A State Theory of Price Levels*

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Abstract

This paper studies whether a state can control the value of its currency by declaring it to be the legal tender for claims between itself and the private sector, and by trading it for desirable commodities according to a mechanism of its choice. In an economy in which all agents are price-setters, we identify when such policies elicit a single equilibrium price level. For policies that fail to do so, for example because different official and unofficial prices may coexist in equilibrium, we still offer tight restrictions on the set of predictable price levels. We offer a parsimonious framework that sheds light on common mechanisms driving various historical and recent forms of monetary or/and fiscal instability.

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1 Introduction

Suppose a state declares a useless good that the rest of society cannot produce to be the legal tender for claims between itself and the private sector: The state accepts this good to settle tax liabilities and, reciprocally, uses it to meet sovereign liabilities. Suppose this state also makes this good the official medium of exchange: It trades it against other commodities according to a mechanism of its choice. When do these two official roles suffice to determine the prices at which private agents trade this good for desirable commodities, if they trade it at all? What happens when, conversely, these roles fail to lead to a unique equilibrium price level? This paper addresses these questions in a setup in which both the strategy of the state and the responses of the rest of society are modeled in novel and more flexible ways than is typically the case in macroeconomics. Our goal is to shed a new light on the determination of the price level by public financial policy.

It has been long noticed that the role of money as legal tender is an important determinant of its value, in the sense that private tax liabilities may affect the price level (e.g., Lerner, 1947; Smith, 1776; Starr, 1974). Symmetrically, the ability of states to issue money to repay sovereign debts potentially affects the price level (Sargent and Wallace, 1981).

The role of money as an official medium of exchange as defined above is of course also an essential determinant of the price level, as it encompasses in particular the conduct of monetary policy. Central banks trade the money that they issue for other stores of value, and typically use the prices at which such trades settle as important nominal anchors and explicit targets for monetary policy—e.g., metallic standards, currency pegs, or currently the targeting of some short-term interest rates.

Still, various historical and recent instances suggest that the response of society to such public plans may not always lead to a stable, well-defined price level. Examples include the pervasive coexistence of official and unofficial exchange rates under currency pegs, the rise of parallel or black markets in the presence of price controls, debt-deflation spirals associated with metallic standards, or even the difficulties met by central banks when trying to control market interest rates. We aim at offering novel insights into the (largely common) reasons such instabilities may arise.

Our framework formalizes public financial policy as a collection of transfers and trades of money between private and public sectors that formalizes the respective roles of money as legal tender and as official medium of exchange. We characterize the policies that lead to a unique equilibrium price level as well as the range of predictable outcomes that arise when, conversely, the price level is not determined.

Our analysis focuses on an elementary one-date economy that features one desirable good and intrinsically worthless money. Public financial policy has three central components: i) a maximum quantity of goods that the state is willing to trade for money, ii) a price at which the state is willing to trade goods for money, and iii) a vector of negative monetary transfers (taxes) and positive ones (e.g., repayment of liabilities issued in the past) from the state to each private agent. The first two components capture the role of money as the official medium of exchange, while the third component captures its role as legal tender. Crucially, all private agents in this economy are free to set price(s) at which they trade goods for money outside the official trading mechanism. Namely, subject to a solvency constraint, each agent can submit any number of buy and sell orders, where an order consists in a quantity of goods and a unit price. Orders with matching prices are executed with proportional rationing of excess supply or demand. A public financial policy determines the price level if and only if all trades occur at the same price level in all the equilibria of this Bertrand-Cournot market game.

Public financial policies that fail to determine the price level this way may in particular lead the official exchange to coexist with a purely private market such that the good trades both at the official price in the former, and at an unofficial one that better reflects the fundamentals of public financial policy in the latter. While our approach remains stylized, we relate these situations to various pervasive empirical patterns, in particular in situations of monetary and fiscal instability. Our main insights can be summarized as follows.

Fixed policies with fiscal creditors. We first study the simple and instructive situation in which the price at which the state is willing to trade is fixed, and in which all agents receive fixed positive transfers from the state that exceed their tax liabilities: They are net fiscal creditors. Such a situation is empirically relevant as a fixed official price resembles a currency peg, a metallic standard, or even some of the allocation mechanisms currently used by central banks in their refinancing operations. Positive fixed transfers correspond to the repayment of nominally safe public liabilities issued in the past such as central-bank reserves.[1] In this situation, all private agents have money that they are willing to sell against goods at any price.

We show that unofficial markets may emerge in such an economy. The central mechanism is that some private agents become intermediaries, selling goods to the others at a high unofficial price and investing the cash proceeds in the official market. By doing so they congest the official market, which pushes their unofficial buyers to accept high prices instead of being rationed at the lower official one. The key insight is that such unofficial markets arise only if private agents have heterogenenous access to the official market. In our minimalist model, such heterogeneity hinges on the conjonction of heterogeneous net transfers across agents with an insufficient official supply of goods. In this case, the marginal return on an official buy order is decreasing in its size because the marginal dollar invested crowds out the inframarginal ones. As a result, cash-poor agents have a better access to the official market in the sense that they earn a higher marginal return in it. Coordinating on the creation of unofficial markets enables them to leverage on this advantage and extract rents from cash-rich agents. We note in passing that all that matters are the *relative* price impacts of private agents in the official market. Thus, our results persists in the negligible limit in which *absolute* price impacts vanish as long as negligible agents remain heterogeneous.

Fiscal debtors. The situation is symmetric when private agents are fiscal debtors: Their tax liabilities exceed their positive transfers from the state and so they must acquire money to extinguish their net liabilities. In this case, if these liabilities are sufficiently heterogeneous, and if the state does not have a sufficiently large (out-of-equilibrium) money supply, unofficial markets with prices below the official one may arise together with a "dash for cash" in the official market. Less indebted agents may coordinate on buying more money than they need in the official market, so that large debtors cannot purchase enough money in the official market. These distressed large debtors may then be willing to offload goods at a low unofficial price, and small debtors can use the cash that they obtain in the official market to snap up these cheap goods.

The common take-away from these symmetric situations is that a fixed policy deter-

[1] Section 6 develops a two-date extension that endogenizes all transfers as resulting from voluntary earlier private decisions to buy government claims.

mines the price level if and only if the state stands ready to trade an out-of-equilibrium volume of goods or money strictly larger than the equilibrium one. Otherwise this creates room for the rise of unofficial markets in which some agents leverage their advantage in the official one, where this advantage is a pure consequence of public financial policy in our setup.

Analogy with bank runs (Bryant, 1980; Diamond and Dybvig, 1983). It is interesting to relate the above rise of unofficial markets to that of equilibria with runs in the banking literature. In Diamond and Dybvig (1983), the patient agents, who benefit the least from going early to the bank, may create rationing if they coordinate on running on it, thereby extracting resources meant for the impatient ones. In our setup, similarly, the agents with the smallest gains from trade with the state may extract rents from the others by coordinating to congest the official market. A difference is that in Diamond and Dybvig (1983), the run equilibria destroy resources through inefficient liquidation and are thus Pareto inferior, whereas unofficial markets merely redistribute a fixed endowment here.

Market-clearing official price. To which extent does such price-level indetermination rest on the assumption that the state trades at a fixed price? To investigate this, we study an alternative policy in which the state commits instead to set a market-clearing official price in and out of equilibrium as in Shapley and Shubik (1977). Interestingly, the same mechanism as under a fixed price may still give rise to parallel markets with high prices. Thus the price level is never determined. Yet an important difference with the fixed-price case is that the absolute price impacts of agents matter for such multiplicity, as opposed to the relative ones when the official price is fixed. Thus all unofficial prices converge to a single value in the limit of negligible agents, and so the price level is determined in the negligible limit in this sense.

Default. We also investigate policies such that transfers are contingent on private strategies as opposed to fixed. We focus on situations in which the state cannot use the money that it creates to meet its transfers, but must instead rely on the money collected from the private sector. This corresponds to situations in which independent central banks do not—neither explicitly nor implicitly—monetize sovereign debt, or to that in which debt is denominated in a foreign currency. We show that this opens up the possibility of self-fulfilling debt crises in which private agents' expectations regarding sovereign default leads them to adjust the money that they sell to the state. The resulting limited proceeds for the state vindicate in turn the expectation of default. We find that in the presence of such defaultable transfers, the price level is never determined away from the negligible limit. The negligible limit sheds light on an interesting interaction between the state's trading protocol and the consequences of default. With a fixed trading price, the price level is determined in the negligible limit as default affects only, and positively, the real resources that the government can spend. By contrast, under a market-clearing official price, a feedback loop unfolds between the price level and the haircut on public transfers, with equilibria with higher haircut featuring lower price levels.

Implications for fiscal and monetary interactions. Our setup sheds light on the out-of-equilibrium policy that justifies the fiscal theory of the price level, in which safe legacy nominal debt and fixed real surpluses pin down the price level. This corresponds in our model to a situation of extreme fiscal dominance such that the state issues whatever money it takes to make good on its nominal commitments, and trades money for goods so as to keep its real spending constant no matter the (out-of-equilibrium) implications for the market-clearing price level. The polar case in which the state trades money for goods at a fixed price, does not monetize its liabilities, and the severity of default and government spending comove positively across equilibria resembles monetary dominance: Monetary policy sets the price level and does not intervene in fiscal matters. In between these two extremes, the policies that may (or may not) lead to unofficial trade at high or low prices can be viewed as intermediate situations in which neither fiscal nor monetary policy fully accommodates the other. Such intermediate situations are typically out-of-reach of Walrasian environments.

Applications. Our findings shed light on multiple historical and also more recent episodes. The equilibria in which financial distress forces large debtors to sell commodities at low unofficial prices are reminiscent of debt-deflation episodes under metallic standards (e.g., the "Long Depression" that started in 1873). Also, our theory of financial repression offers a parsimonious framework to understand the various phases of the "assignats" crisis—paper money issued during the French Revolution. We also relate our setup to the frequent emergence of parallel markets in response to exchange rate pegs or during periods of price controls, to the introduction of in-kind taxation in periods of financial repression, and to the valuation and trade of private monies such as stablecoins, banknotes during the US Free banking era, or money market funds. We elaborate on these various applications throughout the paper. We stress that despite the diversity of their contexts, many of them share the feature that, as in our stylized mechanism, agents with privileged access to official markets use unofficial ones to scale up their rents.

Related literature. The title of this paper is an unsubtle reference to the state theory of money outlined in Knapp (1924). As epitomized by the opening sentence of the book—*"Money is a creature of law."*—the state theory of money contends that the state has a unique ability to impose something as money due to its legislative capacity. Our contribution is to formally study the extent to which this capacity may suffice to determine the price level. Here, the formalization of the distinctive capacity of the state is that it is the only agent which can print money, declare taxes, and expropriate bankrupt private agents.

Bassetto (2002) pioneers the strategic foundations of price-level determination by public financial policy. His goal is to offer an example of an economy in which the fiscal theory of the price level applies. We share with him a strategically closed environment that highlights the importance of credible out-of-equilibrium actions in shaping equilibrium outcomes. By lifting his restriction to centralized markets and market clearing as in Shapley and Shubik (1977), and by considering various types of nominal promises by the state—thus allowing for sovereign default, we also generate a number of additional and, we believe empirically relevant, insights. In particular, we establish a novel and natural connection between the recent literature on the determination of the price level and an older literature that studies the aggregate implications of allocation mechanisms in non-clearing markets—see Bénassy (1993) for a brief exposition of some of its important insights.

Our approach also has points of contact with the literature that endogenizes trading frictions as pure coordination failures in economies that are not plagued by informational nor search frictions. Important contributions include Lagos (2000) and Burdett et al. (2001). One can view our results as identifying public financial policies that eliminate the possibility that the private sector coordinates on other prices than that targeted by the state. We emphasize in particular the central role of the trading protocol selected by the government. In this sense, our approach applies to a context of such endogenous frictions the approach pioneered by Hu et al. (2009), that endogenizes trading mechanisms in the presence of search frictions.

The search literature has like us emphasized that the willingness of the state to back its money by accepting to trade it for desirable goods is important (Aiyagari and Wallace, 1997; Li and Wright, 1998). In our model without exogenous frictions this official trading is simply a necessary condition for price-level determination. In these papers, this source of value for money coexists with its role of mitigating search frictions, and they show that more backing makes it easier to sustain the Pareto-dominant monetary equilibria.

Bassetto and Phelan (2015) find like us that bounds on public interventions at a fixed price in money markets may generate multiple equilibria. In their setup, crises may arise during which the private sector exhausts the public lending capacity, money grows fast, and the private interest rate is higher than the official one. Their contribution highlights that the exact mechanism used to implement monetary policy—e.g., standing ready to trade at a given fixed rate up to a limit versus conducting open-market operations to achieve a target rate—can have significant aggregate implications. One can view our contribution as a related detailed study of the impact of the mechanisms by which the state transfers and trades money on the determination of the price level.

Finally, given the central role of strategic exchange in our framework, we revisit the old and large literature on the strategic foundations of Walrasian equilibrium. A review is beyond the scope of this paper. Important contributions include Dubey (1982) and the references herein, Dubey and Shubik (1980), Schmeidler (1980), and Shapley and Shubik (1977).

The paper is organized as follows. Section 2 outlines our baseline model. Section 3 solves it in the case in which the economy features only fiscal creditors. Section 4 shows how our approach helps clarify—and offers rigorous foundations for—the ones based on the Walrasian equilibrium concept. Section 5 introduces fiscal debtors. Section 6 outlines and solves a two-date model. Section 7 offers an extension to an uncertain environment. Section 8 discusses applications of our setup to historical and current situations for which we believe our insights to be particularly relevant. Section 9 concludes. Most proofs follow the propositions because we find them instructive, yet the paper is written so that they can be skipped in a first reading.

2 One-date model

This section outlines our simple one-date economy. It presents a baseline public financial policy that consists in fixed negative transfers (taxes), in fixed positive transfers that may be interpreted as extinguishments of nominal liabilities issued in an unmodelled past (e.g., reserves with the central bank), and in a commitment to trade a given maximum quantity of goods for money at a fixed official price.

2.1 Setup

The economy comprises a public sector—"the state"—and $I \geq 2$ private agents indexed over $\mathcal{I} \equiv \{1, ..., I\}$. There are two divisible economic goods, one deemed "the good" and the other "money" henceforth. The good is intrinsically desirable to private agents whereas money is not. Each private agent thus ranks any bundles of the good and money using the standard ordering of their respective quantities of the good only.

Each private agent is endowed with $e > 0$ units of the good. The state is endowed with $I\tau > 0$ units of it. The state can produce money. Private agents cannot. All private agents and the state can trade money for the good as described below.

Public financial policy. The state enforces a policy that features monetary transfers, money creation, and trade. We describe each component of a policy in turn.[2]

Negative transfers (taxes). The state requires that each private agent $i \in \mathcal{I}$ pay a tax equal to $T_i \geq 0$ units of money.

Positive transfers. The state makes a cash transfer $L_i \geq 0$ to each agent $i \in \mathcal{I}$.

Money creation. Policy also features the production of $IM \geq 0$ units of money.

[2]Policy could also feature in-kind transfers. This would clutter notations without generating significant insights. Section 8 interprets τ as an in-kind tax levied by the government in order to analyze some historical examples of in-kind taxation under the lens of our model.

Trade. The state posts an order to buy a quantity $I\delta_G \in \mathbb{R}$ of the good at the price level $P^* > 0$, with the convention that this is a sell order if $\delta_G \leq 0$.

In sum, a policy consists in a vector $\mathcal{P} = ((T_i)_{i\in\mathcal{I}}, (L_i)_{i\in\mathcal{I}}, M, P^*, \delta_G)$.[3] The state also consumes $Ic_{G,C} \in \mathbb{R}$ units of the good and $Ic_{G,M} \in \mathbb{R}$ units of money.

