De Pecunia Non Est Disputandum*

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July 18, 2026

Abstract

We study an economy in which money is essential as a superior means to settle private debts. Loans with in-kind repayments are plagued by ex-post extortion by lenders who exploit courts' lack of expertise at assessing the quality of the goods tendered by borrowers. Fiat money is introduced as a superior instrument to clear private debts because it has no intrinsic value and therefore no quality: *Money is indisputable.* Thus courts can swiftly certify repayments in base and inside money. Such monetary settlements are however useful only if money trades in a liquid and transparent market for the goods that agents seek to intertemporally exchange in the first place. In order to create such a market, monetary policy must issue enough base money and adequately trade it. It must in particular depart from the Friedman rule in order to discourage the hoarding of money by lenders that squeezes borrowers out of the money market.

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1 Introduction

Anyone seeking to figure out the purpose of money should first look at what it says on it. Every US dollar bill states that: ”This note is legal tender for all debts, public and private”. This paper offers a theory in which money is essential as a superior instrument for the settlement of debts between private agents.[1] Money overcomes a novel friction in credit markets: The possibility that lenders seek to extort their borrowers when courts cannot assess the quality of in-kind repayments.

Crucially, monetary transactions are not a substitute for dysfunctional and anemic credit markets in our economy. On the contrary, money *enables* credit by directly addressing the frictions that stand between lenders and borrowers. Our monetary theory has four distinctive features:

1. *Our economy is nearly cashless.* Private agents trade inside money between them and net it in equilibrium, where inside money is private debt—a claim to a future monetary payment by a private agent.

2. *Yet the injection of some base money by the issuing authority is essential.* It spurs the circulation of inside claims. This ensures that debtors can avail themselves of inside liquidity to clear their obligations as needed.

3. *Optimal monetary policy departs from the Friedman rule.* Creating an opportunity cost of holding money eliminates equilibria with debt deflation spirals in which lenders hoard money thus driving borrowers into financial distress.

4. *Our theory creates a natural link between monetary policy and financial stability.* The size of the balance sheet of the issuing authority must increase when financial fragility increases.

The extortion problem that motivates the introduction of money to settle debt is a primitive informational friction that can be sketched as follows. We consider a simple endowment economy in which there are gains from intertemporal trade. The future quality of the goods that natural borrowers pledge for current consumption is uncertain.

[1]It is well-known that using money to settle claims between public and private sectors may affect its value. Economists have long acknowledged that tax liabilities create money demand (Lerner, 1947; Smith, 1776; Starr, 1974), and that, symmetrically, the repayment of sovereign debt may spur money supply (Sargent and Wallace, 1981).

All private agents share the same prior about quality, and perfectly observe it ex post. Preferences are such that lenders should not bear all the risk associated with uncertain quality in the optimal contract. The main friction is that the court that seizes borrowers' assets in case of default, unlike private agents, does not observe the quality of the goods used for repayment, but only their quantity. Thus, the court cannot enforce quality-contingent contracts. If the loan contract stipulates a large quantity to be repaid to compensate for the risk of low quality, a lender without commitment power will use this to extract an informational rent. The lender will extort high-quality borrowers by demanding a large repayment that they cannot contest given a poorly informed court. As a result, the only feasible credit arrangements are such that lenders bear excessive quality risk.

The introduction of a monetary system comprised of base and inside money helps overcome this extortion problem. Base money is simply a good that i) is exclusively produced by the government; ii) comes in a single quality, for example because it has no intrinsic value; iii) can be distinguished from other goods by the bankruptcy court. Inside money is an obligation between private agents to repay base money in the future. Given these properties of base money, courts can verify the settlement of debts with inside or/and base money. Notice that this function of money for debt settlement is in principle separate from that of unit of account as studied, e.g., in Doepke and Schneider (2017). If money trades for a desirable commodity at an exchange rate that courts can observe, obligations to repay in money can be indexed by this exchange rate, making this commodity the unit of account.[2]

Such seamless debt settlement is only the first step towards a monetary system that enables the implementation of optimal real intertemporal trades. It must also be the case that money trades for the goods that private agents seek to exchange over time in liquid markets at prices that are transparent in the sense that courts can observe them. The assumed absence of informational frictions between private agents would suggest that setting up such an efficient market is an easy task. Yet the fact that market signals are used as inputs for contracts enforced by a poorly informed court suffices to jeopardize market efficiency.

Liquidity and transparency do not come out of thin air in our economy because lenders

[2]For example, in a commitment to repay USD1M in euros at the prevailing exchange rate at the due date, the unit of account is the US dollar and the means of settlement is the euro.

have ex-post incentives to degrade them. Lenders “gain from no-trade” in the sense that an illiquid equilibrium enables them to reap in the market the extortion outcome that the introduction of money rules out in the courtroom. To generate this equilibrium, lenders ”dash for cash” in the official money market, thereby depriving borrowers from the money they need to repay their debts. Since in-kind payments are penalized by poorly informed courts, such squeezed borrowers are willing to purchase money from the lenders at price levels so low that they are indifferent between cash and in-kind payments, and this implements the extortion payoff. This ”dash for cash” equilibrium leaves lenders with large money holdings at the end of the trade. Monetary policy can thus eliminate this equilibrium by ensuring that holding money comes at an opportunity cost, that is, by deviating from the Friedman rule. Ensuring that base money is an inferior store of value induces debt-free agents to get rid of it, which ignites in turn the circulation of inside money, and allows debtors to purchase the money they need to clear their obligations.

Related literature

Relationship to money as a medium of exchange. The conception of money as a medium of exchange is by far the most influential theory of money. When borrowers can default on their obligations with impunity, there are social gains from trading money, instead of empty promises, against goods: Money as a medium of exchange replaces credit and is all the more useful because frictions in credit markets are important (Gu et al., 2016). It is by contrast the lack of commitment of lenders that is problematic in our theory, in which money conversely enables credit.

In our paper, the introduction of money eliminates inefficient rent extraction by lenders. This makes us close to Lagos and Zhang (2022), in which the option to use money as a medium of exchange reduces inefficient rent extraction by credit intermediaries. In the version of our economy in which the government can control the price level, we find like them a discontinuity in welfare between nearly cashless and cashless economies. The Friedman rule is optimal in Lagos and Zhang (2022) because it makes the option of monetary exchange most valuable. By contrast, its violation is optimal in our setup. Hu and Rocheteau (2026) study a more general environment in which imperfect credit coexists with monetary exchange. They find that the welfare and redistributive implications of the option of using cash crucially depend on the nature of the frictions in the credit market.

In Freeman (1996), money serves both as a medium of exchange and as a debt-settlement device. A matching friction prevents agents from spending their money as they wish, implying that debtors may fail to acquire the cash needed to honor their obligations. In our theory, by contrast, the financial distress of debtors arises in the absence of any trading friction. It is is solely due to the extortion friction that the use of money in debt repayments is intended to mitigate in the first place. Possible shortages of money among debtors are the marketplace manifestation of the same friction that money eliminates in the courtroom.

Informational asymmetry between the private sector and contract-enforcing authorities is the cornerstone of our theory. A sizeable literature has examined how monetary exchange—where, as in our setting, money is an asset of recognizable quality—mitigates the liquidity problems generated by informational asymmetries among private agents (see, among others, Alchian, 1977; Banerjee and Maskin, 1996; Berentsen and Rocheteau, 2004; Brunner and Meltzer, 1971; Rocheteau, 2011; Williamson and Wright, 1994). In environments with endogenous information distribution, Gorton and Ordoñez (2014) and Dang et al. (2017) show that the most suitable candidates for inside money are securities for which private information acquisition is not individually profitable.

