposition 4 applies.

A.7 Proof of Proposition 6

Not trading is weakly optimal if nobody else trades and so no trade is always an equilibrium. A date- t old agent with money holdings L_t strictly prefers to spend all of it at any price. A date- t young agent strictly prefers to sell his entire endowment at a strictly positive price and is indifferent between selling or not at a price of zero. Thus for any price $P_t \geq 0$ there is an equilibrium in which the young (old) go all in selling goods (buying goods) at P_t . If the young are rationed, there is an inefficient equilibrium allocation of goods. If some old are rationed the allocation is efficient as the old consume all the goods at each date.

A.8 Proof of Proposition 7

Claim 1. There are at most two active unofficial trading posts. In the presence of active official trading, active unofficial prices are either all above P^* , or there is a unique one below P^* . Suppose that at a given date there is active trading at the prices P_- and $P_+ > P_-$ both different from P^* . Sellers must be rationed at P_+ so buyers are not and would not buy above P_+ . Buyers must be rationed at P_- so sellers are not and would not sell below P_- . Suppose there is active trading at $P \in (P_-, P_+)$, $P \neq P^*$. Buyers must be rationed and so sellers are not which contradicts that they would sell at P_- . Suppose there is active trading at $P^* > P_-$. We know $P^* < P_+$ and so official buyers must be rationed implying that official sellers are not. This contradicts that there are sellers at P_- .

Claim 2. There can be unofficial trade at prices above P^* if and only if $L > P^*(e - \theta + \sigma)$. Let us construct an equilibrium with active buyers and sellers at $P > P^*$ and P^* . An equilibrium is characterized by $(e_P, e_{P^*}, L_P, L_{P^*}) \in (0, e - \theta)^2 \times (0, L)^2$ such that

$$P \frac{L_P}{Pe_P} = P^*, \quad (30)$$

$$\frac{1}{P} = \frac{1}{P^*} \frac{P^*(e_{P^*} + \sigma)}{L_{P^*}}, \quad (31)$$

$$L_P + L_{P^*} = L, \quad (32)$$

$$e_P + e_{P^*} = e - \theta. \quad (33)$$

The first condition expresses that sellers are rationed at the high price and the second one that so are buyers at the low price. Let $e' = e - \theta + \sigma$. Rearranging yields

$$P = \frac{L - P^* e_P}{e' - e_P}, \quad (34)$$

$$e_P = \frac{P e' - L}{P - P^*}. \quad (35)$$

Thus $P > P^*$ implies $L > P^* e'$, and $e_P \in (0, e - \theta)$ implies $L/e' < P < [L - P^*(e - \theta)]/\sigma$. These conditions imply in turn that $L_P \in (0, L)$, which ensures that the equilibrium exists and establishes the sufficiency of $L > P^* e'$.

To establish necessity, we show that this condition is also required for the two other possible types of equilibria with unofficial trade above P^* . Notice first that there are always official buyers. It may be that there is no official seller and either one or two unofficial prices. In both cases, sellers must be rationed at the highest price and so buyers are not. First consider the case in which there is no official seller and only one unofficial price. The equilibrium is characterized by

$$P \frac{L_P}{P(e - \theta)} \geq P^*, \quad (36)$$

$$\frac{1}{P} = \frac{1}{P^*} \frac{P^* \sigma}{L_{P^*}}, \quad (37)$$

$$L_P + L_{P^*} = L, \quad (38)$$

and rearranging yields that it must be that $L \geq P^*(e - \theta) + P\sigma > P^* e'$. Finally, there might be two unofficial prices $P_2 > P_1 > P^*$. We leave it to the reader to check that rationed sellers at P_2 and buyers at P_1 and P^* yields that necessarily:

$$L_{P^*} = P_2 \sigma, \quad (39)$$

$$L_{P_2} = P_1 e_{P_2}, \quad (40)$$

$$L_{P_1} = P_2 e_{P_1}. \quad (41)$$

and adding and using $e_{P_1} + e_{P_2} = e - \theta$ yields the necessary condition.

Claim 3. **There can be unofficial trade at prices below P^* if and only if $L < P^*(e - \theta + \sigma)$. Trade can be exclusively unofficial if and only if $L \leq P^*(e - \theta)$.** Let us construct an equilibrium with active buyers and sellers at $P < P^*$ and P^* . An equilibrium is characterized by $(e_P, e_{P^*}, L_P, L_{P^*}) \in (0, e - \theta)^2 \times (0, L)^2$ satisfying the same

conditions as in Claim 2 above. Rearranging yields

$$P = \frac{L - P^* e_P}{e' - e_P}, \quad (42)$$

$$e_P = \frac{L - P e'}{P^* - P}. \quad (43)$$

Thus $P < P^*$ implies $L < P^* e'$, and $e_P \in (0, e - \theta)$ implies $[L - P^*(e - \theta)]/\sigma < P < L/e'$. These conditions imply in turn that $L_P \in (0, L)$, which establishes the sufficiency of $L < P^* e'$. The other possible equilibria with unofficial trade below P^* are either such that there are only official buyers (no sellers) or no official trade. In the latter case we have

$$P \geq P^* \frac{L_{P^*}}{P^* \sigma}, \quad (44)$$

$$\frac{1}{P^*} = \frac{1}{P} \frac{P(e - \theta)}{L_P}, \quad (45)$$

$$L_P + L_{P^*} = L, \quad (46)$$

which implies $L \leq P^* e'$.

Finally if there is unofficial trade at $P < P^*$ and no official trade it must be that buyers weakly prefer unofficial trading:

$$\frac{1}{P^*} \leq \frac{1}{P} \frac{P(e - \theta)}{L}, \quad (47)$$

or $L \leq P^*(e - \theta)$.