Remark. We do not model this state consumption as a component of policy but rather as a payoff to the state determined by both policy and by the private sector's strategy profile as detailed below. Bassetto (2002), unlike us, models public spending as a decision that is not contingent on the private sector's strategy, but he posits that taxes, unlike here, are adjusting in response to (in and out of equilibrium) private strategies in order to maintain this fixed spending level. Both approaches are thus equivalent and merely reflect that since the state's surplus depends on voluntary trades by the private sector, either taxes or expenditures (or both) must be modeled as contingent on actions by all agents—as payoffs rather than actions in a game-theoretic setting.

Net transfers. We will make intensive use of the following natural concepts of net transfers associated with a policy $\mathcal{P}$.

Definition 1. ***(Net transfers)*** *For all $i \in \mathcal{I}$, let $N_i = L_i - T_i$. Let*

$$N = \frac{1}{I}\sum_{i\in\mathcal{I}} N_i, \;\; N_+ = \frac{1}{I}\sum_{i\in\mathcal{I}} \max\{N_i, 0\}, \;\; \textit{and} \;\; N_- = \frac{-1}{I}\sum_{i\in\mathcal{I}} \min\{N_i, 0\}. \tag{1}$$

Notice that $N = N_+ - N_-$. In words, N is the net nominal transfer per capita, N_+ is the private net fiscal credit per capita (counting a net debt as zero), and N_- the absolute value of fiscal net debt per capita (counting a net credit as zero). In the following, we will deem "fiscal creditors" the agents such that $N_i \geq 0$ and "fiscal debtors" those for whom $N_i \leq 0$.

Private actions. Taking policy $\mathcal{P}$ as given, private agents play a simultaneous game whereby they make decisions to trade and pay taxes. We describe these decisions in turn.

Taxes. Each private agent $i \in \mathcal{I}$ decides on the amount of cash taxes $\hat{T}_i \geq 0$ that she pays to the state.

[3] We will later introduce the possibility that, unlike in this baseline case of fixed policies, some policy components be contingent on the actions of the private sector.

Trades. Each agent can submit any number of orders to buy or sell a given quantity of goods at a given price. That is, an order features a trading direction (buy or sell), a quantity of goods, and a unit price, and each agent can submit any number of such orders. The only restriction is that the total size of her sell orders—the sum of the quantities of goods over all her sell orders—cannot exceed e. This is essentially a no short-sales constraint, as one cannot sell goods that one needs to buy. We will see below that money can by contrast be sold short.[4]

The trading strategy of agent $i \in \mathcal{I}$ is conveniently described by the functions detailing her cumulative orders. The respective cumulative buy and sell orders at prices (weakly) lower than P, $D_i(P)$ and $S_i(P)$ respectively, are increasing step functions over $[0, +\infty)$ satisfying:

$$D_i(0) = S_i(0) = 0, \tag{2}$$

$$\lim_{+\infty} S_i \leq e. \tag{3}$$

Let us denote, for all $P > 0$, $d_i(P)$ and $s_i(P)$ the respective buy and sell orders of i at the price P:

$$d_i(P) \equiv \int \mathbb{1}_{\{p=P\}} dD_i(p), \;\; s_i(P) \equiv \int \mathbb{1}_{\{p=P\}} dS_i(p). \tag{4}$$

In sum, the strategy of agent $i \in \mathcal{I}$ is $\mathcal{S}_i = (\hat{T}_i, D_i(.), S_i(.))$. Let $\mathcal{S} = (\mathcal{S}_i)_{i\in\mathcal{I}}$ denote the strategy profile of the private sector.

Market clearing, bankruptcy mechanism, and payoffs. We now describe how market clearing and a bankruptcy mechanism shape the payoff of each agent given a policy $\mathcal{P}$ and a strategy profile $\mathcal{S}$.

Market clearing. For all $P > 0$, let $d(P)$ and $s(P)$ denote the aggregate buy and sell orders at the trading post P:

$$d(P) = \sum_{i\in\mathcal{I}} d_i(P) + \mathbb{1}_{\{P=P^*\}}\delta_G^+, \;\; s(P) = \sum_{i\in\mathcal{I}} s_i(P) + \mathbb{1}_{\{P=P^*\}}(-\delta_G)^+ \tag{5}$$

[4] We rule out infinite prices at this stage but will address them in Section 3.

If $d(P)s(P) = 0$, then no trade takes place. Otherwise, the smallest side of the market is fully executed and the other side is rationed pro rata the size of each order. Formally, each private agent $i \in \mathcal{I}$ buys and sells effective quantities $\hat{d}_i(P)$ and $\hat{s}_i(P)$ such that:

$$\hat{d}_i(P) \equiv d_i(P) \min\left\{1, \frac{s(P)}{d(P)}\right\}, \ \hat{s}_i(P) \equiv s_i(P) \min\left\{1, \frac{d(P)}{s(P)}\right\}, \tag{6}$$

and the same proportional rationing rule applies to the state at $P = P^*$. We respectively denote $\hat{D}_i(P)$ and $\hat{S}_i(P)$ the respective cumulative effective purchases and sales of agent $i \in \mathcal{I}$.

The following definition is natural and important. It states that a trading post is active if and only if at least one private agent strictly gains or loses goods in it.

Definition 2. ***(Active trading post, net buyer, net seller)*** *Agent $i \in \mathcal{I}$ is net buyer (respectively net seller) at the trading post P if and only if $\hat{d}_i(P) > \hat{s}_i(P)$ ($\hat{s}_i(P) > \hat{d}_i(P)$ respectively). The trading post P is active if and only if at least one agent is net buyer or net seller at P.*

An active trading post always features both at least one net buyer and one net seller by definition, but one of them can be the state.

Why proportional rationing? The property of proportional rationing that is key to our results is that, on the rationed side of the market, a given bid generates an effective allocation to the bidder that is increasing and concave in the bid size . Another standard scheme that satisfies this natural property, albeit in an extreme form, is uniform rationing, whereby the marginal effective return on an order is equal to one and then to zero beyond a threshold.[5] Our broad results also hold under uniform rationing. Yet, as discussed in more detail later,[6] assuming uniform rationing would lack generality since such binary $0-1$ marginal returns would play a knife-edge simplifying role. We find it useful to study a mechanism with smoother marginal returns on bids that offers more general insights.[7] In particular, such smooth returns on bids highlight an analogy between our setup and other forms of non-price allocations such as random matching, whereby the probability that a given unit trades at a given price is a function of the respective total demand and supply

[5]Under uniform rationing, orders on the large side of the market are fully executed up to a threshold set such that the small side is exhausted.
[6]See end of Section 3.2.
[7]Dubey (1982) and Dubey and Shubik (1980) also assume proportional rationing.

of units at this price. We conjecture that under the same assumption as in Michaillat and Saez (2015)—namely, abstracting from individual randomness—and depending of course on the properties of the matching function, our broad insights would carry over under random matching. Under this simplification of non-randomness, the primary difference with rationing lies in the terminology: Agents trade off degrees of rationing with prices in our setup, whereas under random matching, agents trade off market tightness with prices.

Bankruptcy mechanism and payoffs. Given a policy $\mathcal{P}$ and a strategy profile $\mathcal{S}$, the payoff to agent $i \in \mathcal{I}$ depends on whether she is solvent or not, where we define solvency as follows:

Definition 3. ***(Solvent agent)*** *Agent $i \in \mathcal{I}$ is solvent if and only if*

$$\hat{T}_i \geq T_i, \tag{7}$$

$$\Gamma(D_i, \hat{D}_i, \hat{S}_i) \leq L_i - \hat{T}_i, \tag{8}$$

where Γ is a linear functional described below.

In order to be solvent, agent i must pay her taxes (condition (7)). She must also satisfy a solvency constraint (8) that depends on a linear functional Γ for which we consider three possible values:

1. $\Gamma(D_i, \hat{D}_i, \hat{S}_i) = \int P d\hat{D}_i(P) - \int P d\hat{S}_i(P)$ and the solvency constraint (8) is a standard budget constraint

$$\int P d\hat{D}_i(P) \leq L_i - \hat{T}_i + \int P d\hat{S}_i(P), \tag{9}$$

   stating that agent i must be able to pay for her net *effective* purchases Γ with her (positive or negative) net transfer $L_i - \hat{T}_i$.

2. $\Gamma(D_i, \hat{D}_i, \hat{S}_i) = \int P dD_i(P)$ and the solvency constraint (8) is a cash-in-advance constraint

$$\int P dD_i(P) \leq L_i - \hat{T}_i, \tag{10}$$

stating that the nominal value of i's buy orders Γ cannot exceed her net transfers.[8]

3. $\Gamma(D_i, \hat{D}_i, \hat{S}_i) = \int P dD_i(P) - \int P d\hat{S}_i(P)$ and the solvency constraint (8) is a collateral constraint

$$\int P dD_i(P) \leq L_i - \hat{T}_i + \int P d\hat{S}_i(P), \tag{11}$$

stating that i must cover the nominal value of her buy orders using both her net transfers and her effective sales proceeds as collateral.

One of the goals of our analysis will be to shed light on the respective impacts of these three formulations of the solvency constraint on the equilibria. Notice for now that the cash-in-advance constraint (10) implies the collateral constraint (11). In a way, the cash-in advance constraint can be viewed as an alternative collateral constraint that, unlike constraint (11), does not accept effective sales as valid collateral, only cash. The collateral constraint (11) implies in turn the budget constraint (9). This latter budget constraint imposes only restrictions on effective transactions, not on the posted buy orders.

If agent $i \in \mathcal{I}$ is solvent, then her payoff is given by the respective quantities of goods and money $c_{i,C}$ and $c_{i,M}$ resulting from her transfers and trades[9]:

$$c_{i,C} = e + \int d\hat{D}_i(P) - \int d\hat{S}_i(P), \tag{12}$$

$$c_{i,M} = L_i - \hat{T}_i + \int P d\hat{S}_i(P) - \int P d\hat{D}_i(P). \tag{13}$$

If the agent is insolvent, then the state seizes all the goods and money of that agent ($c_{i,C} = c_{i,M} = 0$) and replaces her in the market. The state creates all the money that is needed to execute this agent's buy orders and/or to make up for $T_i - \hat{T}_i$.

Eliminating default contagion. This simple bankruptcy rule borrows from Dubey (1982). It has the important implication that, for each of the three solvency constraints that we consider, each agent can take other agents' orders as fixed when deciding on her trades because there is no default contagion. An agent does not have to worry that

[8]She thus cannot purchase goods without going bankrupt if she is a fiscal debtor.

[9]Goods consumption (12) is positive for any strategy profile from condition (3). So is money consumption (13) for all the three solvency constraints that we will study, because so is it for the weakest one—the budget constraint (9).

reducing a given buy order may trigger a chain of defaults ultimately affecting her other counterparts.

2.2 Equilibrium concept

We model social interactions as a game between private agents given policy. It is thus natural to adopt Nash equilibrium as our concept of predictable outcome. Formally, given that private agents do not care about their consumption of money, an equilibrium associated with a policy $\mathcal{P}$ is a strategy profile $\mathcal{S}$ such that for every $i \in \mathcal{I}$, strategy $\mathcal{S}_i$ maximizes i's consumption of the good $c_{i,C}$ given other strategies $\mathcal{S}_{-i}$ and policy $\mathcal{P}$. This equilibrium concept yields a natural definition of predictable price levels:

Definition 4. ***(Predictable price levels)*** *A price $P > 0$ is predictable given policy $\mathcal{P}$ if and only if there exists a Nash equilibrium associated with $\mathcal{P}$ with active trading at P. Let $\Pi(\mathcal{P})$ denote the set of predictable price levels associated with a policy $\mathcal{P}$.*

This enables us in turn to characterize whether a public financial policy determines the price level:

Definition 5. ***(Determination of the price level)*** *A policy $\mathcal{P}$ weakly determines the price level if and only if $\Pi(\mathcal{P})$ is a singleton. A policy strongly determines the price level if and only if it weakly determines the price level and every equilibrium features active trade.*

The price level may fail to be determined for three reasons. First, it may be that there exists no equilibrium with active trade. Second, it may be that every equilibrium features active trade at a given equilibrium price, but that this latter price varies across equilibria. Finally, an equilibrium may feature active trades at different prices. We will see that there exist policies leading to each of these three configurations, alongside the ones that actually determine the price level.

2.3 Feasible policies

The consumption of goods and money by the state in the absence of private bankruptcy, $c_{G,C}$ and $c_{G,M}$, are by conservation of quantities:

$$Ic_{G,C} = I(e+\tau) - \sum_{i\in\mathcal{I}} c_{i,C}, \tag{14}$$

$$Ic_{G,M} = IM - \sum_{i\in\mathcal{I}} c_{i,M}. \tag{15}$$

These state consumptions of goods and money are not necessarily positive. The following proposition characterizes policies such that the state consumes positively no matter the private strategy profile. We will deem policies that satisfy these restrictions "feasible":

Definition 6. ***(Feasible policies)*** *A policy is feasible if and only if the state consumes positively goods and money for every private strategy profile.*

We have:

Proposition 1. ***(Characterization of feasible policies)*** *For any of the three solvency constraints considered, a policy is feasible if and only if*

$$\tau + \delta_G \geq 0, \tag{16}$$

$$M \geq N + P^* \min\left\{\delta_G^+, e\right\}. \tag{17}$$

Proof. We prove each inequality in turn.

Positive consumption of goods. The state transfers goods to the private sector only through sales, and not more than $-\delta_G$ per capita, ensuring that condition (16) is sufficient. Suppose that agent $i \in \mathcal{I}$ buys an arbitrarily small quantity at an arbitrarily large price from agent $j \in \mathcal{I}$ whom in turn bids the money, supposed to be larger than $-P^* I\delta_G$, in the official market. Other agents do not trade in the official market. Agent i also sells e at an arbitrarily low price. Then the state must sell $I\delta_G$ units and receives arbitrarily few goods from the possible bankruptcy of i, establishing that (16) is also necessary.

Positive consumption of money. The right-hand side of condition (17) corresponds to the amount of money that the state must transfer to the private sector when the latter sells as many goods as possible and pays its taxes. This is the maximum amount M that the state needs to issue across all private strategy profiles since the state issues additional money when agents are insolvent by assumption. □

Conditions (16) and (17) state that the state has enough real resources $I\tau$ and prints enough money IM to consume positively goods and money given policy $\mathcal{P}$, no matter the strategy profile $\mathcal{S}$ of the private sector. Condition (16) ensures that the state consumes a positive quantity of goods no matter the private strategy profile. Condition (17) ensures that the state consumes a positive quantity of money for all private strategies. It is worthwhile stressing that as soon as the states issues a nominally safe aggregate net promise $N > 0$, then it must stand ready to entirely monetize it—$M \geq N$, as there is no guarantee that (in and out-of-equilibrium) trades with the private sector generate any cash.

In the balance of the paper we will present our results for any policy, whether they are feasible according to the above definition or not. Conditions (16) and (17) make it easy to single out, among the set of policies that we consider, the ones that are feasible.

3 Price-level determination with fiscal creditors

This section first considers the simplest situation in which policy features only fiscal creditors. Section 3.1 studies price-level determination under our baseline fixed policies. We argue in Section 3.2 that the collateral constraint (11) is the most natural solvency constraint. Section 3.3 studies the limiting case of infinitesimal agents. Sections 3.4 and 3.5 study alternative policies in which positive transfers or/and the official price are contingent on the strategy profile of the private sector (outside bankruptcy).

3.1 Financial repression and unofficial markets

Suppose first that the solvency constraint (8) takes the form of the collateral constraint (11). Consider a policy $\mathcal{P} = ((T_i)_{i\in\mathcal{I}}, (L_i)_{i\in\mathcal{I}}, M, P^*, \delta_G)$ in which there are only fiscal creditors—$N_- = 0$ and $N_+ = N > 0$.[10] We have:

Proposition 2. ***(Financial repression and unofficial markets)*** *Under the collateral constraint* (11)*, there are three types of predictable outcomes:*

[10] Proposition 12 tackles the case without transfers $N_+ = N_- = 0$.