Relationship to money as a unit of account. Doepke and Schneider (2017) offers an important rationale for the function of money as a unit of account. Building on the assumption of non-contractible relative prices, they find that a well-chosen common unit of account enhances credit by reducing the impact of price fluctuations on balance sheets. One of our contributions is to endogenize the contractibility of prices denominated in monetary units. Our contract-enforcing authority uses the price level inferred from official markets to determine nominal obligations. This creates incentives for agents to coordinate on squeezing this official market in order to force their credit counterparts to trade at less favorable unofficial prices. This in turn affects the effectiveness of the official price level as a contracting input. We introduce a new role for monetary policy as discouraging such unofficial trades. A related two-way interaction between optimal mechanisms and the relevance of the market values that they use as signals is central in the analysis of fair-value accounting standards in Plantin and Tirole (2018). In our theory of money as a debt-settlement tool, as in mark-to-market accounting, contracting parties free ride on the credible certification of values provided by markets, and this may negatively affect the quality of such certification.

Relationship to money as a store of value. A signature feature of our theory is the prediction that base money should optimally be an inferior store of value. The injection of base money that burns lenders' fingers destroys the equilibria in which they hoard inside money and force borrowers to trade at extortionary prices. This role of base money in equilibrium selection is reminiscent of Dubey and Geanakoplos (2006), who study an economy in which inside money does not determine the price level, whereas so does an arbitrary small injection of outside money. In their economy, outside money serves to pay interests to lenders, and this induces agents to spend all their borrowed money. This bears some similarity with the departure of the Friedman rule that fosters the circulation of money in our economy. In Rojas Breu (2017), the introduction of money as an inferior store of value may improve the borrowing capacity of agents with limited commitment. In this economy in which the cost of default is a reduced future ability to smooth consumption, borrowing against an inferior store of value boosts one's ability to repay by making this reduced future ability more costly.

The paper is organized as follows. Section 2 presents our contracting problem and shows how money can address it. Section 3 examines how monetary policy enforces the role of money as legal tender by fostering a liquid and transparent money market. Section 4 discusses our results. Section 5 applies them to the determination of official interest rates in practice. Section 6 concludes.

2 Extortion and legal tender

This section develops a simple model in which the introduction of money mitigates ex-post extortion by lenders when borrowers' asset quality is not verifiable. Section 2.1 sets up the model. Section 2.2 shows that a bankruptcy court does not need the expertise to assess asset quality if lenders can commit to a contingent suing strategy. Section 2.3 shows that, by contrast, the court's lack of expertise may lead to the extortion of high-quality borrowers if lenders cannot commit. Section 2.4 shows how the introduction of money as a legal tender may mitigate this problem provided this legal tender trades in a liquid and transparent market.

2.1 Setup

There are two dates $\{0,1\}$. There is a single consumption good at date 0, and two consumption goods at date 1, denoted h and l. There are two agents, a lender L and a borrower D. The borrower may be of two types, B or G, where B stands for "bad" and G for "good".

Endowments. Agent $i \in \{B, G, L\}$ has a stream of endowments $(e_{i,0}, e_{i,h}, e_{i,l})$ such that $e_{B,0} = e_{G,0}$, $e_{B,l} = \kappa$, $e_{G,l} = 0$, and $e_{G,h} = e_{B,h} + \kappa$, where $\kappa > 0$. In words, B and G have the same total number of date-1 h and l units, but κ of B's units are comprised of l-goods, whereas G owns only h-goods. This is the only difference between the two types of borrowers.

Beliefs. Both D and L learn D's type at date 1, and share a common prior at date 0 that assigns a strictly positive probability to each type. This setup is thus one of symmetric information among private agents.

Preferences (1/2). Agent $i \in \{D, L\}$ ranks consumption streams $(C_0, \tilde{C}_h, \tilde{C}_l)$ according to the criterion $U_i(C_0, E_0[V_i(\tilde{C}_h, \tilde{C}_l)])$, where the date-1 utility function V_i is strictly and continuously increasing in C_h and constant in C_l. In words, no agents finds the l-good intrinsically desirable.[3]

Preferences (2/2). We suppose that there exist feasible contracts that both agents prefer to autarky. Within this set of contracts, the one that L prefers consists in a date-0 transfer x from L to D followed by a date-1 transfer y of h goods from D to L, where $(x, y) \in [0, e_{L,0}] \times (0, e_{B,h})$.

We lay out the model in a somewhat unusual way by taking this contract (x, y) as a primitive rather than by fully spelling out preferences and deriving the resulting first-best arrangements. We do so because we will study monetary institutions that get as close

[3]We could assume that D's utility depends on his type without affecting the results. All that matters is that l is intrinsically worthless.

as possible to implementing this first-best contract taken as given.[4] Studying a contract such that the borrower bears all quality risk is for simplicity. The frictions that we will introduce would stand in the way of any contract such that at least some quality risk is borne by the borrower. We will not distinguish between the contract (x, y) and payoff-equivalent contracts that feature the transfers (x, y) and also transfers of the l-good. We will not need more restrictions on preferences than assumed above for most of the analysis.

The assumption of a binary distribution $\{h, l\}$ is not important but will simplify the analysis in Section 3 because it entails that we need to study only one market in which the h-good trades for money. That the l-good is worthless is just a normalization.

2.2 Borrower's lack of commitment and the legal system

We first study the situation in which only the lender can commit.

Assumption 1. ***(One-sided commitment)*** *The borrower D, no matter his type, cannot commit at date 0 to a date-1 plan, whereas the lender L can do so.*

We suppose that a legal system mitigates this commitment problem that would otherwise completely preclude D from pledging date-1 goods. It offers the possibility to register a date-1 obligation at date 0 and to have it enforced by a bankruptcy court if need be. At date 0, D and L can agree on recording an obligation by the borrower D to make a payment at date 1. At date 1, D offers a payment and L can sue the borrower for default based on this offer and on the recorded obligation. When seized of the case, if the court finds that D has defaulted, it seizes goods from the borrower to implement the obligation. This process destroys a random fraction of D's endowment.[5] If it finds that there is no default, the court validates D's proposed payment.[6] This legal system is imperfect in the following sense:

Assumption 2. ***(The court lacks expertise to assess quality)*** *The court cannot tell apart h and l goods.*

[4]This optimal arrangement could be optimal risk sharing. We could also derive it as a second-best in the presence of frictions such as a hidden date-0 effort by the borrower that affects his date-1 endowment. This would not affect the analysis nor generate useful insights.

[5]We suppose that the borrower's endowment is sufficiently large and this fraction sufficiently small that all obligations in the relevant range can be met despite this deadweight loss.

[6]Although this would play no role in the analysis, we could assume that L gets penalized for a frivolous lawsuit in the absence of default.

Assumption 2 posits informational asymmetry between the court and private agents. The natural interpretation of the model is that the same date-1 good (or service) comes in two qualities, h and l. Private agents observe quality whereas the court lacks the expertise to do so. We study whether despite this informational asymmetry, this legal system makes it possible to implement the contract (x, y) as a subgame-perfect Nash equilibrium of a game that has the following extensive form:

- At date 0, L makes a take-it-or-leave it offer to D comprised of a date-0 transfer and of the registration of a date-1 obligation. If D turns it down the game ends.
- Otherwise, at date 1, L and D learn D's type and:
    1. D posts goods offered as a payment.
    2. L decides whether to sue or not.
    3. If the case is brought before the court, it assesses whether the borrower has defaulted based on his proposed payment and on the obligation. In case of default the court seizes goods from D in order to execute the obligation. The court gives the benefit of the doubt to the borrower.