1. ***No active trade.*** *If* $\delta_G \geq 0$*, then* $\Pi(\mathcal{P}) = \emptyset$*.*

2. ***Strong price-level determination.*** *If* $N < -P^*\delta_G$*, then* $\Pi(\mathcal{P}) = \{P^*\}$ *and the policy strongly determines the price level. Furthermore, the consumption of the government is given by*

$$c_{G,C} = \tau - \frac{N}{P^*},\ c_{G,M} = M. \tag{18}$$

3. ***Financial repression.*** *If* $N \geq -P^*\delta_G > 0$*, then there is strong determination of the price level if and only if* $N > -P^*\delta_G$ *and* $N_i = N$ *for all* $i \in \mathcal{I}$*. Otherwise, there also exist equilibria with multiple active trading posts, with all unofficial prices strictly above* P^**. Whether the price level is determined or not, it is always the case that:*

$$c_{G,C} = \tau + \delta_G \geq \tau - \frac{N}{P^*}, \tag{19}$$

$$c_{G,M} = M - N - P^*\delta_G \leq M. \tag{20}$$

Proof. We first present useful preliminary results that we will repeatedly apply throughout the paper. We then prove the proposition.

Some preliminary results. The following lemma first shows that one can offset trades by the same agent at a given price in the following sense.

Lemma 3. ***(Netting)*** *Consider a strategy profile such that agent* $i \in \mathcal{I}$ *is a non-bankrupt net buyer at the trading post* P*. If she deviates and sets* $s'_i(P) = 0$*,* $d'_i(P) = d_i(P) - d(P)s_i(P)/s(P)$ *then she does not affect her allocation nor that of other agents. Symmetrically, suppose she is net seller at* P*. If she deviates and sets* $d'_i(P) = 0$*,* $s'_i(P) = s_i(P) - s(P)d_i(P)/d(P)$ *then she does not affect her allocation nor that of other agents.*

Proof. See Appendix A. □

This result is useful because it implies that whenever an agent is net seller or net buyer at one post, we can assume that she nets her trades this way before entering into a profitable deviation so that we do not have to worry about the impact of small deviations from her larger effective order on her potential order on the other side.

We now state two lemmas showing how the collateral constraint (11) imposes limits on arbitrage. To this aim, we first introduce a convenient measure of congestion in rationed markets. For any active trading post P and any $i \in \mathcal{I}$, let

$$\Delta_i(P) \equiv \begin{cases} \frac{s(P)(d(P)-d_i(P))}{d(P)^2} \text{ if } s(P) \leq d(P), \\ 1 \text{ otherwise.} \end{cases} \tag{21}$$

The coefficient $\Delta_i(P)$ measures the marginal return from increasing a buy order in a market in which buyers are (weakly) rationed ($s(P) \leq d(P)$). On one hand a marginal increase ϵ in i's order generates $\epsilon s(P)/d(P)$ additional marginal units. On the other hand it crowds out her own outstanding order $d_i(P)$, thereby costing a marginal reduction $\epsilon(d_i(P)/d(P)) \times (s(P)/d(P))$ in the return on this outstanding order. The following lemma first tackles strategies of selling dear and buying cheap:

Lemma 4. ***(Selling high to buy low)*** *Suppose that in an equilibrium that features (at least) two active trading posts with price levels P and P', a non-bankrupt agent $i \in \mathcal{I}$ is net seller at P' and net buyer at P. Then $P' > P$, and if $s(P) < d(P)$,*

$$P'\Delta_i(P) \geq P. \tag{22}$$

Proof. See Appendix B. □

Intuitively, selling high to buy low is profitable only if the marginal redeployment of the sales proceeds to buy in the cheap trading post does not crowd out the outstanding order at this post. Very much like the collateral constraint may limit how much agents want to sell at a high price, it may also lead some to be willing to buy at such a high price. The following lemma offers a necessary condition for a private agent being willing to buy at the highest of two prices.

Lemma 5. ***(Buying high instead of low)*** *Suppose that an equilibrium features (at least) two active trading posts with price levels P and $P' > P$. If a non-bankrupt agent $i \in \mathcal{I}$ is net buyer at P' then*

$$P'\Delta_i(P) \leq P\Delta_i(P'). \tag{23}$$

Proof. See Appendix C. □

Intuitively, an agent is willing to be net buyer at $P' > P$ if her order at P is sufficiently large that she would crowd herself out by rebalancing some of her expensive order P' towards P. We are now equipped to prove Proposition 2.

Proof of Proposition 2. Notice first that fiscal creditors can always avoid bankruptcy by not trading, and find it strictly preferable to going broke, so any equilibrium is without bankruptcy. The proof takes five steps.

Step 1: $\Pi(\mathcal{P}) = \emptyset$ when $\delta_G \geq 0$. Suppose otherwise that there is an active trading post. There has to be an active private net seller since the state buys. At the lowest price at which there is a private net seller, this net seller does not buy at a higher price from Lemma 4, and cannot by definition buy at a lower price. She would thus be strictly better off reducing her order, a contradiction.

Suppose for the rest of the proof that $\delta_G < 0$. There is no equilibrium with no trade in this case as one agent could deviate and buy goods from the state.

Step 2: All predictable prices are weakly larger than P^*. There exists a "P^*-equilibrium" in which each private agent $i \in \mathcal{I}$ places a buy order for N_i/P^* units at P^*. Suppose there exists an equilibrium with active trading at another price. Let us denote $\underline{P}$ the lowest unofficial price. There has to be an active net seller at this price. She must be net buyer too somewhere else otherwise she would be strictly better off cutting her order. She must buy below $\underline{P}$ from Lemma 4, and by definition cannot do so from a private net seller, so she does so at $P^* < \underline{P}$.

Step 3: The P^*-equilibrium is unique when $N + P^*\delta_G < 0$. In this case buyers at P^* cannot be rationed since the private sector as a whole cannot bid more than N at P^* in an equilibrium without bankruptcy. Condition (23) then implies that there cannot be a private net buyer at $\underline{P} > P^*$ defined above.

Step 4: Equilibrium with unofficial trade when $N + P^*\delta_G = 0$ or $N + P^*\delta_G > 0$ and $N_i \neq N$ for some $i \in \mathcal{I}$. Without loss of generality, we suppose that $(N_i)_{i\in\mathcal{I}}$ is (weakly) increasing. Let us construct an equilibrium in which there is active trade at two prices, P^* and $P > P^*$.

Case 1: Suppose $\{i \in \mathcal{I} \mid N_i < N_I\} \neq \emptyset$. Let $k \geq 1$ such that $I - k$ is the supremum of this set. We construct the equilibrium as follows. Each agent $i > I - k$ submits a buy order with nominal value $B > 0$ at $P > P^*$ and a buy order with nominal value $N_I - B$

at P^*, where B and P are defined below. Each agent $i \leq I-k$ posts a sell order e at P and submits a buy order at P^* with nominal value equal to the effective proceeds from this P-order plus N_i. We select B sufficiently small that

$$\frac{kB}{I-k} < P^* e, \tag{24}$$

$$\frac{kB}{I-k} < N_I - N_{I-k} - B, \tag{25}$$

and for such a B set

$$P \equiv \frac{N}{(-\delta_G)} \times \frac{I}{I - \frac{N_I - B}{N}}. \tag{26}$$

We have $P\Delta_I(P^*) = P^* = P^*\Delta_I(P)$. The first inequality stems from the definition of P given by (26). The second one holds because (24) and $P^* < P$ imply that unofficial sellers at P are rationed. Applying Lemma 5 thus yields that agents $i > I-k$ find their position optimal. The reason is that their utility is concave in their orders, and so these agents invest optimally when condition (23) holds with an equality. Condition (25) implies that $\Delta_i(P^*) > \Delta_I(P^*)$ for $i \leq I-k$, and so applying Lemma 4 yields that agent $i \leq I-k$ finds her position optimal. The reason is that (22) holds strictly and they exhaust their short-sales constraint on goods.

Case 2: $\{i \in \mathcal{I} \mid N_i < N_I\} = \emptyset$: Net transfers are identical across agents, and so $N + P^*\delta_G = 0$. For any $B \in (0, \min\{N, (I-1)P^*e\})$, let

$$P \equiv P^* \frac{I}{I - 1 + \frac{B}{N}} > P^*. \tag{27}$$

Suppose that one agent $j \in \mathcal{I}$ places buy orders with nominal value B at P and $N - B$ at P^*. Agent $i \in \mathcal{I}\backslash\{j\}$ sells e at P and invests the proceeds plus N_i at P^*. This strategy is optimal for j from Lemma 5 and $P\Delta_j(P^*) = P^*$. So is it for the other agents from Lemma 4 applied to the case in which the low-price post clears because $s(P^*) = d(P^*)$.

Step 5: The P^*-equilibrium is unique when $N_i = N$ for all $i \in \mathcal{I}$ and $N + P^*\delta_G > 0$. Suppose by contradiction that there is unofficial active trade, and let $\bar{P}$ and (again) $\underline{P} > P^*$ respectively denote the largest and smallest unofficial active prices. Suppose $i \in \mathcal{I}$ is net seller at $\underline{P}$ and $j \in \mathcal{I}$ is net buyer at $\bar{P}$. Suppose the official market is rationed on the buy side ($d(P^*) > s(P^*)$), so that condition (22) applies to i. Conditions (22) and

(23) together imply $\Delta_i(P^*) \geq P^*/\underline{P} \geq P^*/\bar{P} \geq \Delta_j(P^*)$, requiring that i is unofficial net buyer above $\underline{P}$ or/and j is unofficial net seller below $\bar{P}$, either way a contradiction given Lemma 4.

It remains to show that the official buyers are indeed rationed in an equilibrium with unofficial trading. For future use, we actually show that this always holds under financial repression, even when transfers are heterogeneous. Suppose otherwise that $s(P^*) \geq d(P^*)$. It cannot be that the official sell side is rationed otherwise a net unofficial buyer could strictly benefit from shifting part of an unofficial buy order to the official market from (23), and so $s(P^*) = d(P^*)$. If no buyer is rationed, $D_i(P) = \hat{D}_i(P)$ for all i, P, and summing up collateral constraints and netting buy orders from resources yields an aggregate slack $N + P^*\delta_G$. That $N + P^*\delta_G > 0$ thus implies that at least one collateral constraint is slack, which is suboptimal, a contradiction. Thus at least one unofficial market has rationed buyers. Condition (23) and $s(P^*) = d(P^*)$ require that these rationed buyers all have a strictly smaller share in the bids than they do in the official market, which contradicts that these shares must add up to 1. □

Proposition 2 first states that there is no active trade if the state does not sell goods. Essentially, no private agent is interested in being net seller in this case, so nor can there be any net private buyer. Notice that we ruled out the posting of infinite prices when describing the action space of private agents. Yet they could arise as a variant of a no-trade equilibrium in which some agents transfer money without counterparts to others. By contrast, such "infinite price levels" are out of range when all equilibria feature active trade, as is the case when $\delta_G < 0$, because money has strictly positive value in this case.

Proposition 2 then states that in the presence of heterogeneous net transfers $(N_i)_{i\in\mathcal{I}}$, the state must supply strictly more goods $-P^*\delta_G$ than the equilibrium average net demand N ($N < -P^*\delta_G$) in order to determine the price level.

In the case of insufficient backing $N \geq -P^*\delta_G$ that we deem "financial repression", heterogeneity among net transfers $(N_i)_{i\in\mathcal{I}}$ creates room for unofficial trading among creditors.[11] Whereas there still exists an equilibrium without unofficial trades, there also exist equilibria with unofficial prices strictly above P^*. In these equilibria, small creditors are willing to acquire money at a low cost (at a high price level) in order to be able to scale

[11]As stated in the proposition, heterogeneity is not required in the particular case in which $N+P^*\delta_G = 0$.

up their trades in the official market. Conversely, agents with large claims on the state accept to sell some money to them at such high unofficial prices rather than buying goods in the yet cheaper official market.

Thus, agents with low cash holdings arise as endogenous intermediaries between cash-rich agents and the state, selling goods dear and buying cheap. In the absence of unofficial trading, these small fiscal creditors earn a higher marginal return on money in the official post than the large creditors. By using borrowed money in the unofficial market to crowd large creditors out, small creditors coordinate on extracting rents from them: Unofficial markets reduce consumption inequality. We will see that the fact that unofficial markets benefit agents with high returns in the official one this way is a pervasive feature across the very diverse empirical applications that we discuss in Section 8. The following numerical example simply illustrates these insights.

Numerical example. Suppose $I = 2$, $\delta_G = -0.5$, $P^* = 1$, $N_1 = 0$, $N_2 = 2$, and $e > 0.25$. There are equilibria in which only agent 2 trades in the official market and gets the consumption unit sold by the state.[12] For $B \in (0, 1)$, there also exists an equilibrium with an unofficial market with price $P = 4/B > 4$. Agent 2 bids B units of cash in the unofficial market and $2 - B$ in the official one. Agent 1 fulfills the unofficial buy order and uses the collected B to bid in the official market. To see that it is an equilibrium, notice that if 2 shifts money $P\epsilon$ to the official from the unofficial market, she remains solvent and her net gain is

$$-\epsilon + \frac{2 - B + \frac{4\epsilon}{B}}{2 + \frac{4\epsilon}{B}} - \frac{2 - B}{2} = o(\epsilon), \tag{28}$$

whereas if 1 reduces her effective unofficial sale by $\epsilon > 0$ and her official bid by $P\epsilon$, she remains solvent and her net consumption gain is

$$\epsilon + \frac{B - \frac{4\epsilon}{B}}{2 - \frac{4\epsilon}{B}} - \frac{B}{2} = -2\left(\frac{1}{B} - 1\right)\epsilon + o(\epsilon). \tag{29}$$

In this equilibrium, agent 2 strictly benefits from selling dear and buying cheap, whereas agent 1 is indifferent between buying cheap or dear because the former earns a higher marginal return on her incremental official bid B than the latter.

[12] Given that 2 is alone in the market, she is indifferent over all bids within $[1, 2]$.

Homogeneous agents. Proposition 2 also shows that in the case of strict insufficient backing $N > -P^*\delta_G$, there is strong price-level determination if all the transfers N_i are identical because there is no room for unofficial trading. An inspection of the proof shows that this result is actually continuous in essence, in the sense that unofficial trading volume shrinks as private cash holdings become more homogeneous.[13] This is a direct consequence from the fact that unofficial trade is a form of rent extraction by small creditors enjoying a relatively high marginal return on the official market. The volume of money that these cash-poor agents collect in the unofficial market from the cash-rich ones with low marginal official returns cannot be so large that the former agents end up with lower official marginal returns than the latter. Thus, the more similar the initial holdings across agents, the less room there is for unofficial transactions until official marginal returns converge. In other words, if all creditors earn similar marginal returns on money in the official market, there is only limited room for rent extraction via unofficial markets across them. In this case there is limited deviation from the official price despite insufficient backing.

Other implications. Two additional implications of Proposition 2 are worth mentioning.

Velocity. Our setup offers a direct observation of velocity defined as the average number of times a monetary unit is traded for goods. In the equilibria that we construct with a single unofficial market, the larger the unofficial trading volume, the larger the velocity as this means that more cash gets traded twice, both unofficially and officially. Thus we predict a positive comovement between the rise of unofficial markets and velocity.

Money neutrality. A change in policy whereby the state expands the money in circulation but does not adjust the price-level target P^* in due proportion has potential real effects via the redistributive role of unofficial markets. To see this, suppose that all transfers of money are multiplied by λ such that $N < -P^*\delta_G$ and $\lambda N \geq -P^*\delta_G$. In this case, the expansion by λ may imply the rise of unofficial markets with distributional implications at the expense of the agents with the largest cash holdings.

[13]In the above elementary example, unofficial trade B would have to tend to 0 as $N_1 \uparrow N_2$.

3.2 The role of the collateral constraint

It is interesting to compare the outcomes in Proposition 2 to the ones resulting from the alternative formulations of the solvency constraint (8) either as a budget constraint (9) or as a cash-in advance constraint (10). We focus for brevity on the situations that yield multiple predictable prices in the case of constraint (11) studied in Proposition 2.

Corollary 6. ***(Alternative solvency constraints)*** *Suppose that $N > -P^*\delta_G > 0$ and that the transfers N_i are heterogeneous. If agents are subject to the cash-in-advance constraint* (10)*, there is strong determination of the price level at P^*. If agents are subject to the standard budget constraint* (9)*, there is no equilibrium.*

Proof. No matter the nature of the solvency constraint, all agents can and do avoid bankruptcy and there is trade in equilibrium. In the presence of cash-in-advance constraints, any unofficial net seller is strictly better off cancelling her sale as she remains solvent and consumes more, thus there cannot be unofficial trading. In the presence of a budget constraint, there can be only one active market. Suppose otherwise that there are active buyers at two prices. Unless she is the only low-price buyer, a high-price buyer can always get more bang for the buck relocating part of her high-price order to the low-price market diluting the other low-price buyers as needed. If she is the only low-price buyer, then another high-price buyer can do just that. In a single active market, given an equilibrium, every agent is strictly better off increasing her buy orders arbitrarily to dilute the others, a contradiction. □

The formulations of the solvency constraint other than the collateral constraint (11) make the outcome trivial and unreasonable in opposite directions. The cash-in-advance constraint (10) eliminates any possibility of arbitrage against officially overvalued money, and so a state willing and able to enforce it can impose any price level with arbitrarily little backing.[14] There is by contrast no equilibrium at all under the assumption of a mere budget constraint. This is so because the action space is unrealistically unbounded and each agent wants to bid more than the others.

Away from these extremes, we argue that the collateral constraint (11) is a natural way of introducing plausible limits to arbitrage in the form of bounds on buy orders

[14] As detailed in Section 8, Sargent and Velde (1995) describe a regime that has this flavor during the French Terror.

in our model. In fact, Section 7 offers a simple version of this economy with uncertain execution of trades in which the equilibria that we obtain here under perfect foresight and a collateral constraint (11) can be sustained with a mere budget constraint (9). In this economy, ambiguity-averse agents seek to ensure that they can avoid bankruptcy and thus satisfy their budget constraint under all contingencies. Since the state may abandon the peg P^* and offer a market-clearing price,[15] agents endogenously fully back their official buy orders with cash and effective sales: Constraints (11) are self-inflicted. In other words, collateral constraints arise endogenously when agents subject to standard budget constraints face execution uncertainty in a richer model. The perfect-foresight nature of our model implies that we must assume them instead. This foundation for the collateral constraints (11), together with the fact that they generate equilibria with empirically plausible features[16], leads us to adopt them as solvency constraints for the rest of the paper.

Assumption 1. *In the remainder of the paper, the solvency constraint is the collateral constraint* (11)*.*

Another pass on uniform rationing. The extreme 0-1 marginal return on bids induced by uniform rationing has the knife-edge implication that a budget constraint (9) and a collateral constraint (11) become equivalent in this case. The reason is that a marginal increase in an order is either fully executed, in which case it must be fully backed by cash, or irrelevant, in which case it is weakly dominated. Thus, had we assumed uniform rationing, we would have bypassed this analysis of the role of trading limits and merely imposed a budget constraint, albeit for spurious reasons. Trading limits matter as soon as marginal returns on bids are within $(0, 1)$.

3.3 Asymptotically atomistic economies

It is interesting to separate out, among the results in Proposition 2, the ones that survive in the limit in which each private agent becomes negligible. To be sure, the results that hinge on private agents' absolute price impact are interesting in their own right, as there is ample evidence that the large institutions that participate in the primary

[15] Section 3.5 introduces such a market-clearing official price.

[16] Section 8 details the empirical applications of our setup.

markets for public liabilities have some price impact in practice. Yet assessing our results in the limiting case of negligible agents is also instructive. Here we show that when the economy converges to one in which each agent becomes negligible, the unofficial prices that may arise in the presence of financial repression all tend to $N/(-\delta_G) \geq P^*$.

Corollary 7. ***(Negligible agents)*** *Consider a sequence $(\mathcal{P}^I)_{I\in\mathbb{N}}$ of policies with financial repression each associated with an economy of size I, and each such that the net transfers are not all identical. Suppose that $(P^{*I}, \delta_G^I, N^I) \underset{I\to+\infty}{\to} (P^*, \delta_G, N)$ such that $-P^*\delta_G > 0$, and that*

$$\max_{i\in\mathcal{I}} \left\{ \frac{N_i^I}{IN^I} \right\} \underset{I\to+\infty}{\to} 0. \tag{30}$$

For every $\epsilon > 0$, there exists $n \in \mathbb{N}$ such that for all $I \geq n$, the unofficial prices that are predictable given $\mathcal{P}^I$ belong to $(N/(-\delta_G) - \epsilon, N/(-\delta_G) + \epsilon)$.

Proof. The proof is in three steps.

Step 1: All official buy orders become negligible in the negligible limit. Formally, denoting $d^I(P^{*I}) = \sum_{i\in\mathcal{I}} d_i^I(P^{*I})$ the official buy orders of the size-I economy in a generic equilibrium, we have

$$\max_{i\in\mathcal{I}} \left\{ \frac{d_i^I(P^{*I})}{d^I(P^{*I})} \right\} \underset{I\to+\infty}{\to} 0.$$

To see why, notice first that any net buyer $i \in \mathcal{I}$ in unofficial markets invests less than N_i^I at P^*. Second, we have shown in the proof of Proposition 2 that the official post is always rationed in equilibrium—$d^I(P^{*I}) > I(-\delta_G^I)$. This and condition (30) imply that net buyers in unofficial markets place negligible official buy orders in the negligible limit. Comparing conditions (22) and (23) at a given unofficial price P shows that any buyer in an unofficial post has a larger official position than any seller in an unofficial post. Thus all agents must have negligible buy orders in the negligible limit.

Step 2: The rationing of buyers in unofficial posts becomes negligible in the negligible limit. We write

$$P^I \Delta_s(P^{*I}) \geq P^{*I}, \tag{31}$$

$$P^I \Delta_b(P^{*I}) \leq P^{*I} \Delta_b(P^I). \tag{32}$$

the generic conditions (22) and (23), where s and b respectively stand for seller and buyer at P^I. From Step 1, $\Delta_s(P^{*I})/\Delta_b(P^{*I})$ tends to 1 in the negligible limit and thus so must $\Delta_b(P^I)$.

Step 3: The lower and upper bound of unofficial prices converges to $N/(-\delta_G)$ in the negligible limit. Steps 1 and 2 imply that

$$\Delta_s(P^{*I}),\ \Delta_b(P^{*I}) \underset{I\to+\infty}{\to} \frac{N}{P^*(-\delta_G)}, \tag{33}$$

which applied to the above conditions (22) and (23) yields the result. □

Condition (30) encodes that agents become negligible in the limit. Notice that this proposition implies in particular than when the state backs money with the exact real amount given P^*—$N + P^*\delta_G = 0$—then all equilibrium prices converge to the official target P^* in the negligible limit. If $N + P^*\delta_G > 0$, there still are multiple equilibria with varying trading volume at unofficial prices, including possibly no unofficial trade. Yet all unofficial prices become arbitrarily close to $N/(-\delta_G)$ in the negligible limit.

The rest of this section studies policies such that transfers or/and official price-setting are contingent on the actions of the private sector in empirically relevant ways. We first introduce the possibility that the positive transfers of the state are defaultable. In this case, the state uses only the proceeds from its sales of goods to fund net transfers. It does not use money creation to make them (nominally) safe as in the above fixed policies. Thus transfers resemble sovereign debt that cannot be repaid by money printing. We then also open up the possibility that the state commits to quote an official price that clears the official market in-and-out of equilibrium. This will help compare our economy to Walrasian environments in which price setting is delegated to an unmodelled auctioneer.

3.4 Contingent policy: Defaultable security

The policy assumed thus far comprises a fixed nominal payment L_i from the state to private agent $i \in \mathcal{I}$. As already mentioned, feasibility requires in this case that the state uses sufficient money creation M to fund transfers in all contingencies. This section considers policies such that, by contrast, the state does not use money creation M to make good on its net transfers. In the case $\delta_G < 0$ that we will consider, this implies that $c_{G,M} \geq M$ must hold no matter the private strategy profile because M is neither used to fund transfers nor for good purchases. Combining equations (15) and (13) then yields that the aggregate net transfer to the private sector cannot exceed the amount of money that the government purchases, neither in nor out of equilibrium. In other words, policies such that transfers are never monetized must be such that transfers are contingent on the amount of money that the state collects through its transactions. This admits a natural interpretation as the state issuing defaultable securities.

Formally, we modify the baseline policy with only fiscal creditors studied in Proposition 2 as follows. For simplicity, we assume away taxes: $T_i = 0$ for all $i \in \mathcal{I}$. More important, we posit that the positive cash transfer from the state to agent $i \in \mathcal{I}$ depends on the amount of money that the state collects in the market. In game-theoretic language, this transfer is no longer an action of the state but rather a payoff vector $(L_i(\mathcal{S}))_{i\in\mathcal{I}}$ that depends on the private strategy profile $\mathcal{S}$. Denoting $(B_i)_{i\in\mathcal{I}}$ a vector with strictly positive components, we suppose that that the transfer to agent $i \in \mathcal{I}$ is

$$L_i(\mathcal{S}) = B_i \min\left\{1, \frac{\left(\sum_{i\in\mathcal{I}} P^* \hat{d}_i(P^*) - P^* \hat{s}_i(P^*)\right)^+}{IB}\right\}, \tag{34}$$

where $B = \sum_{i\in\mathcal{I}} B_i/I$. Thus, B_i admits a natural interpretation as the nominal promise to agent i. In case of default, creditors are treated pari passu—They receive a share in the available cash proportional to the size of their claim B_i. This is to fix ideas, all our insights carry over with alternative seniority rules.

The rest of the policy is unchanged and so are the trading and bankruptcy mechanisms. For brevity we consider only the case $\delta_G < 0$. We have:

Proposition 8. ***(Defaultable security and price-level determination)*** *The policy is feasible if and only if condition* (16) *holds. For all* $D \in [(B + P^*\delta_G)^+, B]$*, there exist equilibria in which the state pays* $B - D$ *per capita. Across such equilibria,*

$$c_{G,C} = \tau - \frac{B - D}{P^*}, \; c_{G,M} = M. \tag{35}$$

There is no price-level determination. However, all active price levels converge to P^* *as agents become negligible in the sense of Proposition 7, where we replace condition* (30) *with* $\max_{i\in\mathcal{I}} \left\{B_i^I/IB^I\right\} \underset{I\to+\infty}{\to} 0$.

Proof. Notice first that agents can always avoid bankruptcy by not trading and are strictly better off this way, so that any equilibrium must be without bankruptcy. Notice also that there exists a no-trade equilibrium, so that determination of the price level is at best weak. We leave it to the reader to check that for every $D \in [(B + P^*\delta_G)^+, B)$, there exists a P^*-equilibrium in which only the official trading post is active. Agent $i \in \mathcal{I}$ bids $B_i(1 - D/B)$ at P^* and collects the same transfer from the state. The rest of the proof is in three steps.

Step 1: Feasibility. By construction the state transfers only what it receives, and so it does not need to create M to consume money positively (but of course can always do so), that is, $c_{G,M} = M$ in and out of equilibrium. The reasoning leading to condition (16) is identical to that in Proposition 1.

Step 2: Unofficial prices tend to P^* as agents become negligible. Suppose that an equilibrium features active unofficial trading. Suppose that there is a net buyer at $P^b > P^*$. Lemma 5 applies with $P' = P^b$ and $P = P^*$. To see this, notice that the deviation used to prove the lemma—shifting part of the P'-bid towards P—strictly improves solvency as it increases the transfer received by the deviating agent. Thus condition (23) is still necessary for equilibrium. Official orders become negligible in the negligible limit for the same reasons as that outlined in the proof of Corollary 7. Furthermore, the nominal value of aggregate transfers is by construction equal to the effective sales of the government, so that the official market clears and buyers are not rationed in unofficial markets. Condition (23) thus implies that P^b becomes arbitrarily close to P^* as agents become negligible. Suppose then that there is a net buyer below P^*, and let P^b denote the smallest price at which there is one. There must be net sellers at this price. Lemma 4 applies between P^b and any other unofficial price $P \neq P^*$, implying that these net sellers, who must be buying somewhere, buy at P^*. Suppose that such a net seller $i \in \mathcal{I}$ reduces her effective sales at P^b by $\epsilon > 0$. Her order at P^* must then shrink by x such that

$$P^*x = \epsilon P^b + \frac{B_i P^* x}{IB}, \tag{36}$$

where the second term on the right-hand side reflects that her smaller bid reduces her net transfer from the state, and thus tightens her solvency constraint. Equilibrium requires that $x \geq \epsilon$, or $P^b \geq P^*[1 - B_i/(IB)]$, and so, as $P^b < P^*$, P^b must become arbitrarily close to P^* as agents become negligible.

Step 3: There exist equilibria with unofficial trade. For D such that $B - D +$

$P^*\delta_G < 0$, let $i \in \mathcal{I}$ an agent with a minimum value of B_i— i need not be unique. Let this agent buy a sufficiently small quantity at $P = P^*[1 - B_i/(IB)]$ and all the others sell their e at this post and invest the proceeds at P^*. It is an equilibrium as the unofficial buyer is indifferent between buying at P and P^* from the same reasoning as that used in step 2 and sellers may strictly prefer to sell more (unless their claim is equal to B_i in which case they are indifferent) but cannot. □

Within the limits of a static model, the equilibria are reminiscent of self-fulfilling debt crises. Equilibria with default here are gridlocks whereby the private sector does not bid much cash for goods because it expects sovereign default in the form of a small transfer, and these small bids in turn vindicate the small transfer. A key difference with the case of a nominally safe security is that there is no longer room for strict financial repression since the aggregate transfer of the state by construction never exceeds the amount of money that it is willing to purchase at P^*. As a result, the situation bears similarities with that in which $N + P^*\delta_G = 0$ with safe securities: Unofficial trades are made possible only because of absolute price impacts. Thus unofficial prices all become arbitrarily close to the official one when agents become negligible. Yet there are multiple equilibria with varying default severity even in this negligible limit, but only the goods consumption of the state varies across them ($c_{G,C}$ depends on D in (35)). The price level does not. A higher loss given default creates additional real resources for the state.

Interpretation. A natural interpretation of this setting is the one of a dollarized economy in which the state has to repay its debt in dollars that it has to purchase from private agents against local currency.[17] By imposing a peg on the dollar/local currency exchange rate, the state is at risk of falling short of the amount of dollars it needs to honor its liabilities—a form of liquidity crisis. The inability to print money to repay nominal debt and a peg are also features of monetary unions, and there as well the connection was made with default risk.

More on out-of-equilibrium monetization. We find this result that the absence of any monetization of the transfer creates self-justified default to be interesting. Still, it is

[17]Under this interpretation, the "good" is the local currency that agents intrisically value for some unmodelled reason such as liquidity services, and "money" is dollar.

a knife-edge one, to the extent that the state has the possibility to create some money—unlike in the interpretation above. By committing to arbitrarily small out-of-equilibrium financial repression, the state can eliminate all the equilibria but the one without default. To see this, suppose that for $\epsilon > 0$ arbitrarily small, one replaces (34) with

$$L_i(\mathcal{S}) = B_i \min \left\{ 1, \frac{\left(\sum_{i \in \mathcal{I}} P^* \hat{d}_i(P^*) - P^* \hat{s}_i(P^*) \right)^+}{IB} + \epsilon \right\}, \tag{37}$$

which requires a money creation of ϵ to be feasible in the sense of Proposition 1. Then it cannot be that an agent receives less than B_i in equilibrium as this would imply that the private sector does not bid its aggregate cash holdings in the official market, which is inconsistent with the rationality of at least one private agent.

3.5 Contingent policy: Market-clearing policy

The goal of this section is to compare the outcomes when the state's trading strategy consists in posting a fixed price as above with those when the state acts as an auctioneer à la Shapley and Shubik (1977), setting a price that absorbs all the private demand for goods. This will highlight the crucial role of the official trading protocol on the set of predictable price levels.