Formally, giving the benefit of the doubt to the borrower means that the court trusts a report by D about his type and the type of his goods as long as the report is consistent with the common prior—for example, D cannot claim to be of type B and have strictly less than l-goods. As detailed below, all that matters is that the court does not give the entire benefit of the doubt to the lender.[7]

The following proposition shows that this imperfect legal system suffices to implement the optimal contract (x, y) provided L can commit.

Proposition 1. ***(Implementation of*** (x, y)***)*** *The contract* (x, y) *is the outcome of a subgame-perfect Nash equilibrium. In this equilibrium, L registers an obligation to repay* $y + \kappa$ *date-1 units of any type. L and D agree at date 0 to bring the obligation to court at date 1 if and only if D repays strictly less than* y *h-goods.*

Proof. A G-borrower repays y h-goods because bankruptcy would entail repaying $y + \kappa$ of them and getting some more destroyed. The B-borrower repays with y h-goods, and

[7] For example, the court could randomly select which side it trusts when receiving inconsistent reports.

may also add up to κ l-goods but that is payoff irrelevant. This is because he would have to use at least y h-goods to repay in case of bankruptcy and would incur the deadweight loss. $\square$

Assumption 1 introduces a standard failure in credit markets, the rationing of a borrower because he cannot credibly commit to repay ex-post as much as is ex-ante optimal. Proposition 1 shows that a bankruptcy court need not be as well informed as the private sector in order to mitigate this friction and implement the contract (x, y). It suffices that L commit to make this informational asymmetry irrelevant at date 1. The reason is that an obligation that is incentive-compatible with a B-borrower is so, with more slack, with a G-borrower. We now show that conversely, lack of commitment leads to ex-post extortion by L by exploiting this extra slack.

2.3 Lender's lack of commitment and extortion

The remainder of the paper drops Assumption 1 and posits instead two-sided lack of commitment:

Assumption 3. ***(No commitment)*** *Neither L nor D can commit at date 0 to a date-1 plan.*

Proposition 2. ***(Ex-post extortion prevents the implementation of*** (x, y)***)*** *An obligation such that a B-borrower repays at least y h-goods must be such that a G-borrower repays at least $y + \kappa$ goods. In particular, the implementation of (x, y) is out of reach. There is either autarky or contracts such that G ends up paying more h-goods than B depending on agents' preferences.*

Proof. Given that the borrower has the benefit of the doubt, the only way L can obtain at least y h-goods when facing a B-borrower is by being able to produce an obligation of at least $y + \kappa$ total units. But then given his lack of commitment, L would extort a G-borrower with such an obligation. $\square$

Extortion and quality-risk sharing. We focus on the implementation of the particular contract (x, y) to fix ideas. Yet it should be clear to the reader that the assumed frictions more broadly imply that only the contracts in which the lender has equity exposure on quality risk can be enforced. That is, the lender always receives a contingent payment of the form $(z - \mathbb{1}_{\{D=B\}}\kappa)^+$ for some $z > 0$. In words, implementable contracts feature cross-subsidies from G to B, implying that L bears all or part of quality risk ex-ante.

Remark on the benefit of the doubt. Whereas the assumption that the court gives the entire benefit of the doubt to the borrower D is for simplicity, it is important that it does not give it entirely to L. Otherwise the tension between enforcement and information that Proposition 2 showcases would vanish. L and D could just write an obligation to repay y h-goods. L could then claim default and request the seizure of y goods he identifies as being of h-type as soon as the proposed payment features strictly less than y h-goods. Whereas giving the full benefit of the doubt to the lender is exogenously precluded here for brevity, it would be easy to rationalize such ex-post borrower protection under simple extensions of the model. For example, some borrowers could own superior goods of quality higher than h that lenders would try to extort, or some lenders would materially benefit from the liquidation of their borrower, and the court would want to restrict such outcomes. We leave such extensions towards fully endogenizing the court's decision process given a social objective for future research.

The rest of the paper studies how money as a superior instrument for clearing obligations can restore the implementation of L's preferred contract (x, y) despite this extortion problem due to the conjonction of non verifiable quality and limited commitment.

2.4 Money as legal tender

We suppose that the government can mitigate the extortion problem triggered by the non verifiable quality of in-kind payments as follows. Suppose the government is a monopolist producer of a date-1 good—"base money"—that comes in only one quality. The court can distinguish it from any other good. We suppose that private agents must consume base money positively at date 1 and, to fix ideas, that they do not find it intrinsically desirable. We deem "inside money" a date-0 obligation between private agents to make a transfer of base money at date 1.

Fiat versus commodity money. Whereas intrisically worthless money is a natural candidate for base money as we define it, any good that satisfies the two following properties can actually play this role in our theory. It must be impossible to counterfeit, and its quality must be observable by courts. Metallic monies stamped by a political authority historically played this role—albeit imperfectly due to well-known issues related to debasement and tampering with metal content. A mere reliable recording of private claims on the government denominated in an arbitrary unit of account is another candidate for base money as we define it.

By introducing money defined this way, the government can eliminate informational asymmetries between the private sector and the court provided private agents use inside money to contract. It is however only a first step towards enabling the implementation of the optimal contract (x, y). It must also be that agents can exchange money for h-goods at prices that are observable by the court. The following proposition illustrates this straightforward yet essential point in a simple partial-equilibrium environment. Proposition 3 is a heuristic presentation that abstracts from important aspects such as the treatment of default on inside money that Section 3 will fully rigorously address.

Proposition 3. ***(An efficient legal tender requires a liquid and transparent money market)*** *Suppose that after the type of D is revealed and before they meet again at date 1, D and L both have access to an exchange in which they each can trade the h-good for P units of base money, where $P > 0$. If all agents including the court observe P at date 1 then they can implement (x, y).*

Conversely, if D or/and L cannot trade, or if at least one of them trades money for the h-good at a price that is uncertain as of date 0 and not observed by the court, then (x, y) cannot be implemented.

Proof. If P is observable by all agents including the court at date 1, no matter the (possibly heterogeneous) beliefs of all parties involved at date 0, D and L can implement (x, y) by recording an obligation contingent on the date-1 realization of P equal to Py monetary units. D sells y h-goods at date 1 to acquire the money it needs whereas L buys the same quantity against money. Conversely, if at least one agent either cannot trade or faces price uncertainty that the court cannot observe, then (x, y) cannot be implemented.

For future reference, when P is publicly observable, notice that an obligation that leaves D the option to pay either Py monetary units or $y + \kappa$ units of the good also implements (x, y) as a G-borrower strictly prefers the monetary payment whereas a B-borrower is indifferent. $\square$

Proposition 3 illustrates simply that in the presence of perfect markets, the role of money in this economy is to separate out the function of enforcement performed by the court and that of quality certification that is delegated to the market. Proposition 3 also states that the contract (x, y) cannot be implemented with money if the agents either cannot trade or trade at an uncertain price that the court cannot observe. Otherwise stated, the implementation of (x, y) requires that money trades for the h-good in a liquid and transparent market. By "liquid" we mean that all agents can trade, and by "transparent" that they do so at prices observable by the court.

Given the assumed absence of informational frictions between private agents at date 1, the assumption that such an efficient market may arise seems reasonable. Yet the following section will show that the fact that market signals are inputs to contracts enforced by an uninformed court suffices to adversely affect market liquidity and transparency. It also studies how monetary policy can mitigate this problem.