We modify again the baseline policy with fiscal creditors only as follows. First, for brevity, we restrict again the analysis to policies such that $T_i = 0$ for all $i \in \mathcal{I}$ and $\delta_G < 0$.

Second, the official trading post no longer operates as the unofficial ones, but rather as a "sell-all" market à la Shapley and Shubik (1977). Private agent $i \in \mathcal{I}$ bids a positive quantity of money $C_i \geq 0$. The official price is then a function of the private strategy profile $\mathcal{S}$ defined as

$$P(\mathcal{S}) \equiv \frac{\sum_{i \in \mathcal{I}} C_i}{-I\delta_G}. \tag{38}$$

Finally, since our main goal is to highlight how defaultability and the trading protocol jointly determine the price level or fail to do so, we posit that the transfers to the private sector may feature both a safe and a defaultable component similar to that introduced in the above Section 3.4. There exists a positive sequence $(l_i, B_i)_{i \in \mathcal{I}}$ such that

$l = \sum_{i\in\mathcal{I}} l_i/I > 0$ and such that the net transfer to agent $i \in \mathcal{I}$ is

$$L_i(\mathcal{S}) = l_i + \frac{B_i}{B} \min\left\{B, \left(\frac{1}{I}\sum_{i\in\mathcal{I}} C_i - l\right)^+\right\}, \tag{39}$$

where $B = \sum_{i\in\mathcal{I}} B_i/I$. By convention, $B_i/B = 0$ for all $i \in \mathcal{I}$ when $B = 0$. For brevity, we restrict the analysis to the cases in which $l_i \neq l$ for some $i \in \mathcal{I}$, and in which at least two B_i are strictly positive if $B \neq 0$.

Proposition 9. ***(Market-clearing policy and price-level determination)*** *The policy is feasible if and only if condition* (16) *holds and $M \geq l$. Furthermore,*

- *The price level is not determined.*
- *When $B = 0$, there exists a unique equilibrium without unofficial trade. The official price is $-l/\delta_G$.*
- *When $B > 0$, there exists a continuum of equilibria without unofficial trade. These equilibria are indexed by their official prices $-(l + B - D)/\delta_G$ for $D \in [0, B]$.*
- *All unofficial prices uniformly converge towards official prices across all equilibria as agents become negligible in the sense of Proposition 7. Thus there is determination of the price level in the negligible limit if and only if $B = 0$.*

Proof. By construction the state transfers only what it receives beyond Il and positive consumption of money thus only requires $M \geq l$. The reasoning leading to condition (16) is identical to that in Proposition 1. Notice that there is always trade in equilibrium as $l > 0$ and $\delta_G < 0$. We leave it to the reader to check that the equilibria with only official trading are such that each agent $i \in \mathcal{I}$ bids $C_i = l_i + B_i(1 - D/B)$ at the official post and the price is $-(l + B - D)/\delta_G$ for every $D \in [0, B]$.

Price-level indetermination. Consider first the case $B = 0$. A key remark is that holding other official nominal bids fixed, if an agent changes her official order by a nominal amount c then it affects her official allocation in the same way as if she was modifying her order by a quantity c/P^* in a rationed official market with fixed price and the same $-\delta_G$. This is because proportional rationing is equivalent to the Shapley-Shubik market clearing rule. Thus, one can construct the very same equilibrium with unofficial price as in Proposition 2 with $N + P^*\delta_G > 0$ and heterogeneous transfers. This establishes indetermination when $B = 0$.

Suppose now $B > 0$. Indetermination is warranted by the multiplicity of official prices with varying default $D \in [0, B]$. The reader may still wonder if equilibria with unofficial trading exist in this case as well. Whereas it is relatively easy to construct such equilibria in some cases,[18] a generic construction is more complex and beyond the scope of this paper. The important result however is the following one that unofficial prices all converge to the official ones in the negligible limit.

Negligible agents. Suppose that there is unofficial trade at a price $P > 0$ and that agent i is buyer. Then it must be that her official bid C_i maximizes

$$\frac{l_i + \frac{B_i}{BI}\min\{C_i + C_{-i} - lI, BI\} - C_i}{P} - \frac{\delta_G I C_i}{C_i + C_{-i}}. \tag{40}$$

An interior solution satisfies

$$P = \frac{(C_i + C_{-i})^2}{-\delta_G I C_{-i}}\left(1 - \mathbb{1}_{\{C_i + C_{-i} - lI < BI\}}\frac{B_i}{BI}\right). \tag{41}$$

As C_i/C_{-i} and B_i/BI tend to zero, which has to be the case in the negligible limit for the same reasons as in Corollar 7, P must therefore tend to the official price $C/(-\delta_G I)$. In the absence of an interior solution, the equality becomes a "<" and $C_i = 0$. P is smaller than the official price. In this case applying the same reasoning to an unofficial seller yields that P must become close to the official price as well in the negligible limit. □

Overall, a market-clearing price does not rule out unofficial markets, except in the negligible limit. In addition, a market-clearing price policy still allows for self-fulfilling debt crises, with larger haircuts D associated with lower price levels. This is consistent with debt deflation on the side of the public sector.[19] Taken together, self-fulfilling debt crises arise notwithstanding the trading mechanism selected by the state—fixed or market-clearing price: In both cases, the state may end up being short in money to honor its liabilities, either because it is rationed or because the real value of its debt has increased.

[18] We hold examples available upon request. One case in point is one value of B_i being small relative to the others.

[19] The evidence are much more in favor of sovereign defaults associated with inflation, but we find interesting that episodes of deflation are also associated with pressures on public finances. See the cross-country evidence on the evolution of debt-to-GDP ratios during deflation episodes by End et al. (2015).

The change in trading protocol flips the relationship between default and price-level determination. The salient implication of Proposition 9 is that shifting the official trading protocol from fixed price to fixed quantity completely flips the relationship between the defaultable nature of public liabilities and price-level determination in the negligible limit. Proposition 7 shows that the fixed-price trading protocol may fail to determine the price level in the presence of a non-defaultable security because financial repression opens up the possibility of trade at multiple prices in equilibrium. Proposition 8 shows that by contrast, this fixed-price protocol determines the price level in the presence of a defaultable security in the negligible limit because only state consumption varies across equilibria with varying haircuts on public debt. The market-clearing trading protocol generates the exact opposite prediction in the negligible limit. The price level is determined in the negligible limit if and only if the security is non-defaultable ($B = 0$). In the presence of defaultable securities, it is the price level that absorbs all the fluctuations in the haircut on public debt across equilibria with varying default severity, whereas state consumption remains unaffected.

In sum, in the negligible limit, safe securities warrant price-level determination in the presence of a market-clearing official price whereas defaultable ones do so when the official price is fixed. This study of how the interplay between transfers and official trades shapes predictable price levels is novel to our knowledge. The following section shows how it can to some extent be captured in Walrasian models.

4 Getting the Walrasian approach right

The above analysis showcases the reasons the use of the Walrasian equilibrium concept is ill-suited to study whether the legal-tender and official medium of exchange functions of money determine the price level. First, out-of-equilibrium actions are crucial to pin down the equilibrium, and it is not clear what "out-of-equilibrium" even means in the Walrasian framework. Second, the state's trading protocol crucially matters and interacts with the nature of sovereign liabilities to determine (or not) the price level, and so forcing delegation of price-setting to a Walrasian auctioneer is an important (and counterfactual) restriction.

This section applies the insights from our strategic model to analyze price determination in a Walrasian version of it. We perform a translation exercise: For each of the main policies studied in our strategic setup, we design a Walrasian counterpart that delivers outcomes that are similar to the ones we obtain in the limit of strategic but negligible agents. This exercise highlights the dimensions of price-level determination that are neglected under the Walrasian approach relative to the strategic approach in which the state may default, and markets do not necessarily clear. More specifically, we deliver the four following insights.

Insight #1: The fiscal theory of the price level is an extreme form of fiscal dominance. For notational simplicity, we study a version of our economy populated by $I = 1$ private agent—one needs only one representative agent in a Walrasian environment. Consider a policy comprised of a nominal transfer to that agent L, an in-kind tax τ levied on her, and government consumption of the good $\tau - \sigma$, all strictly positive real numbers. σ is then the surplus run by the government. A Walrasian equilibrium associated with this policy (L, τ, σ) is comprised of consumption of money C_M and goods C_C by the private agent and of a price level $P > 0$ such that the price-taking private agent optimally consumes:

$$(C_M, C_C) = \arg\max C_C \tag{42}$$

$$s.t.\ C_M + PC_C + P\tau \leq Pe + L, \tag{43}$$

$$C_M,\ C_C \geq 0, \tag{44}$$

and, from Walras' law, such that the market for money clears:

$$L - C_M = P\sigma. \tag{45}$$

Condition (45) states that the private supply of money (endowment minus consumption of money) equals the state's demand of money (the nominal value of the goods it trades for money). Individual rationality requires that (43) binds and $C_M = 0$. Injecting this in the market-clearing condition (45) yields a unique equilibrium price level P associated with (L, τ, σ) that solves $L = P\sigma$: This is the so-called fiscal theory of the price level. In our setup, the case $B = 0$ in Proposition 9 corresponds (in the negligible limit) to such a policy with a fixed nominal liability and a fixed real quantity sold by the state for money no matter the actions of the private sector. This policy with fixed L and σ is associated with two important assumptions. First, the state creates money and stands ready to use it to make good on its liability if it does not collect enough money in the market. We highlight that the state must stand ready to fully monetize the entire value of its liability this way in the (out-of-equilibrium) event that it does not collect any money in the official market. Second, the state adjusts the official price level in response to private demand so that its real surplus and in turn its consumption remain constant. In other words, our full-fledged model shows that an extreme form of fiscal dominance underlies the fiscal theory of the price level. The state prints money as needed to make sure that its liabilities are all perfect substitutes with money, standing ready to monetize all of it, and manipulates the price level so as to insure the government's consumption from the fluctuations of private demand. We now relax these two assumptions in turn and describe the Walrasian policy that generates the same outcomes as the strategic one in the absence of these assumptions.

Insight #2: Defaultable nominal debt is akin to real debt for price-level determination. We first relax the assumption that the transfer is nominally safe, while sticking to the one that the state trades at a market-clearing price. This corresponds to the other polar case in (the negligible limit in) Proposition 9 in which $l = 0$ and $B > 0$.[20] In this case, our setup predicts that there is price-level indetermination. All price levels within $[0, -B/\delta_G]$ can be sustained as equilibria with increasing levels of default and deflation. In the Walrasian setup, it is straightforward to see that these outcomes obtain when the policy (L, τ, σ) becomes $(\min\{L, P\sigma\}, \tau, \sigma)$. In words, the transfer is contingent on the equilibrium outcome as it depends on P. As in the strategic case, all price levels within $[0, L/\sigma]$ are then equilibrium outcomes. This basically shows that when debt is defaultable in our setup in the sense that it is not monetized, it is as if it was real in the Walrasian environment (up to an upper bound). As is well-known, the fiscal theory of the price level does not hold if debt is real.

Insight #3: An indexed surplus and defaultable debt make a hidden peg. The situation in which the two implicit assumptions in the fiscal theory of the price level—

[20]Proposition 9 ruled out $l = 0$ only to avoid a price level equal to 0 without active trade, which we allow for here.

debt monetization and market-clearing price—are relaxed is covered in (the negligible limit of) Proposition 8, in which debt is defaultable and the state trades at a fixed official price. In this case, in the negligible limit, the price level is determined at P^* but there are a continuum of equilibria with varying default and surpluses. The more the state repays the less it consumes. This situation can be viewed as one of monetary dominance since there is no money creation to honor public liabilities, and policy sets a fixed price level no matter the consequences for state consumption. It is again possible to obtain these equilibria in the Walrasian environment by defining an appropriate contingent policy. The policy has to be contingent on two equilibrium outcomes, the real value of private money supply that we denote s, and the price level P. A policy that generates the outcomes in Proposition 8 is $(\min\{L, Ps\}, \tau, P^*s/P)$. In words, holding the real value of private money supply s fixed, the state makes its real surplus decreasing in the price level so that the only equilibrium price is P^*. Since the state cannot explicitly set an official price level in the Walrasian environment, it uses a contingent surplus that eliminates all possible equilibrium price levels but its target P^*. A potential connection with this strategy is the approach followed by Levy-Yeyati and Sturzenegger (2003) or Reinhart and Rogoff (2004) and the literature thereafter to identify "hidden pegs": these pegs correspond to a low variability in the exchange rates and a large one in the amount of FX reserves. As in this Walrasian policy, there is no explicit price setting, but the trades amount to it.

Insight #4: The Walrasian auctioneer bans the state from partially controlling the price level with financial repression. A final situation that is interesting to bring to the Walrasian environment is that of financial repression—$N > -P^*\delta_G > 0$ in Proposition 2. In the negligible limit, assuming heterogeneous transfers, there are multiple equilibria in which the trading volume is split between the official market at P^* and the unofficial one at $-N/\delta_G$. The unofficial volume has an upper bound that is decreasing when agents have more homogeneous money holdings. A situation with two markets for the same good is of course out-of-reach of Walrasian environments. The Walrasian model prevents financial repression, and predicts given N and $-\delta_G$ a unique equilibrium price equal to the unofficial one under financial repression $N/(-\delta_G)$. Yet, the coexistence of official and unofficial prices is commonplace in practice.

5 Fiscal debtors

This section studies the baseline fixed policies defined in Section 2 that feature fiscal debtors—private agents such that $N_i \leq 0$. We first study policies such that all agents are fiscal debtors, and then that in which debtors and creditors coexist.

5.1 Only fiscal debtors: $N_+ = 0$ and $N_- = -N > 0$

Proposition 10. ***(Fiscal debtors and debt deflation)*** *Suppose that a policy $\mathcal{P}$ is such that $N_+ = 0$ and $N_- = -N > 0$. There exist equilibria without bankruptcy if and only if*

$$N_i + P^* e \geq 0 \text{ for all } i \in \mathcal{I} \text{ and } N + P^* \delta_G \geq 0. \tag{46}$$

In any equilibrium without bankruptcy, active prices are in $(0, P^*]$.

Condition (46) does not suffice to ensure price-level determination. A sufficient condition for strong price-level determination with $\Pi(\mathcal{P}) = \{P^*\}$ is

$$N_i + P^* \min\{\delta_G, e\} \geq 0 \text{ for all } i \in \mathcal{I}. \tag{47}$$

Proof. We proceed in four steps.

Step 1: In any equilibrium without bankruptcy, active prices are in $(0, P^*]$. Suppose that an equilibrium is without bankruptcy. As agents are fiscal debtors, this implies that there must be active trading. Suppose that the highest active-trading price is strictly above P^*. Any net buyer at this price is a private agent and is not net seller anywhere from Lemma 4. But then she must be bankrupt, a contradiction.

Step 2: There exist equilibria without bankruptcy if and only if (46) holds. There always exists an equilibrium with a single trading post at P^* in which agent $i \in \mathcal{I}$ sells $\min\{e; -N_i/P^*\}$. This "P^*-equilibrium" features no bankruptcy if condition (46) holds because every agent can afford her taxes in this case. If (46) does not hold, any equilibrium without bankruptcy would require that the agents that are bankrupt in the P^*-equilibrium can sell goods at a strictly higher price than P^*, a contradiction from the above point.

Step 3: Condition (46) does not suffice to ensure price-level determination.

We build a simple counter-example. Suppose that the number of agents satisfies $I > 2$, and, without loss of generality, that $(N_i)_{i\in\mathcal{I}}$ is increasing in i. Suppose that condition (46) holds, so that the P^*-equilibrium involves no bankruptcy. Suppose however that there exists $n \in [1, I-2]$ such that $(I-n+1)N_n + IP^*\delta_G < 0$ – such an n may exist only when condition (47) does not hold. Notice that the existence of such an n given condition (46) implies that the fiscal debts of agents $i > n$ be sufficiently small in absolute values and $N + P^*\delta_G$ be sufficiently close to 0. If e is sufficiently large other things being equal, there also exists an equilibrium in which all agents $i \leq n$—the "large" fiscal debtors—are bankrupt and sell their entire endowments at an arbitrarily small unofficial price to agents $j > n$—the "small" fiscal debtors. These latter small debtors bid their entire endowment at the official post and reinvest the proceeds at this unofficial low price. To see why this is an equilibrium, notice first that for e sufficiently large, the small fiscal debtors squeeze the official market in this equilibrium. Thus a large fiscal debtor i would have a strict gain from deviating and buying cash on the official market to get out of bankruptcy if $IP^*\delta_G/(I-n+1) \geq -N_i$, which does not hold. Small fiscal debtors strictly benefit from this trade for e sufficiently large as they give up $\delta_G I/(I-n)$ consumption units in the official market and get $en/(I-n)$ in the unofficial one. Any of them would thus be strictly worse off just buying money in the official market to pay taxes and consuming strictly less than e.

Step 4: There is strong price-level determination if (47) **holds.** We show that the P^*-equilibrium is the only equilibrium if condition (47) holds. In this case, notice first that there is no equilibrium without active trading otherwise any agent such that $N_i < 0$ would be better off deviating and escaping bankruptcy by selling $-N_i/P^*$ at P^*. In any equilibrium in which there is trade at another price than P^*, there has to be a private net buyer and a private net seller. Let $\underline{P}$ denote the lowest price at which there is a private net seller i and $\bar{P}$ denote the highest price at which there is a private net buyer j. Net buyer j cannot sell at any lower price than $\bar{P}$ from Lemma 4 but must sell somewhere to avoid bankruptcy, which she could always achieve from condition (47). Thus she must sell at $P^* > \bar{P}$, which implies $\underline{P} \leq \bar{P} < P^*$. But then i, who is not net buyer at any post from condition (22) and $P^* > \underline{P}$, would be strictly better off selling only at P^* the amount required to pay her taxes, a contradiction. Condition (47) warrants that she is never too diluted by the other orders to achieve this. $\square$

To grasp the intuition for the results behind Proposition 10, it is useful to start with the remark that there always exists an equilibrium with a single trading post at P^*, in which agent $i \in \mathcal{I}$ sells $\min\{e; -N_i/P^*\}$. We deem this equilibrium the "P^*-equilibrium".

If condition (46) holds, this equilibrium is without bankruptcy since i) each private agent has enough goods to sell to pay her net taxes ($-N_i > P^*e$), and ii) and their aggregate demand for money $-N$ is within the state's maximum supply $P^*\delta_G$. Proposition 10 shows that condition (46) is actually necessary for the existence of a bankruptcy-free equilibrium.

Debt-deflation equilibria. Interestingly, in terms of price-level determination, whereas condition (46) warrants that all predictable prices are smaller than the official target P^*, it does not suffice to rule out lower unofficial prices. As showcased by the example constructed in the proof, if (46) holds but i) fiscal debts are sufficiently heterogeneous, and ii) the maximum supply of money $P^*\delta_G$ is sufficiently close to the P^*-equilibrium demand N, then equilibria that we deem ones of "debt deflation" may arise. In these equilibria, the agents with low fiscal debt coordinate on squeezing the official market, purchasing more money than they need so as to ration the larger debtors. The small debtors can then redeploy this cash in an unofficial market in which they snap up goods sold by the distressed large debtors at a low price. Section 8 relates the conditions under which such equilibria arise in our model—limited money supply and large dispersion in private debts—to the conditions that led to the "Long Depression" episode between 1873 and 1896.

Fixed-rate full allotment auctions. The interpretation of condition (47) ensuring price-level determination is that the state commits to do whatever it takes to ensure that each single fiscal debtor can purchase money to honor her liabilities regardless of the (in-or-out-of equilibrium) actions of the rest of the private sector. This implies standing ready to sell possibly much more money than the equilibrium quantity IN. An example of such an elastic money supply by the public sector is the Eurozone's post-Lehman shift to a fixed-rate full-allotment operational framework, implemented to regain control over market rates as the ECB began to lose its grip.

Symmetry with financial repression. Debt-deflation equilibria are such that agents with low fiscal debt corner the official market by flooding it with goods thereby forcing the more indebted ones to sell goods at a low unofficial price. These equilibria are thus symmetric to financial-repression ones in which agents with little cash coordinate on squeezing the official market by bidding acquired cash. This forces agents holding more cash to sell it at a high unofficial price level, thereby financing the squeezing strategy.

In sum, in both cases, agents with small trading needs squeeze agents with big ones out of the official market, and this forces the latter to accept less favorable unofficial trades. In both cases, the state ensures price-level determination by committing to trade larger volumes than the equilibrium one. The only difference is that this excess backing can be made arbitrarily small in the case of fiscal creditors, whereas it can be quite large in the case of fiscal debtors depending on the distribution of fiscal debts.

Finally, it is worthwhile noticing that in both cases, more unofficial trades yield more redistribution from the agents with the largest cash positions in absolute values towards the others, as the former force the latter to trade at unofficial prices that are less favorable than the official one.

Remark on multiple goods. Our parsimonious setup features only one desirable good. Extending the analysis to the case of multiple goods is an interesting route for future research. By construction, our setup can only predict the coexistence of several prices for the same good. With, say, two goods such that only one of them is traded by the government, there could be a private post in which agents trade the other good for money, and the sellers use the proceeds to bid in a rationed official market for the "official" good. In this case, any insufficient backing by the state would affect the relative prices of goods or assets in a two-period version of the model. Thus we conjecture that the forces that we study in this one-good model could potentially deliver interesting predictions on the impact of monetary policy on relative goods or/and asset prices, e.g., liquidity premia.

5.2 Creditors and debtors: $N_+N_- > 0$

We now turn to the case in which there are both fiscal creditors and fiscal debtors.

Proposition 11. ***(Private gains from trade preclude the determination of the***

price level)

(i) *If a policy is such that $N_+ N_- > 0$—in words, it creates both fiscal creditors and debtors—then it does not determine the price level because $\{P^*\} \subsetneq \Pi(\mathcal{P})$.*

(ii) *If the state sells $-\bar{\delta}_G$ at $\bar{P}$ such that $N_+ + \bar{P}\bar{\delta}_G < 0$, and buys $\underline{\delta}_G$ at $\underline{P} < \bar{P}$ such that $N_i + \underline{P}\min\{\underline{\delta}_G, e\} \geq 0$ for all $i \in \mathcal{I}$, then the predictable prices must be within $[\underline{P}, \bar{P}]$, an interval that can be made arbitrarily small.*

Proof. **Point (i).** Without loss of generality, we suppose that $(N_i)_{i\in\mathcal{I}}$ is increasing. We denote I_- the fiscal debtor with the smallest debt (the smallest absolute value of $N_i < 0$). Notice first that there always exists an equilibrium with a single active trading post at P^* in which agent $i \in \mathcal{I}$ submits a buy order N_i/P^* if $i > I_-$ and sells $\min\{e, -N_i/P^*\}$ otherwise. This "P^*-equilibrium" features no bankruptcy if and only if i) $P^*e + N_i \geq 0$ for all $i \in \{1, ..., I_-\}$, and ii) $N + P^*\delta_G \geq 0$. Condition i) states that every fiscal debtor has enough goods to sell to acquire $-N_i$ of money. Condition ii) ensures that the demand of money by fiscal debtors is covered by public and creditors' supply at P^*.

We construct another equilibrium in which there is active trade at two prices, P^* and $P > P^*$. We construct the equilibrium supposing that $N_I > N_{I-1}$. We explain how to adapt the analysis to the case in which several agents share this same highest value of net transfers N_I in Step 3 below.

Step 1. Suppose first that the P^*-equilibrium features no bankruptcy. We construct an equilibrium in which agent I places a buy order with a sufficiently small (in a sense made precise below) nominal amount B in a trading post $P > P^*$. All the other agents are selling at P. The other fiscal creditors (if any) redeploy in the P^*-post the proceeds from selling their entire net endowment e at P. The fiscal debtors mix sales at P^* and at the rationed higher price P so as to meet their liabilities at the lowest cost.

We first define these fiscal debtors' strategies. For $B > 0$ sufficiently small, define $S(B)$ the positive solution to

$$\frac{I_-e - S(B)}{(I-1)e - S(B)}B + P^*S(B) = IN_-. \tag{48}$$

For B sufficiently small, for every $i \in \{1, ..., I_-\}$, there exists a strictly positive solution

$s_i(P^*)$ to

$$\frac{e - s_i(P^*)}{(I-1)e - S(B)}B + P^* s_i(P^*) = -N_i, \tag{49}$$

and by definition

$$\sum_{i=1}^{I_-} s_i(P^*) = S(B). \tag{50}$$

The equilibrium is then such that fiscal debtor $i \in \{1, ..., I_-\}$ sells $s_i(P^*)$ at the P^*-post and $e - s_i(P^*)$ at the P-post where P is defined below.

Let us now define fiscal creditors' strategies. Agent $j \in \{I_- + 1, ..., I-1\}$ (if any) sells e at P and invests a nominal amount equal to the proceeds plus N_j at P^*. Agent I invests a nominal amount $N_I - B$ at P^* and B at P. The supply at P^* is thus $s(P^*) = I(-\delta_G)^+ + S(B)$, the demand $d(P^*) = I\delta_G^+ + IN_+/P^* - B(I_-e - S(B))/[P^*[(I-1)e - S(B)]] = I\delta_G^+ + IN_+/P^* - IN_-/P^* + S(B)) \geq s(P^*)$. Let us define

$$P = \frac{P^* d(P^*)^2}{s(P^*)\left(d(P^*) - \frac{N_I - B}{P^*}\right)}. \tag{51}$$

Suppose B is sufficiently small that $N_I - B > N_{I-1} + Be/[(I-1)e - S(B)]$ and that $s(P) > B$. Then I's trade is optimal from (51) and Lemma 5. So are the trades of the other fiscal creditors because Lemma 4 and (51) imply that they would like to sell more at P to reinvest at P^* but they hit their maximum supply e at P. Finally, fiscal debtors cannot meet their net liabilities at a lower cost as they sell as much as possible at $P > P^*$ subject to being solvent.

Step 2. Suppose now that the P^*-equilibrium features at least one bankrupt agent because there exists $i \in \{1, ..., I_-\}$ such that $P^*e < -N_i$ or because $N + P^*\delta_G < 0$. We re-create essentially the same equilibrium as in Step 1. First, for any fiscal debtor $i \leq I_-$ such that $P^*e < -N_i$, replace $-N_i$ with P^*e. Second, take one bankrupt agent, and make him add a buy order larger than N_I/P^* (which of course will be executed by the state) at P^* such that overall $N' + P^*\delta_G \geq 0$ where the new aggregate transfer per capita N' factors in the revised sell and buy orders of the bankrupt agents. It is easy to see that replacing this buy order of the bankrupt agent by another one split between P^* and P defined as in Step 1 for B sufficiently small is an equilibrium.

Step 3. In order to adapt the proof to the case in which $k > 1$ agents share the same maximum transfer N_I, we leave it to the reader to check that one only needs to let each of them invest a nominal amount B/k in a P-post defined as in Step 1.

Point (ii). Point (ii) is a direct implication from Propositions 2 and 10. □

Proposition 11 states that if a policy opens up potential gains from trade between private agents because the transfers create both fiscal creditors and fiscal debtors, then it cannot determine the price level if the state is only on one side of the market (either buy or sell side). Price level determination obtains only if the policy features two official trading posts in opposite directions.

The essential reason private gains from trade make it impossible to peg the value of money with a single official trade is that, in the absence of a rationed official market, money ultimately serves no other purpose than dodging bankruptcy in this economy.[21] Thus fiscal creditors are happy to trade money for goods at any price. Symmetrically, debtors are happy to trade goods for money at any price provided this makes them solvent. (They also are indifferent between any trade in the absence of any way out of bankruptcy.) The single trade of the state is thus not sufficient to coordinate the private sector on its price-level target P^*. In the presence of gains from trade between them, private agents can always simultaneously trade on this official market and on unofficial ones at different price levels.

By using two trading posts, the state restricts the incentives for agents to engage in all these trades and then puts bounds on the price levels. The post on the buy side puts a floor on the price level as no agents will accept to trade at a lower price level. The post of the sell side puts a cap on the price level as no one has the incentive to trade at higher price.

The only situation that we have not covered so far is that in which policy features only a trade and no net transfers—$N_i = 0$ for all $i \in \mathcal{I}$.

Proposition 12. ***(No price-level determination without net transfers)*** *If a policy is such that $N_+ + N_- = 0$—in words, it features neither fiscal creditors nor debtors—there is no determination of the price level because there is no equilibrium with active trading: $\Pi(\mathcal{P}) = \emptyset$.*

[21] Section 6 develops a two-date version of the model in which money may also be desirable as a store of value.

Proof. By definition, $N_+ + N_- = 0$ implies that $N_i = 0$ for all $i \in \mathcal{I}$. As a result, agents can avoid bankruptcy by not trading and thus any agent actively trading in equilibrium cannot be bankrupt. Suppose by contradiction that an equilibrium is such that agent $i \in \mathcal{I}$ is active in at least one trading post. She cannot be net buyer in every post in which she is active since she would then be bankrupt from $\int PdD_i(P) \geq \int Pd\hat{D}_i(P) > \int Pd\hat{S}_i(P) + N_i$ with $N_i = 0$. This implies that if there exists at least one active trading post, at least one private agent is net seller somewhere. Let $\underline{P}$ denote the smallest price at which there is a private net seller. She must be net buyer somewhere else otherwise she would be strictly better off not trading. From Lemma 4, it has to be at a lower price, but since $\underline{P}$ is the smallest price at which there is a private net seller, the only possible net seller facing her is the government, and she buys at $P^* < \underline{P}$. But then this means there is a private net buyer at $\underline{P}$, as it cannot be the state which buys at this price. Let $\bar{P} \geq \underline{P}$ denote the largest price at which there is a private net buyer. A net buyer at this price cannot be net seller at any lower price from Lemma 4. But then she must be bankrupt, a contradiction. □

In this static model, the absence of net transfers implies indetermination of the price level because there exists no equilibrium with active trading in this case. Section 6 will show that by contrast, a policy with only trades and no transfers may suffice to determine the price level in a dynamic environment in which money gains intrinsic value as storage.

6 Two-date model

This section studies a simple extension of our model with two dates $\{0, 1\}$. The economy is still populated by a state and by $I \geq 2$ private agents. These agents value only a date-1 consumption good that is obtained out of the storage of a date-0 consumption good at a linear rate $\rho > 0$ between 0 and 1. Each agent $i \in \mathcal{I}$ is endowed with $e_i > 0$ units of the date-0 consumption good, where $(e_i)_{i \in \mathcal{I}}$ is increasing without loss of generality, strictly so for brevity. We denote $e = 1/I \sum_{i \in \mathcal{I}} e_i$. We focus for brevity on the following simple policies.

Policy. A policy $\mathcal{P} = (\delta_{G,0}, P_0^*, R, \delta_{G,1}, P_1^*)$ consists in two trades and one contingent transfer:

- **Trades.** The state stands ready to buy up to $I\delta_{G,0} > 0$ units of the date-0 good at a price P_0^*, and to sell up to $-I\delta_{G,1} > 0$ units of the date-1 good at a price P_1^*.

- **Transfers.** The state multiplies any outstanding net position in money by a private agent at the end of date 0 by $R > 0$, and this defines her net position at the outset of date 1.

Notice that, except for the fact that the interest rate applies to quantities that have been decided at date 0, we study only policies that are not contingent at date 1 on the date-0 actions of the private sector.

Private trades and bankruptcy. At each date $t \in \{0, 1\}$, private agents can submit any number of buy or sell orders of the date-t good, with the restriction that they cannot place sell orders for a total quantity larger than their endowment at the outset of each date. Trading posts clear with proportional rationing as in the one-date model.