3 Monetary policy for a liquid and transparent money market

This section studies how monetary policy can cement the role of money as an efficient instrument to settle debts by supporting a liquid and transparent money market in the sense of Proposition 3. A meaningful analysis of this question requires a setup in which such liquidity and transparency do not come out of thin air, unlike in a Walrasian setup in which they are directly assumed. Accordingly, we model trading as a market game. We proceed in two steps, first studying the determinants of the money-market's liquidity and then that of its transparency. More precisely, in order to focus on liquidity, Section 3.1 first assumes that the government can control the price level, and studies whether agents trade at all at this price. Section 3.2 relaxes this stark price-control assumption. It posits that private agents are free to trade at whichever prices they want in unofficial markets that the court cannot observe. We identify the conditions under which monetary policy can deter them from doing so. To fix ideas, this section restricts the analysis to the case $y \geq \kappa$.[8]

[8] All our results hold when $y < \kappa$.

3.1 Making a liquid official money market

We suppose that the economy is populated by unit masses of lenders and borrowers identical to those studied in Section 2. We suppose that the fraction of G-borrowers is fixed and denote it $\gamma \in (0, 1)$. We also posit that agents value each monetary unit that they hold at the end of date 1 as $v \geq 0$ units of good h. In the case of linear date-1 preferences that we will adopt in Section 3.2, v admits a literal interpretation as the present value of the future purchasing power of money at an unmodelled date 2. The extensive form of the interactions between private agents is as follows.

Date 0: Pairs of lenders and borrowers are formed. Lenders make take-it-or-leave it offers on date-0 transfers and date-1 obligations. Obligations can be purely in-kind, purely monetary, or give the borrower the option to pay all or part with money. We deem the last two types "inside money".

Date 1, after types are revealed:

- The official market opens and settles as described below.
- The date-0 pairs meet again and interact as in Section 2.2. The court observes the official price if any, and enforces obligations as described below.

Structure of the official market. The official market functions as follows:

- The government announces the price P^* at which all transactions take place.
- Agents post sell orders listing the goods they wish to sell and buy orders indicating the number of goods they wish to purchase.
- Sell orders and buy orders are randomly arranged in separate queues. Orders are matched sequentially walking down the queues. Each buyer may decline goods and keep proceeding down the queue until his order is fully matched or he reaches the end of the queue. Rejected goods remain available for subsequent buyers.
- If there remains unmatched sell orders after all private agents have had a chance to buy, the government walks down the queue and matches them purchasing a total of up to δ units, where $\delta \geq 0$. Any residual unfilled sell orders are not served.

- All the matched orders generate an in-kind obligation for the seller and a monetary obligation valuing the quantity bought at the official price P^* for the buyer.

An agent's total sell orders cannot exceed his endowment net of his pure in-kind obligations from date 0. The government, like the court, cannot observe the type of private agents' date-1 goods. For brevity, we restrict the analysis to the case $(1-\gamma)\kappa > \delta$ in which there are more l-goods than the government can purchase.

Legal enforcement. Agents can use either base money or inside money to clear their obligations when they have the option to pay in money. If an agent defaults on an obligation, the court seizes his holdings of goods and base money. It uses the goods to settle the pure in-kind obligations, which is always possible given the assumed restriction on sell orders. The government creates base money to make good on any net monetary obligation that agents fail to repay.

Equilibrium concept. Our solution concept is subgame-perfect equilibrium. In addition, we adopt the tie-breaking rule that, whenever an agent is indifferent among multiple trading strategies, he selects the one that minimizes the time spent in the market. For instance, if indifferent among several units of h-goods available for purchase, he buys the first ones encountered in the queue. This refinement can be naturally interpreted as capturing either the opportunity cost of time devoted to trading, or small costs of turning down goods. This merely prunes equilibria of limited economic interest that would crucially rely on buyers' indifference.

Remarks on the market structure. Moving away from the Walrasian limit forces a choice among many possible market structures. Notice first that markets à la Shapley and Shubik (1977), in which a market maker sets the price that clears the bids of quantities of money and goods, would not be suited to our environment with heterogeneous goods and uninformed authorities. If a market maker is able to segment markets per good quality, he should just run the court in the first place. The setup that we use here aims at isolating the pure impact of the extortion friction on market liquidity, abstracting from any standard trading frictions. The assumption that buyers can sequentially direct their matches as they see fit minimizes matching restrictions and eliminates coordination failures such as the coexistence of crowded and unfilled trades (e.g., Burdett et al., 2001; Lagos, 2000).

Whereas this market structure will make the implications from the extortion friction on liquidity stark and transparent, we discuss below how alternative settings would affect our results.

Our first result is that in the absence of base-money injection, the extortion problem stands in the way of a liquid market.

Proposition 4. ***(Characterization of liquid money markets)*** *Suppose agents have signed an obligation to repay either a real value y in money at the official price or $y + \kappa$ if all or part is repaid in-kind. Suppose $P^*v \leq 1$.*

- *There exists an equilibrium in which some borrowers repay with money if and only if $\delta > 0$.*
- *Suppose $\delta > 0$ and $P^*v < 1$.*
  - *There is a unique equilibrium, dominance-solvable, in which all borrowers pay with money, and the government buys a quantity δ of l-goods.*
  - *All pairs implement the payoff (x, y) except for a mass δ/y of B-borrowers who pay only $y - \kappa$ in real terms and an equal mass of lenders stuck with money with value $P^*vy < y$.*

Proof. See Appendix A. □

The first point in Proposition 4 states that some injection of money ($\delta > 0$) is necessary to foster a liquid money market. The second point shows that it is sufficient provided $P^*v < 1$. We discuss each of these points in turn.

Inside money merely moves the extortion problem from the courtroom to the exchange. The first point in Proposition 4 states that the government must inject base money in the economy ($\delta > 0$) if it wants any chance that the private sector uses money to clear obligations. Otherwise, it is strictly dominant for each lender to not place any buy order as this implies that his associated borrower will not be able to receive inside money from any other lender in equilibrium. This is because any active buyer must be the date-0 lender to an active seller so that he funds his bid with a monetary repayment of his date-0 obligation. Together with the fact that inside money is by definition in zero net supply, this implies that all active sellers must be borrowers from active buyers.

Role of the market structure. The assumption that buyers can perfectly direct their orders makes this no-trade result particularly stark (in particular, it is independent from agents' preferences). We conjecture that some random matching of sell and buy orders could actually help sustain a liquid equilibrium in addition to this illiquid one because this would give a borrower a chance to execute his sell order even when his lender is not in the market. In any case, a robust insight is that for any market structure, as long as all money is inside money, there exists an illiquid equilibrium because lenders strictly "gain from no trade". By all refraining from trading, lenders earn their highest possible payoff and thus have no individual incentives to deviate.

The injection of outside money jump-starts the market. The second point of the proposition shows that the government must also commit to sell base money for lemons to cement the role of money as legal tender. By committing to supply money in case all sell orders are not executed ($\delta > 0$) and setting $P^*v < 1$, the government warrants a liquid money market. Both G and B borrowers trading is the only equilibrium that survives iterated dominance in this case. Intuitively, even if they expect that no lender buys, borrowers all strictly benefit from trying and selling goods since the government commits to buy some ($\delta > 0$). G-borrowers seek to sell y h-goods and B ones strictly benefit from shedding their l-goods on top of $y - \kappa$ h-goods. This implies that even in the absence of private buy orders, some borrowers, the ones ahead in the queue, will manage to obtain money. Their respective lenders anticipating this find it strictly dominant to place buy orders since $P^*v < 1$. This implies that more borrowers will be able to pay with money, triggering more buy orders, and so on until all h-goods are sold.

Role of the market structure. Proposition 4 exhibits a stark discontinuity at $\delta = 0$, in the sense that an arbitrarily small trade by the government suffices to ignite a chain reaction of trade provided $P^*v < 1$. This result owes to a market structure which entails that a given buyer can perfectly foresee the trades in the continuation equilibrium subsequent to his decision. We conjecture that δ would have to be in general bounded away from zero to spur liquidity if, for example, there was more uncertainty as to which borrowers would benefit from the government intervention holding private trading strategies fixed. Still, even if the interplay of our extortion friction with additional trading frictions would presumably make our results less stark, we believe that there would still be a multiplier on the injection of base money because the lenders to the borrowers who receive it spend their inside claims on other borrowers, and so on...

Friedman rule. We interpret a market price P^* strictly smaller than $1/v$ (which always holds when $v = 0$) as a deviation from the Friedman rule because it create an opportunity cost of holding money at the end of date 1. The Friedman rule is typically introduced in economies in which money provides liquidity services in addition to being a store of value. It consists in ensuring that the demand for liquidity services is saturated and agents are indifferent between holding money and other stores of values. The equivalent definition here is that agents who are debt-free and thus do not need money incur no opportunity cost of holding money, which requires that the government sets $P^*v = 1$. By contrast, setting $P^*v < 1$ is a departure from the Friedman rule that makes money burn these agents' fingers. This spurs liquidity because it induces each of them to sell their money which allows more borrowers to buy money thereby inducing their lenders to sell money and so on. Thus departing from the Friedman rule is a way of spurring the efficient circulation of money, a novel rationale to our knowledge.

Distance to the first-best. The only reason some pairs of lenders and borrowers fail to implement (x, y) is that a mass δ/y of B-borrowers get away with getting money from the government against their l-goods, and use it to repay their debt. These borrowers thus pay only $y - \kappa$ in real terms. But then, their lenders seek to spend this money on h goods and there is too much money chasing too few h-goods. As a result, the last δ/y lenders who have a chance to buy fail to do so. Notice that this deviation from the first-best can be made arbitrarily small. The ex-ante benefits to the borrowers and costs to the lenders can be made arbitrarily small by taking δ sufficiently small. Holding δ fixed, if $v > 0$, the costs to the lenders can also be made arbitrarily small by taking P^* sufficiently close to $1/v$. The following section will show that the assumption of perfect price control plays an important role in this result of arbitrarily small costs.

A nearly cashless economy. A remarkable feature of the unique equilibrium is that private agents trade only inside money for goods between them and then net their claims. Base money trades in equilibrium only through the trades of the government aimed at jump-starting the money market. Such a result is reminiscent of Tirole (2012) and

Philippon and Skreta (2012), who also obtain that the government purchases low-quality asset to restore market functioning. In their case, such purchases alleviate the asymmetric information problem regarding asset quality faced by private agents. In our case, private agents are perfectly informed about the quality of assets and the public purchases of low-quality assets is the unintended consequence of igniting liquidity.

How can fiat money be useful in a finite-horizon model? The above results hold in particular in the case $v = 0$ in which selling money is strictly dominant for all agents without outstanding liabilities. But then, that unbacked fiat money can circulate and be socially useful in a finite-horizon economy flies in the face of existing monetary theory. There are two ingredients that make it possible here. First, money is a medium of intertemporal exchange and agents commit ex ante to accept money for debt extinction ex post. Borrowers sell goods for money at the terminal date only because they can get rid of the money for sure by extinguishing their debt: Lenders have credibly committed ex ante to accept this monetary transfer as a debt settlement. They can only indirectly and collectively default on this commitment by making the market illiquid. Second, we allow the government to use a trading mechanism with fixed price that does not necessarily clear the market. Notice that this trading protocol in which the price need not clear the market is in line with the way central banks conduct many operations in practice (limited allotment). The only way we could ensure that the market clears at P^* is if the government accepted to sell δ h-goods to meet unfilled buy orders. But then this would mean that money is fully backed and it is no longer puzzling that it can be useful at short horizon given its recognizability. Proposition 5 in the following section states in fact that such full backing is necessary to avoid unofficial trading at prices above P^* when the government cannot impose that all trades take place at P^*.

What if the government must follow the Friedman rule? If, for reasons outside the model, the government was compelled to follow the Friedman rule,[9] then the only way it could eliminate extortion is by committing to purchase a quantity of goods $\delta \geq y$ that covers both the acquisition of all l-goods and that of the h-goods tendered by the borrowers. We rule this out given our assumption that $(1-\gamma)\kappa > \delta$. In particular, imple-

[9] For example because money also serves as a medium of exchange for other (unmodelled) agents in this economy.

menting the Friedman rule comes at a significant cost for the government of purchasing all the l-goods. In standard models, the Friedman rule is implemented at no cost by exchanging all the money demanded by the private sector for goods that are not subject to informational issues. Here by contrast, the very friction that makes money essential also makes the Friedman rule not only undesirable because it stifles debt settlement, but also costly to implement.

Taking stock, this study of monetary policy under perfect price control generates two interesting insights:

- The liquidity of the money market is not warranted because our primitive extortion friction creates "gains from no-trade" for lenders who earn the extortionary payoff if their G-borrowers fail to secure money by selling goods.

- The government can foster liquidity by injecting base money in the economy and departing from the Friedman rule. The injection sparks a chain reaction whereby purchases of money by borrowers spur sales of money by their lenders who expect this money to be transferred to them, which induces more sales of money by lenders and so on. The departure from the Friedman rule is central to propagate the initial impact of base money and ensuring that lenders keep offloading their expected monetary transfers in the money market.

3.2 Making a liquid and transparent money market

This section relaxes the assumption that the government can control the price level. To this aim, we grant private agents the ability to trade with each other with a free hand at selecting prices and quantities. We suppose that the court cannot observe such unofficial trades. In this case, monetary policy faces the additional task to deter unofficial trading in order to warrant the verifiability of the effective price level.

More precisely, we modify the setup in Section 3.1 as follows. We suppose that agents are risk-neutral over date-1 consumption: $V_i(C_h, C_l) = C_h$ for $i \in \{D, L\}$. Their total date-1 endowments are identical, equal to $e > y + \kappa$. Risk neutrality plays no other role than simplifying the analysis of the trading decisions of private agents given rationing and thus execution risk. The extensive form of the game is now as follows, where the incremental stages are in italics.

Date 0: Pairs of lenders and borrowers are formed. Lenders make take-it-or-leave it offers on date-0 transfers and date-1 obligations.

Date 1, after types are revealed:

- *Unofficial trades settle (if any).*
- The official market opens and settles.
- *Unofficial trades settle (if any).*
- The date-0 pairs meet again and interact as in Section 3.1.

The official market has the same structure as in the previous section. At the end of the official market, the government still stands ready to buy up to $\delta \geq 0$ units to fill unmatched sell orders. The official price satisfies $P^* < 1/v$. We only add to the previous section that the government also commits to sell up to $\sigma \geq 0$ units of h-goods to fill unmatched buy orders.[10]

Structure of unofficial markets. One session takes place before the official market opens and another one after it closes. At each session, private agents simultaneously submit any number of unofficial trading orders specifying a direction (buy or sell), a quantity, and a price. Orders that share the same price are matched in random sequence as in the official market. Namely, sell and buy orders are arranged in queues. Each buyer may decline goods and proceed down the queue of unmatched sell orders until his order is fully matched or he reaches the end of the queue. Sellers are subject to the same short-sell constraint as in the official market.

Such Bertrand-Cournot unofficial trading allows in principle for many equilibria because any price at which some agents are willing to buy and some willing to sell is an equilibrium outcome. We deliberately impose little structure on trade this way to make it as difficult as possible for the government to control the price level.