With a straightforward extension of the one-date notations, the strategy of agent $i \in \mathcal{I}$ is $\mathcal{S}_i = (D_{0,i}(.), S_{0,i}(.), D_{1,i}(.), S_{1,i}(.))$. While it does not show in notations for parsimony, the date-1 orders are conditional on history, that is, on date-0 actions. Agent $i \in \mathcal{I}$ is bankrupt at date 0 if and only if

$$\int PdD_{0,i}(P) > \frac{1}{R}\left(\int Pd\hat{S}_{1,i}(P) - \int Pd\hat{D}_{1,i}(P)\right) + \int Pd\hat{S}_{0,i}(P), \tag{52}$$

and at date 1 if and only if for all history,

$$\int PdD_{1,i}(P) > R\left(\int Pd\hat{S}_{0,i}(P) - \int Pd\hat{D}_{0,i}(P)\right) + \int Pd\hat{S}_{1,i}(P). \tag{53}$$

Equilibrium concept. A profile $\mathcal{S} = (\mathcal{S}_i)_{i\in\mathcal{I}}$ is a predictable outcome given $\mathcal{P}$ if and only if it is a subgame-perfect Nash equilibrium.

This setup departs from the one-date model in three interesting dimensions. First, apart from interest payments, there are no transfers of any sign at any date imposed on private agents. Their date-1 net cash positions result only from their voluntary date-0 trades. Proposition 12 shows that this precludes any trade in the one-date model, we will see that it is no longer the case here. Second, money may deliver consumption at date 1, and thus serves as a store of value. In this sense, date 0 is an extension of the one-date model in which money may be intrisically desirable. Finally, the absence of cash-in-advance constraint opens up the possibility of inside-money creation, whereby an agent can pay for goods at date 0 with "money"—IOUs—backed by anticipated date-1 sales of goods. The following proposition characterizes price-level determination.

Proposition 13. ***(Price-level determination with two dates.)*** *Let $r^* \equiv RP_0^*/P_1^*$. The date-0 price level is weakly determined when $r^* = \rho$. It is strongly determined if and only if $r^* > \rho$ and*

$$\delta_{G,1} + r^* \min\{e, \delta_{G,0}\} \leq 0, \tag{54}$$

which holds if $\delta_{G,0}$ is sufficiently small or/and $-\delta_{G,1}$ sufficiently large other things being equal. The date-1 price level is determined if and only if the date-0 one is.

Proof. Notice that if $r^* \leq \rho$, no trade is an equilibrium. It is not if $r^* > \rho$, as one agent could deviate and strictly benefiting from selling in the date-0 official post and buying in the date-1 one with the saved proceeds. Notice also that agents can avoid bankruptcy by simply not trading. The proof is in five steps.

Step 1: There is no date-0 price-level determination if $r^* < \rho$. Suppose that $r^* < \rho$. Let $P_0 \in (P_0^*, \rho P_0^*/r^*)$. An agent $i \in \mathcal{I}$ buying some goods at P_0 from an agent $j \in \mathcal{I}$ at date 0, storing them and then selling them back to j at $P_1 = RP_0/\rho = (r^*/\rho)(P_0/P_0^*)P_1^* < P_1^*$ is an equilibrium because j cannot gain from selling in the official post at date 0, i cannot benefit from buying in the official post at date 1, and no other agent can strictly increase her date-1 consumption by intervening in the official or unofficial posts.

Step 2: There is strong date-0 price-level determination if condition (54) holds. Let us start by showing that there cannot be an equilibrium in which the date-1 official post is rationed on the buy side. Otherwise, suppose that there is some rationing. First, there should be inside money creation: with only state-issued money, condition (54) implies that no rationing occurs at date 1. That there is rationing at date 1 implies that some agents sold goods against private money at date-0 and invested the proceeds at P_1^*. They do so only if this gives them a return of at least ρ. The counterparts of this inside money creation must finance their date-0 bids by selling goods at date 1. These agents do so only if the real return on money is at most ρ. At date 1, there are two possibilities. First, issuers of money buy back money at a different price than P_1^*. Rationing at the post P_1^* then implies than agents issuing money cannot receive enough cash, a contradiction. Second, issuers of money buy back money at P_1^*. In this case, they are able to buy back all the money that they have issued but this contradicts the fact that the official post is on the buy side.

When $\delta_{G,1} + r^* e$ and $\delta_{G,0} > e$, the absence of rationing at date 1 implies that if there exists an active unofficial trading post with price P_0 at date 0, sellers must earn r^* on their investment. Buyers can only generate that by investing their acquired goods in the date-0 official market. Thus it must be that $P_0 < P_0^*$. But then sellers should pivot to the official post, a contradiction. The absence of unofficial trading at date 0 implies that at date 1 from Proposition 10 case 2) applied at date 1.

This result extends to the case in which $e > \delta_{G,0}$. In this case, the official trading post at P_0^* is rationed. However, agents are better off posting all their endowment at this price and not to engage in private money creation. Such a strategy generates a return strictly larger than ρ: agents obtain a return r^* on the money that they succeed to buy and a return of at least ρ on the goods that they were unable to sell due to rationing.

Step 3: There is weak price-level determination if $r^* = \rho$. Notice that the date-1 official market cannot be rationed on the buy side. The rationed buyers would have to have sold at a price above P_0^* at date 0 to earn at least ρ. But then their counterparts could not fund their date-1 bids with sales above P_1^*. Suppose the lowest unofficial price is $P_0 < P_0^*$. Sellers must earn at least ρ and thus must invest the cash below $RP_0/\rho < P_1^*$ at date 1. But it cannot be that date-0 buyers sell them enough goods at this date-1 price. This would mean they break even at ρ. They would then be strictly better off selling some of their goods at P_0^* to fund their acquisitions, possibly reducing their order, thereby earning a strictly positive NPV. Suppose the largest unofficial price is $P_0 > P_0^*$. Buyers could not fund their date-1 bids with sales above P_1^* since sellers must invest part of their proceeds at P_1^*. The only case in which they could potentially not is if the P_1^*-buy side was exactly clearing from bids funded by P_0^*-investments. But in this case one agent would be strictly better off cutting her zero-NPV official trade and earning positive NPV by selling a bit at P_0 and buying at P_1^*.

Step 4: There is no date-0 price-level determination if $r^* > \rho$ but condition (54) does not hold. For brevity, we present the proof in the case in which $I = 2$. When condition (54) is not satisfied, we have $r^* e > -\delta_{G,1}$, which implies $r^* e > -\delta_{G,1}\left(2e - e_2\right)/(2e)$.

We start with the (most involved) case in which $e \leq \delta_{G,0}$ and $\rho < -\delta_{G,1}(2e - e_2)/(2e^2) < r^*$. In this case, in the equilibrium without unofficial trade, all agents are all-in in the official markets. Let r_2 and $r_1 > r_2$ denote the respective marginal return that agents 2 and 1 respectively earn on their last unit. Let $r' \in (r_2, r_1)$. Suppose agents 1 and 2 agree on a trade whereby 2 sells an arbitrarily small quantity to 1 at $P_0 = r' P_1 / R < P_0^*$ instead of selling to the state. 1 bids her cash in the official market at date 1, and sells what it takes to agent 2 at P_1 to finance her date-0 purchase. For $P_1 > P_1^*$ but sufficiently small, 2 is happy to buy at P_1 from Lemma 5, and 1 must sell to finance her date-0 acquisition. The case in which $e \leq \delta_{G,0}$ and $\rho \geq -\delta_{G,1}(2e - e_2)/(2e^2)$ is similar. The only difference is that 1 uses the storage technology ρ instead of the official market in this unofficial trade. The cases in which $\delta_{G,0} < e$ are also similar, the only difference is that all agents earn the same marginal return in the official market. Finally, in the case of n agents, the unofficial trade has the same structure. The agent with the largest marginal return in the official market is the unofficial date-0 buyer and sells to all the others at date 1 to fund her acquisition. The construction of the equilibrium is just more cumbersome because all these other agents must be happy to buy at P_1, and a system of equations determines the respective sizes of their bids that achieves this.

Step 5: Date-0 and date-1 price level determinations are equivalent. This follows from the steps above: Date-0 determination holds when the date-1 official post is not rationed and so the date-1 price level is determined from Proposition 2. All the equilibria with multiple date-0 prices that we constructed feature also multiple date-1 prices. □

The state sells money at date 0, and so official markets are naturally inactive when money carries a low real return $r^* < \rho$. Still, money can be privately issued and traded above the official price at date 0 and below the official price at date 1. Price levels are then indeterminate at both dates but a Fisher equation determines the inflation rate given the official nominal interest rate R and the private return ρ.

Condition (54) ensuring strong price-level determination has a very simple interpretation. The state should not only promise a return larger than the private one ($r^* > \rho$) but it should also provide a sufficient backing to give a positive net present value to money issued at the promised real interest rate r^*. This guarantees that the official post is not rationed at date 1 given the amount of money issued at date 0.

In our setting, date-1 price-level determination also guarantees date-0 price level determination, even when the date-0 official post is rationed. This happens, for example, when the state does not issue enough money, that is when $\delta_{G,0} < e$. But, in this case, agents are still better off not to engage in private trades. These trades would yield at most the return ρ, while posting goods in the official post yields a strictly higher yield as the rationed demand for money may still be invested at a rate ρ.

That the state can determine price levels in the absence of exogenous transfers contrasts with the static case in Proposition 12. It also does so with the results in Niepelt (2004) that the fiscal theory of the price level requires the presence of exogenous legacy debt. This result owes to the assumption that the state is price-setter here.

7 Uncertainty and endogenous collateral constraint

This section constructs an equilibrium with unofficial trade in the presence of financial repression in a version of our economy in which the budget constraint (9) replaces the collateral constraint (11) in the definition of solvency. Policy uncertainty induces ambiguity-averse agents to self-impose such a collateral constraint: Some agents find it worthwhile selling goods in the unofficial market in order to fully cover their official orders with cash.

Formally, we modify here our baseline model along three dimensions. First, agents are ambiguity-averse and seek to maximize the minimum value of their consumptions across states of nature. Second, they define their strategies before policy is revealed, and policy can be of two types. Either it is a policy that leads to financial repression and possibly unofficial markets as in 3. in Proposition 2, or it is a policy with a market-clearing price as in Proposition 9. In this latter case, the state sets the price that clears the market given its demand $\delta_G < 0$ and the nominal value of buy orders at P^*. The other dimensions of policy are unchanged. Finally, we substitute constraint (9) for constraint (11) in our definition of solvency.

The equilibrium with unofficial trading and binding collateral constraints that we construct in the proof of Proposition 11 is still an equilibrium in our environment with policy uncertainty. To see this, notice that whether the official market clears or whether there is proportional rationing does not affect the allocation of goods across bidders. The only difference is that they must pay up their entire orders in the former case and only the effective part in the latter. Thus the possibility of market clearing only implies that if agents do not impose on themselves a collateral constraint (11) in the official market, they go bankrupt in the market-clearing case. We leave it to the reader to check that private agents find it endogenously optimal to fully cover their bids in the official market, so that the construction of the equilibrium is verbatim that in the presence of an exogenous collateral constraint (11).

The assumption of uncertain price-setting admits a natural interpretation as a situation in which currency-markets participants are unsure about the willingness of a country to maintain a peg. We would obtain the same results if uncertainty was about the quantity $-\delta_G$ instead—holding the other ingredients of policy including the fixed official price P^* unchanged—equal either to its value under financial repression or arbitrarily large so that there is no rationing in the official market.

8 Applications

This section discusses several applications of our framework.

Fiscal backing: Assignats during the French Revolution. In 1790, during the Revolution, the French state issued paper money—"assignats"—to reimburse debts (N), and at the same time was selling the National Estates ($-\delta_G > 0$) consisting in assets seized from the Church.[22] Sargent and Velde (1995) describe the "rise and fall of the assignat" as a sequence of three periods: a "real-bills" period, a "legal restriction" period and a "hyperinflation" period. Each of these periods corresponds to specific regimes in Proposition 2.

During the "real bills" period, the value of National Estates was about 2,400 millions livres and exceeded the value of the debt that the state had to honor, which was around

[22] Interestingly, Bolt et al. (2024) comment on an episode of insufficient backing by the Bank of Amsterdam that is contemporaneous to the Assignats debacle discussed here.

2,000 millions livres. In the terms of our model, $N + P^*\delta_G < 0$: Assignats were then fully backed. Here, P^* is the face value of assignats.

During the following two periods, a large quantity of assignats were issued to compensate for limited fiscal resources due to war and internal chaos. We interpret these two periods as situations in which $N + P^*\delta_G \geq 0$: the state does not back its currency. However, we note important differences between the two periods.

During the "legal restriction" period, under the Terror, the state implemented very harsh restrictions on hoarding assets, closed markets, and imposed wage and price controls. As Sargent and Velde (1995) note, these restrictions led to a "guillotine-backed currency":

> Under the Terror, any citizen accused of violating these laws could expect swift and arbitrary proceedings. The law on parity of the assignat called for arraignment and trial within 48 hours of the offense. The law encouraged denunciations from informants and gave extravagant powers to local authorities to enforce the restrictions. In a few dozen instances, the death penalty was imposed for crimes against the assignat or for hoarding.

The outcome is that the price level was determined and no inflation arose, consistently with our model in which the absence of private trades leads to price determination with financial repression in the case $N + P^*\delta_G \geq 0$.

With military successes, the Terror was overthrown, legal restrictions were alleviated, and markets reopened. In this "hyperinflation" period, private agents tried to sell all their holdings of assignats for goods and specie. This period opened up arbitrage opportunities. As reported by Sargent and Velde (1995):

> In terms of gold, prices were lower than in 1790, creating trading opportunities for the savvy. A Swiss visiting Paris hastened to change his gold for paper; bought hundreds of shoes, stockings, and hats; shipped them off to Switzerland; and lived in Paris like a king for a month.

This kind of trade is very similar to that in our setup in which agents with high marginal returns in the official market scale up these returns by buying money at a high price level in the unofficial market from agents with lower official returns on money. In this historical example with two "goods", gold and Parisian luxury goods, low-return agents may simply value relatively more gold over the goods available in Paris that they have already stocked up on.

Exchange rate pegs and parallel market exchange rates. Parallel foreign exchange markets may exist for multiple reasons (e.g., in order to evade capital controls or for illegal transactions). They may also emerge from countries maintaining an overvalued official exchange rate and rationing the supply of foreign currency. In particular, Gray (2021) notes that, among the motives for such a policy decision:

> It may also be promoted by those who can profit from privileged access to FX at the official exchange rate (rent seeking behavior).

In our model, financial repression creates room for rent extraction in the parallel market by the agents with privileged access to the official market in the form of a higher marginal return on investment. These are the agents with low cash holdings in our parsimonious setup, but the higher marginal return could be for other reasons than price impact.

Gray (2021) also provides a list of 19 countries with official and parallel markets in the 2010-2020 period. More generally, by using estimates of export misinvoicing practices,[23] Reinhart and Rogoff (2004) show that parallel markets are widespread in fixed exchange rate regimes and concern more than half the pegs, since at least World War II. As they show, parallel market exchange rates are better indicator of monetary policy stance and they even predict realignments in the official exchange rate. This aspect of parallel markets connects with our results in the negligible limit that the prices on parallel markets reflect the actual backing of currency $N/(-\delta_G)$.

Price controls. Another application of our model regards price controls. With price controls, the government imposes the prices at which private agents trade goods for money. Imposing a price below the equilibrium one may lead to rationing. Such a rationing may be reinforced by lower production by firms facing lower prices. In this case, the post of the government can be interpreted as the supply of goods $-\delta_G > 0$ supplied by firms at the controlled price P^*. Notice that, with this interpretation, the feasibility conditions described in Section 2.3 do not apply, as the supply of goods $-\delta_G$

[23] According to IMF (1991), the ways to circumvent official markets are "smuggling, over-invoicing of imports and under-invoicing of exports, workers' remittances from abroad, and tourism".

does not stem from the taxation power of the government.[24]

The connection between price controls and parallel markets is well documented, and examples of black markets associated with price controls abound. The US during World War II (e.g., Rockoff, 2004), Argentina in 1973-1975 (e.g., Chu and Feltenstein, 1978), Chile in 1973 (e.g., Edwards, 2023) are well-known examples. This connection is so well established that the Wikipedia page on black markets even mentions "Common motives for operating in black markets are to trade contraband, avoid taxes and regulations, or evade price controls or rationing." Notice that, in our model, black markets take place between natural net buyers of goods (net sellers of money). With price controls, another important dimension of black markets is that they take place between natural net sellers of goods (net purchasers of money) and natural net buyers of goods (net sellers of money): Firms may participate themselves in black markets to sell their production at better prices compared with the controlled one.