Finally, we assume that lenders start out at date 1 with $M > 0$ units of base money inherited from an unmodelled past. This will be useful in the application to the determination of the nominal rate in Section 5 as it captures that banks have on aggregate strictly positive reserve holdings.

[10] The goods possibly sold by the government could stem from lumpsum taxation. In this case the government would then collect a mix of h and l goods, and would adjust the quantities sold accordingly.

We now show that unless the government stands credibly ready to buy and sell sufficiently large quantities in the official market, the fact that contracts use the official price to index obligations may lead to the rise of unofficial trades. These trades reflect the respective collective attempts of lenders and borrowers at respectively increasing or reducing the real value of the obligations. The following proposition first studies equilibria in which borrowers force lenders to accept unofficial prices at which their nominal obligation is inflated away.

Proposition 5. ***(Incomplete backing and debt inflation)*** *Suppose agents have signed an obligation to repay either a real value y in money at the official price or $y+\kappa$ if all or part is repaid in-kind. There exists no equilibrium with trades at unofficial prices strictly above P^* if and only if $P^*\sigma \geq M + P^*\delta$.*

Proof. See Appendix B. □

In the presence of incomplete backing ($P^*\sigma < M + P^*\delta$), borrowers can exploit the rationing of lenders in the official market in order to purchase money at an unofficial price above P^*. In order to do so, they seek to sell goods in unofficial markets that take place before the official one. Lenders are indifferent between purchasing goods at such bad unofficial terms and trading in a rationed market at a better price.

In order to deter borrowers from entering into this debt-inflation trade, the government must make sure that base money is fully backed—$P^*\sigma \geq M + P^*\delta$. The following proposition studies the symmetric problem of debt deflation.

Proposition 6. ***(Debt deflation)*** *Suppose agents have signed an obligation to repay either a real value y in money at the official price or $y+\kappa$ if all or part is repaid in-kind. There exists no equilibrium in which some G-borrowers may buy money at unofficial prices below P^* if and only if*

$$P^*\delta > 2e\frac{\kappa\gamma P^* - (1-P^*v)M}{\kappa\gamma + 2e(1-P^*v)}. \tag{1}$$

Proof. See Appendix C. □

If condition (1) fails to hold, lenders can coordinate to exploit the fact that G-borrowers are willing to trade up to $y+\kappa$ units of h-goods for money. To do so, they first bid for money in the official market alongside borrowers so as to squeeze them. Then they recycle part of the proceeds in an unofficial market with an unofficial price below P^* in which the borrowers who were squeezed out of the official market scramble for cash. This strategy entails that lenders/buyers are rationed in the unofficial market and so they end up this trade with money on hand. As a result, the government can discourage such debt deflation by making these terminal money holdings sufficiently costly. Condition (1) shows that this is the case when the government issues enough base money M and $P^*\delta$, and deviates a lot from the Friedman rule (P^*v small). It is instructive to rewrite this condition as:

$$\kappa\gamma\left(1-\frac{\delta}{2e}\right)\leq\left(\frac{M}{P^*}+\delta\right)(1-P^*v). \tag{2}$$

The left-hand side are the maximum excess resources that lenders can generate with the debt-deflation strategy since a mass $\gamma(1-\delta/2e)$ of G-borrowers are rationed in the official market and thus willing to spend up to κ more goods for money. The right-hand side is the collective cost for lenders to hold a real amount of base money $M/P^*+\delta$ at a unit opportunity cost $1-P^*v$ which is larger, the larger the deviation from the Friedman rule.

Distance to the first-best. Section 3.1 has shown that under the assumption of perfect price control, the government can make the cost to promote liquidity in the official money market arbitrarily small. It suffices to buy an arbitrarily small amount of l-goods δ. By contrast, Propositions 5 and 6 together show that the cost of fostering a liquid and transparent money market is bounded away from zero when the government must make trading at the official price incentive compatible. The total cost to the government is indeed

$$(1-P^*v)\frac{M}{P^*}+2e\frac{\gamma\kappa-(1-P^*v)\frac{M}{P^*}}{\gamma\kappa+2e(1-P^*v)}. \tag{3}$$

We deem the first term in (3) the L-cost and the second one the B-cost. The L-cost is the rent that the government grants to lenders by purchasing their money M at a price $1/P^*$ higher than its fundamental value v. The B-cost is the cost of buying δ l-goods from B-borrowers and then repurchasing the issued money against h-goods.

Expression (3) exhibits a tradeoff between these two costs. Holding P^* fixed, reducing the L-cost by reducing M raises the B-cost. The intuition is that a reduction in the rent paid to lenders tendering their money makes it easier for them to coordinate on the debt-deflation equilibrium. This must be offset by an increase in the amount of money injected in the economy via government purchases in the official market, which in equilibrium translates into an increase in the number of l-goods δ that the government swaps with h-goods. Also, holding the L-cost fixed by adjusting M, the total cost (3) increases in P^* which thus should be made as small as possible. The intuition is that a reduction in P^* raises the unit cost of terminal holdings of money.

The optimal policy depends on the type of departure from the first-best that the government seeks to minimize. Suppose that for unmodelled reasons, the government seeks to minimize its purchases of l-goods so that borrowers bear all quality risk.[11] In this case the government can actually set the B-cost δ arbitrarily close to 0 by setting the L-cost arbitrarily close to $\gamma\kappa$.[12] Interestingly, $\gamma\kappa$ is exactly the rent from extortion that lenders could obtain in court. Implementing optimal risk bearing requires that the government transfers them this rent.

Alternatively the government may simply seek to minimize the total cost of liquidity and transparency (3). In this case, it sets P^* as small as possible and $M = 0$. The total cost is in this case the B-cost of swapping $\delta = \gamma\kappa/[1 + \gamma\kappa/(2e)] < \gamma\kappa$ l-goods into h-goods.

4 Discussion

Alternative interpretations with multiple goods. In our simple setup with a unique desirable consumption good, the rise of unofficial markets just means that the law of one price may fail to hold for this good. There are many empirical examples of the coexistence of official and unofficial markets when governments seek to control prices. Reinhart and Rogoff (2004) document the pervasiveness of unofficial forex markets in the presence of pegs. Yet we believe that our unofficial markets also admit a broader interpretation related to what we conjecture would prevail in a version of our economy with multiple goods. In a richer model with multiple goods/assets, we could assume that the government trades only one item (safe and liquid fixed-income claims in practice).

[11] A natural rationale would be to provide incentives to borrowers if the quality of their date-1 allocation depended on a date-0 hidden effort.

[12] Recall that $\delta > 0$ is required to rule out the illiquid equilibrium.

The unofficial markets here would simply be the markets for other goods in which it does not intervene. The departure from the law of one price that we sometimes observe in equilibrium here would then become a departure of relative prices from their fundamental values. To see this, consider the extreme situation in which two goods are perfect substitutes and yet only one of them trades in the official market. In this case the equivalent of the unofficial market would be a distorted relative price of the second good. We leave this interesting route for future research.

Contracting externalities and bankruptcy law. In an equilibrium with a liquid and transparent money market, each individual pair is indifferent between writing down a date-0 obligation with the option to repay in-kind, as we assume, and a purely monetary one. Yet the in-kind option that we have assumed throughout Section 3 has an important equilibrium impact because this puts a lower bound $P^*y/(y+\kappa)$ on the unofficial price at which G-borrowers are ready to sell in debt-deflation equilibria. This reduces in turn the amount σ that the government must commit to buy. One way to internalize this contracting externality is a bankruptcy law that puts a limit on the seizures of collateral by creditors to illiquid firms with valuable real assets.

Private leverage and public balance sheet. Our theory establishes a natural connection between financial fragility and the balance sheet of the issuing authority. To see this, notice that the amount of money $P^*\delta$ that the government must issue to deter debt-deflation dynamics is increasing other things being equal in $\gamma\kappa$ and decreasing in M. Thus, if the private sector starts date 1 without much cash on hand (M small), and features many borrowers (γ large) who are prone to severe financial distress (κ large), then the issuing authority must stand ready to expand its balance sheet a lot to avoid a dash for cash.

Could contracting be more efficient? For simplicity we have directly assumed that contracts are indexed using the official price, and so the mere existence of unofficial prices implies that (x, y) cannot be implemented. The reader may object that if unofficial trading is anticipated as of date 0 then optimal contracting can take it into account. There are two reasons such more sophisticated contracts would not be very helpful. First, there is rationing and thus execution uncertainty in unofficial markets that contracts cannot offset. Second, unofficial prices can take several values, each of them with strictly positive probabilities in stochastic equilibria, and the court cannot infer these values from observing only the official market.[13] Thus as soon as there is room for unofficial trades, the court cannot insulate transfers from the fluctuations induced by unofficial trade.

Where is the seigniorage income? Maintaining a liquid and transparent money market involves two date-1 costs, overpaying for low-quality assets or underselling high-quality assets. But since money creates surplus by implementing the optimal contract, the issuer of money could extract some of it by requiring at date 0 that lenders meet the liquidity requirement M and auctioning off the money.[14] Still, our theory of money requires that this issuer commit to date-1 value-destroying trades, and this offers a rationale for the public nature of money.

5 Application to interest-rate setting

The conduct of monetary policy consists both in practice and in mainstream macroeconomic models in setting short-term nominal interest rates. A reinterpretation of our model offers interesting insights into some recent challenges facing interest-rate setters.

We adapt the model to this application as follows. We add a terminal date 2 to the economy. What was a date-1 good in the main model is now a bond—the promise of a date-2 nominal payment. A high-quality (h) bond pays off 1 monetary unit and a low-quality one (l) pays off $1/(1+\lambda)$, where $\lambda > 0$.

Private agents are banks. They observe bond quality. They value consumption at dates 0 and 2 and take the date-2 price level as given. Each borrowing bank owes 1 monetary unit to a lending one at date 1. Lending banks can split their claim at date 1 into two uses, either deposit it in reserves at the central bank thereby earning a nominal return R^R set by the central bank, or lend it in the interbank market. Borrowers can fund their date-1 payment either by borrowing in the interbank market or by emergency borrowing at the central bank. Borrowers can default with impunity at date 2 and so they must pledge one h-bond for each monetary unit they promise to repay at date 2.

[13] The multiplicity of unofficial prices is clear from our construction of the equilibria with unofficial trades in Sections B and C.

[14] Banks are no longer subject to reserve requirements in major jurisdictions but to prudential liquidity requirements that are often partly met with reserves.

The central bank cannot tell bonds apart. It is a market-maker in the interbank market in the same way as in the main model. It quotes the rate R^*, and offers to lend $\Delta M > 0$ in base money and to sell $B > 0$ h-bonds in case all private trades are not executed. It lends without limits at the emergency rate $R^*(1+\lambda)$. Finally, lenders enter date 1 with M units of base money.

In sum, this setup is essentially a re-interpretation of the main model in which lenders' ex-post informational rent stems from the central bank's inability to tell apart good and bad bonds. Otherwise it is merely re-labelling $R^R = v,\ R^* = 1/P^*,\ \Delta M = P^*\delta,\ B = \sigma,\ R^*\lambda = \kappa$. We now interpret unofficial markets simply as official trading that takes place either before or after the central bank's intervention. When such markets are active, this means that the effective rate is a volume-weighted average over all trades. Proposition 5 thus means that the effective rate may be below R^* if and only if $B < R^*(M + \Delta M)$. In this case, the good bond is overly scarce and banks are willing to lend at rates below R^* in order to put their hands on it as collateral. Thus, a central bank that does not have enough high-quality assets backing the monetary base is unable to avoid the risk of an overly loose effective monetary policy.

Symmetrically, applying Proposition 6 yields that there exist no equilibria with official rates above R^* if and only if

$$\Delta M > 2e \frac{\gamma\lambda - M\left(1 - \frac{R^R}{R^*}\right)}{\gamma\lambda R^* + 2e\left(1 - \frac{R^R}{R^*}\right)}. \tag{4}$$

The central bank must commit to create enough money to purchase bonds in order to avoid dashes for cash in the repo market. This commitment increases with respect to $\gamma\lambda$ because this means that more banks are subject to the extraction of informational rents. This corresponds to situations of spikes in interest rates following increases in the uncertainty in collateral values as during the 2008 crisis, the 2010 Eurozone debt crisis, or to a lesser extent the 2019 rate spike in US money markets.

6 Concluding remarks

Money as a medium of exchange typically arises to replace credit when borrowers cannot commit to offering sufficiently large repayments. By contrast, money as a debt-settlement device arises here to enable credit when lenders cannot commit to asking sufficiently small repayments. Unsurprisingly, this topsy-turvy approach generates very complementary predictions. Since money is essential for private-debt settlements, our theory is primarily one of inside money. Yet, debtors and creditors collectively seek to manipulate the price level in their respective interests, and this makes appropriate injections of base money essential to discipline these attempts. In particular, ensuring that base money is an inferior storage so as to spur the circulation of all monies is a natural and important implication of our theory. Finally, our theory connects the financial stability of the private sector to the stance of monetary policy.

A natural route for future research consists in studying the interplay of this new role of money as a debt-settlement device with its best-known one as a medium of exchange. On one hand, recognizability is an instrumental feature of money for both functions. This creates economies of scope justifying that the same instrument plays a dual role. On the other hand, it is well-known that an efficient medium of exchange must in general be a reasonably efficient store of value, whereas we have shown that a good debt-settlement device must discourage hoarding. We conjecture that the resulting tradeoff could deliver a theory of optimal inflation.

References

Alchian, A. A. (1977): "Why Money?" *Journal of Money, Credit and Banking*, 9, 133–140.

Banerjee, A. V. and E. S. Maskin (1996): "A Walrasian Theory of Money and Barter," *The Quarterly Journal of Economics*, 111, 955–1005.

Berentsen, A. and G. Rocheteau (2004): "Money and Information," *Review of Economic Studies*, 71, 915–944.

Brunner, K. and A. H. Meltzer (1971): "The Uses of Money: Money in the Theory of an Exchange Economy," *American Economic Review*, 61, 784–805.

Burdett, K., S. Shi, and R. Wright (2001): "Pricing and Matching with Frictions," *Journal of Political Economy*, 109, 1060–1085.

Dang, T. V., G. Gorton, B. Holmström, and G. Ordoñez (2017): "Banks as Secret Keepers," *The American Economic Review*, 107, 1005–1029.

Doepke, M. and M. Schneider (2017): "Money as a Unit of Account," *Econometrica*, 85, 1537–1574.

Dubey, P. and J. Geanakoplos (2006): "Determinacy with Nominal Assets and Outside Money," *Economic Theory*, 27, 79–106.

Freeman, S. (1996): "The Payments System, Liquidity, and Rediscounting," *The American Economic Review*, 86, 1126–1138.

Gorton, G. and G. Ordoñez (2014): "Collateral Crises," *The American Economic Review*, 104, 343–378.

Gu, C., F. Mattesini, and R. Wright (2016): "Money and Credit Redux," *Econometrica*, 84, 1–32.

Hu, T.-W. and G. Rocheteau (2026): "Breaking Up With M: Cashless Limits Under Limited Commitment," Working paper.

Lagos, R. (2000): "An Alternative Approach to Search Frictions," *Journal of Political Economy*, 108, 851–873.

Lagos, R. and S. Zhang (2022): "The Limits of Onetary Economics: On Money as a Constraint on Market Power," *Econometrica*, 90, pp. 1177–1204.

Lerner, A. P. (1947): "Money as a Creature of the State," *American Economic Review*, 37, 312–317.

Philippon, T. and V. Skreta (2012): "Optimal Interventions in Markets with Adverse Selection," *American Economic Review*, 102, 1–28.

Plantin, G. and J. Tirole (2018): "Marking to Market versus Taking to Market," *American Economic Review*, 108, 2246–76.

Reinhart, C. M. and K. S. Rogoff (2004): "The Modern History of Exchange Rate Arrangements: A Reinterpretation," *Quarterly Journal of Economics*, 119, 1–48.

Rocheteau, G. (2011): "Payments and liquidity under adverse selection," *Journal of Monetary Economics*, 58, 191–205.

Rojas Breu, M. (2017): "Debt enforcement and the value of money," *Journal of Economic Theory*, 168, 237–251.

Sargent, T. J. and N. Wallace (1981): "Some unpleasant monetarist arithmetic," *Quarterly Review*.

Shapley, L. and M. Shubik (1977): "Trade Using One Commodity as a Means of Payment," *Journal of Political Economy*, 85, 937–968.

Smith, A. (1776): *An Inquiry into the Nature and Causes of the Wealth of Nations*, London: W. Strahan and T. Cadell.

Starr, R. M. (1974): "The Price of Money in a Pure Exchange Monetary Economy with Taxation," *Econometrica*, 42, 45–54.

Tirole, J. (2012): "Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning," *American Economic Review*, 102, 29–59.

Williamson, S. and R. Wright (1994): "Barter and monetary exchange under private information," *American Economic Review*, 84, 104–123.

A Proof of Proposition 4

Proof that there is no trade when $\delta = 0$. Suppose a lender does not place any buy order. He obtains his unconditional maximum feasible payoff in the continuations in which his borrower fails to obtain money in the market to repay his debt because he earns $y + \kappa$ if it is a G-borrower. If it is a B-borrower, he receives y h-goods and does not have to spend time in the market. The following claim shows that these are the only possible equilibrium continuations.

Claim. If a lender does not participate in the market then his associated borrower cannot sell goods for money in it.

Proof. In any equilibrium, active buyers are lenders to an active seller with a larger executed order because otherwise the active buyer would be bankrupt. Any strategy with a strictly positive probability of bankruptcy is strictly dominated because one can always buy the same quantity from his own borrower if he is still unmatched. Together with the zero net supply of inside money, this implies that all active sellers must be borrowers from active buyers. Thus if the agent does not buy from his borrower nobody else will in the continuation equilibrium. □

Proof that all G-borrowers repay with money when $\delta > 0$. Anticipating that the government buys some goods if private agents do not match all sell orders, G-borrowers find it strictly dominant to sell y goods, and B-ones sell $y - \kappa$ h-goods and κ l-goods. Anticipating this, lenders find it in turn strictly dominant to place buy orders in case their respective borrowers can pay back with money. One the queues are formed, the lenders to the δ borrowers in the front of the queue find it strictly dominant to spend, and do so by buying the first h-goods they come across. But this implies that the lenders to the sellers of these goods, either observing or perfectly anticipating this, spend similarly, and so on until all h-goods are sold.

Finally, notice that no agent can strictly gain from a buy order when $P^* v = 1$ and so borrowers receive only money from the government in this case.

B Proof of Proposition 5

Let us deem for brevity simply Gs and Bs the good and bad borrowers and L_Gs and L_Bs the lenders to good and bad borrowers. Let $Y \equiv P^*y$. We construct an equilibrium in which all borrowers place a sell order Y at $P \in (P^*, 1/v]$ and lenders place a buy order $L < M$ at this price, where all orders are expressed in monetary units instead of number of goods. Only h-goods trade in this unofficial market. Afterwards, in the official market, the Gs who have not obtained money unofficially this way sell Y. The Bs sell officially their l-goods and $Y - P^*\kappa$ worth of h-goods if they were not successful in the unofficial market. L_Gs seek to buy $M + Y - L$. L_Bs buy $M - L$, plus Y if their borrower got unofficial money or if he did not but got official money from the government. Finally, Bs who paid off their debt with unofficial money but sold their l-goods to the government seek to reinvest the cash to get h-goods.

This implies that private official sell orders of h-goods are $\gamma Y(1-L/Y)+(1-\gamma)(1-L/Y)(Y-P^*\kappa)$ and official buy orders are $M-L+\gamma Y+(1-\gamma)YL/Y+(1-\gamma)(1-L/Y)(Y-P^*\kappa)+P^*\delta$. This is an equilibrium if agents are indifferent between buying at P and at P^*. It must be that there is rationing at P^*: $M+P^*\delta > P^*\sigma$, and that P satisfies

$$\frac{P^*\sigma + Y - L - (1-\gamma)\left(1-\frac{L}{Y}\right)P^*\kappa}{M+Y-L-(1-\gamma)\left(1-\frac{L}{Y}\right)P^*\kappa + P^*\delta}\frac{1}{P^*} + \frac{M+P^*\delta - P^*\sigma}{M+Y-L-(1-\gamma)\left(1-\frac{L}{Y}\right)P^*\kappa + P^*\delta}v = \frac{1}{P}. \tag{5}$$

which admits exactly one solution.

Conversely, there cannot be unofficial trade at an unofficial high price if $M + P^*\delta \leq P^*\sigma$. This would require the rationing of buyers in the official market. But if no agent gets bankrupt there is no net inside money creation in equilibrium, so this cannot be.

C Proof of Proposition 6

Let us deem for brevity simply Gs and Bs the good and bad borrowers and L_Gs and L_Bs the lenders to good and bad borrowers. Let $Y \equiv P^*y$. Let us construct an equilibrium in which all agents first sell all their goods in the official market, and then trade as follows in an unofficial market with price $P = P^*y/(y+\kappa)$, where we express the bids in monetary units rather than in goods for simplicity:

Unofficial buy orders:

- $L_G s$ bid all their official gains of money and $Y + M$. Their orders are thus $\gamma(M + Y + P^*\delta/2)$.
- $L_B s$ bid all their official gains of money plus M. They also bid Y if their borrower received money in the official market. Their orders are thus $(1-\gamma)(M + Y\delta/(2e) + P^*\delta/2)$.
- Bs and Gs bid all their official gains of money net of Y if any and nothing otherwise. Their orders are thus $P^*\delta/2 - Y\delta/(2e)$.

This adds up to buy orders equal to $M + P^*\delta + \gamma Y(1 - \delta/(2e))$.

Unofficial sell orders. Gs who were unsuccessful getting official money sell Y, their orders are thus $\gamma Y(1 - \delta/(2e))$. Agents do not find it profitable to sell goods dear and buy them cheap this way if and only if:

$$\frac{\gamma Y\left(1 - \frac{\delta}{2e}\right)}{M + P^*\delta + \gamma Y\left(1 - \frac{\delta}{2e}\right)} \times \frac{1}{P} + v\frac{M + P^*\delta}{M + P^*\delta + \gamma Y\left(1 - \frac{\delta}{2e}\right)} < \frac{1}{P^*}. \tag{6}$$

Rearranging yields condition (1). This condition is sufficient because any active market at a low unofficial price must be such that agents find the same trade—selling officiallly and buying unofficially—profitable, and we have selected P as low as possible so that we have constructed the most profitable of these trades.