In-kind taxation in periods of financial repressions. Proposition 2 also has some implication for the use of in-kind taxation in periods of financial repressions: In-kind taxation boosts the return on financial repression for the state. The mix between in-kind and nominal taxes matters in such situations, unlike under full backing. To see this, we interpret here the real resources of the state $I\tau$ as in-kind taxes. Under full backing ($N + P^*\delta_G < 0$), condition (18) relates the real value of state liabilities to the real surplus in the absence of financial repression as follows:

$$\frac{1}{P^*}\sum_{i\in\mathcal{I}} L_i = f - Ic_{G,C}, \tag{55}$$

where $f \equiv I\tau + (\sum_{i\in\mathcal{I}} T_i)/P^*$ are the real fiscal resources of the state. Thus, among the policies that determine the price level, those that differ only along the modalities of tax payment—in-kind versus in cash—but not along the real value of taxes f lead to the same real allocations. By contrast, the modalities of tax payment are no longer irrelevant under financial repression. As is transparent from expressions (19) and (20), a reduction in $\sum_{i\in\mathcal{I}} T_i$ and increase in τ holding f fixed shifts real resources from the private sector towards the state since this reduces the private demand for money for

[24] A full model of price controls would include the endogenous supply of goods by firms $-\delta_G(P) > 0$ as a function of the controlled price P, with all the potential tools that the government may have to force firm production.

tax-payment motives, and thus increases households' forced money holdings.

This is reminiscent of historical situations in which a financially distressed public authority imposed in-kind payments for some taxes, such as the Confederacy during the US civil war or the USSR in the 20s.[25]

Private monies. While our main set of implications pertains to official currencies, our framework may also be used to think about privately-issued monies.

Stablecoins. Our model provides a useful framework for analyzing the determination of stablecoin prices. Major stablecoins aim to maintain a peg to the US dollar by backing tokens with USD-denominated assets, such as US Treasury bonds. Typically, issuers exchange tokens for dollars in a primary market and promise to redeem them at par value. Stablecoins then trade in secondary markets.[26] In the context of our model, the stablecoin represents "money", while the US dollar is the "good". In the primary market, the issuer (analogous to the state in our model) posts a sell order $-\bar{\delta}_G$ at price $\bar{P}$ and a buy order $\underline{\delta}_G$ at price $\underline{P}$, with the spread $\bar{P} - \underline{P}$ representing transaction fees. The feasibility conditions involve the dollar value of reserve assets held by the stablecoin issuer. According to our model, stablecoin prices on the secondary market should fluctuate within this band as long as token issuance $\underline{\delta}_G$ exceeds all the potential demand ($\underline{\delta}_G \geq \max_{i \in \mathcal{I}}(-N_i)^+/\underline{P}$) and the backing remains sufficient, i.e., $-\bar{\delta}_G > N_+/\bar{P}$. If the backing is perceived as insufficient, prices may breach the upper bound $\bar{P}$, as seen during the brief depeg of USDC following the SVB failure in March 2023 (Aldasoro et al., 2023).

While fiat-backed stablecoins have been the most successful, they are not the only type. Before the Terra Luna collapse in May 2022, algorithmic stablecoins were gaining popularity.[27] In this ecosystem, the stablecoin UST was circulating and differed from

[25] More precisely, the Confederacy faced important difficulties to levy taxes and had to rely on money printing to finance its war effort. Through the lens of our model, the fraction of new money used to pay for the wages of the army, for example, can be understood as nominal transfers N. The large issuance of money led to hyperinflation. In particular, no fiscal backing was supporting the value of money. As noted by Nielsen (2005), "The Treasury bills issued during the war had a peculiar feature: They were redeemable for gold two years after the war ended, which meant that the value of the bills was partially tied to expectations of victory for the Confederacy." This lack of backing can be understood as a situation of financial repression in which $N + \delta_G P^* > 0$. Consistent with this situation of financial repression, the mix between nominal and in-kind taxes did matter for the Confederacy: As documented by Burdekin and K. (1993), in-kind taxes contributed more than 50% of the Confederate revenue for example in the ten first months of 1863.

[26] Numerous studies have explored stablecoin stability: Lyons and Viswanath-Natraj (2023), d'Avernas et al. (2022), and Routledge and Zetlin-Jones (2022), among others.

[27] The Terra Luna ecosystem was the third largest in the market, with a capitalization of $50 billion,

fiat-backed stablecoins in its pegging mechanism. A smart contract allowed the exchange of one UST for $1 worth of LUNA—the native token circulating on Terra and a claim on Terra's transaction fees and block rewards, but also a token giving access to Terra applications. This setup resembles the primary market described earlier, except that δ_G was not fixed in USD but varied with LUNA's dollar value, which depended on the overall value of the Terra system. The collapse of UST was triggered by a sharp decline in LUNA's price, stemming from a sudden loss of confidence in the system's sustainability (see Liu et al., 2023, for further details). Liu et al. (2023) highlight heterogeneity in the use of the primary versus secondary markets during the May 2022 crisis:

> Interestingly, we find that Alameda Research, a cryptocurrency trading firm closely affiliated with the FTX exchange, conducted the largest amount of UST-LUNA swaps among Anchor depositors. It seems that the swap fees and uncertainty about the execution price of LUNA on exchanges discouraged most other Anchor depositors from utilizing the native swap contract as an exit strategy. But Alameda Research, with its advantageous access to the FTX exchange, had a competitive advantage over other market participants.

This observation suggests again that some agents, such as Alameda Research, had a competitive edge in trading on the primary market due to their superior access and resources to resell LUNA, while other participants opted for the secondary market, even at significantly discounted prices. In this scenario, market segmentation arose not from differences in price impacts due to rationing and order sizes (as modelled in our setting), but from other sources of superior returns in the primary market.

Another noteworthy aspect of the crash was the role of Anchor, a lending platform for UST. TerraForm Labs, the creator of the Terra network, subsidized the interest rate on UST deposits in Anchor to stimulate demand. Until May 2022, the interest rate stood at 19.5%.

Free banking era. Under Free Banking, as experienced by the US between 1837 and 1863, banks' liabilities were in form of banknotes redeemable in specie. These banknotes, in contrast with deposits,[28] were massively exchanged on secondary markets. In

which collapsed to nearly zero in just three days.

[28]As noted by Gorton (1985), deposits are "double claims" that is 'a claim on a specific agent's account at a specific bank.

particular, brokers specialized in trading these banknotes. As Gorton (1999) showed (see also Jaremski, 2011), the prices at which these banknotes traded reflected banks' default risk, which relates to the option to redeem the banknote (and the transportation cost to travel to the bank to redeem it). "Wildcat" banking, i.e., banks which intentionally overissue money compared with their ability to redeem it, is considered to have been at most marginal. To be more precise, Jaremski (2011) notes that banknotes were traded at par locally, unless the bank was closed or suspended. From the perspective of our model, money is banknotes while the good stands for species. The commitment of the bank to redeem banknotes at par is consistent with a fixed-price order. The situation in which there is no redemption risk is akin to one in which backing is sufficient ($\delta_G P^* + N < 0$), while the one in which banknotes are traded at a discount, even in their place of issuance, is one in which backing is insufficient, ($\delta_G P^* + N \geq 0$).

Money market funds and other asset-backed funds. Similarly, our model applies to money market mutual funds (MMFs). These funds implicitly guarantee that the value of their shares is 1\$. Interpreting the government in our model as the MMF and the private sector as the MMF's shareholders willing to withdraw their funds from the MMF, this guarantee can be modeled as a fixed-price order, and $-\delta_G$ is the measure of resources that the MMF can use to redeem its shares. When this backing is sufficient ($N + P^*\delta_G < 0$), the price of shares is pegged at P^*. When this backing is insufficient ($N + P^*\delta_G \geq 0$), the price of shares may fall in secondary markets. Such situations of insufficient backing can be connected to the runs that MMF experienced in the aftermath of the 2008 financial crisis (see Gorton and Metrick, 2010, among many others).

In the case where the fund redeems a function of the value of its resources $-\delta_G$, the situation is then akin to the market-clearing price case that we discuss in Section 3.5.

Debt-deflation equilibria and the causes of the "Long Depression". In the presence of fiscal debtors, the basic mechanism behind the debt-deflation equilibria in Proposition 10 is reminiscent of the explanation for recessions under a metallic standard, according to which the financial distress of nominally indebted firms, a low price level and a "dash for cash" amplify each other (Fisher, 1933). Our stylized model of an endowment economy has of course nothing to say about the economic contraction resulting from such debt-deflation dynamics. But the conditions under which we can construct debt-deflation equilibria echo analyses of the so-called Long Depression between 1873 and 1896. Such equilibria arise in the presence of two ingredients: the presence of large debtors[29], and a sufficiently small money supply, so that it is possible to squeeze these large debtors out of the official money market. Two related stylized facts are commonly identified to be at the origin of the Long Depression. First, the "Crime of 1873", whereby the US de facto ended the bimetallic standard via the Coinage Act of 1873, contracted the money supply. Second, a long credit and financial boom on either side of the Atlantic preceded the outburst of the financial or/and banking panics that are viewed as the starting point of the Long Depression.

The state can eliminate debt-deflation equilibria in our model by increasing δ_G other things being equal, namely, by committing to buy more goods thereby injecting enough money in the economy so that condition (47) holds. One case in point in which this is not feasible is that in which the "good" that the state is willing to trade for money is in limited supply. Accordingly, the limited supply of gold until new discoveries and improvements in extraction technologies in the 1880s and 1990s is commonly described as an important reason the 1873 morphed into a protracted recession.

9 Conclusion

We argue in this paper that Walrasian environments are ill-suited to study the determination of the price level by public financial policy. This is because this imposes arbitrary and materially important trading bans on the private sector, and because it counterfactually describes the way private and public sectors trade money. In addition to characterizing policies that do determine the price level in a more robust sense, we also offer a description of the set of predictable price levels even when this is not a singleton. We obtain realistic predictable outcomes such as the rise of unofficial prices and that of endogenous intermediaries in this case of price-level indetermination. Our strategically closed framework also unveils the important out-of-equilibrium dimensions of policies that shape the equilibrium outcomes behind the curtains in Walrasian environments.

Our focus has been on economies in which neither money nor other public liabilities play an important role at overcoming frictions. A natural route for future research is to

[29]We focus on fiscal debt here but debt could be owed to another private agent.

incorporate such a role in the analysis. This would in particular allow us to develop a normative analysis, assessing for example the welfare costs of price-level indetermination. Other situations that our strategically closed model is well-suited to study are that of the coexistence of multiple (private or/and public) monies. We also leave this for future work.

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Appendix

A Proof of Lemma 3

First, such deviations by i do not affect the payoffs of the other agents. This stems from the fact that these deviations do not affect the rationing coefficients since when $s(P) \neq s_i(P)$ and $d(P) \neq d_i(P)$,

$$\frac{d(P) - \frac{d(P)si(P)}{s(P)}}{s(P) - s_i(P)} = \frac{d(P) - d_i(P)}{s(P) - \frac{s(P)di(P)}{d(P)}} = \frac{d(P)}{s(P)}.$$

Second, these deviations are payoff-irrelevant for i as well. To see this, suppose that i is net buyer. The reduction in her buy order $d(P)s_i(P)/s(P)$ is weakly larger than that of her effective sales $s_i(P)\min\{d(P)/s(P), 1\}$, and so the deviation leaves her solvent. Furthermore,

$$\hat{d}_i'(P) - \hat{s}_i'(P) = \left(d_i(P) - d(P)\frac{s_i(P)}{s(P)}\right)\min\left\{1, \frac{s(P)}{d(P)}\right\} = \hat{d}_i(P) - \hat{s}_i(P),$$

leaving her allocation unchanged. The same reasoning applies to a net seller.

B Proof of Lemma 4

We show that if $P' \leq P$, or if $s(P) < d(P)$ and condition (22) does not hold, i can strictly increase her utility by simultaneously reducing her sell and buy positions. Let us define:

$$\delta(P, \epsilon) = \min\left\{1, \frac{s(P)}{d(P) + \epsilon}\right\} \text{ and } \sigma(P, \epsilon) = \min\left\{1, \frac{d(P)}{s(P) + \epsilon}\right\}.$$

We let i modify her orders as follows. She first nets her positions as in Lemma 3. If she is the only seller at P' and is rationed, she also reduces her order up to the total buy order. For notational simplicity we maintain the notation $s_i(P')$, $d_i(P)$ for these new net orders.

For $\epsilon > 0$ sufficiently small, define $\eta(\epsilon)$ as

$$-P\eta(\epsilon) = P'[(s_i(P') - \epsilon)\sigma(P', -\epsilon) - s_i(P')\sigma(P', 0)]. \tag{56}$$

In words $\eta(\epsilon)$ is the reduction in the buy order at P that has the same monetary value $P\eta(\epsilon)$ as that of the reduction in effective sales at P' when the sell order is reduced by ϵ. In particular, $\eta(\epsilon) = \epsilon P'/P$ when $\sigma(P', 0) = 1$. Suppose that i reduces her sell order at P' by ϵ and her buy order at P by $\eta(\epsilon)$. This leave her solvent by construction of $\eta(\epsilon)$, and brings a net change in consumption:

$$\begin{aligned}&(d_i(P) - \eta(\epsilon))\delta\left(P, -\eta(\epsilon)\right) - d_i(P)\delta\left(P, 0\right) - (s_i(P') - \epsilon)\sigma(P', -\epsilon) + s_i(P')\sigma(P', 0)\\ =&d_i(P)(\delta\left(P, -\eta(\epsilon)\right) - \delta(P, 0)) + \eta(\epsilon)\left(\frac{P}{P'} - \delta\left(P, -\eta(\epsilon)\right)\right).\end{aligned}$$

At first-order in $\eta(\epsilon)$, this is equal to

$$\eta(\epsilon)\left(\frac{P}{P'} - \delta(P) + \mathbb{1}_{\{\delta(P)<1\}}\delta(P)\frac{d_i(P)}{d(P)}\right),$$

Thus this deviation yields a strict benefit if $P' < P$ or if $\delta(P) < 1$ and (22) does not hold.

C Proof of Lemma 5

We show that if condition (23) does not hold, i can strictly increase her utility by moving some of her P'-order at P. Notice that Lemma 4 ensures that i cannot be a net seller at P as $P' > P$. We let i modify her orders as follows. She first nets her positions as in Lemma 3. Then she reduces her buy order at P' by ϵ and increases her buy order at P (possibly equal to 0) by $\epsilon P'/P$, where $\epsilon > 0$ is sufficiently small. Her collateral constraint still holds since the total cash value of her buy orders is unchanged. The net change in consumption units resulting from this deviation is

$$\begin{aligned}&(d_i(P') - \epsilon)\delta(P', -\epsilon) - d_i(P')\delta(P') + \left(d_i(P) + \epsilon\frac{P'}{P}\right)\delta\left(P, \epsilon\frac{P'}{P}\right) - d_i(P)\delta(P)\\ =&\epsilon\left(\frac{P'}{P}\delta\left(P, \epsilon\frac{P'}{P}\right) - \delta(P', -\epsilon)\right) + d_i(P)\left(\delta\left(P, \epsilon\frac{P'}{P}\right) - \delta(P)\right)\\ &+ d_i(P')\left(\delta\left(P', -\epsilon\right) - \delta(P')\right).\end{aligned}$$

At first-order w.r.t. ϵ this is equal to

$$\epsilon\frac{P'\delta(P)}{P}\left[1-\mathbb{1}_{\{\Delta_i(P)<1\}}\frac{d_i(P)}{d(P)}\right]-\epsilon\delta(P')\left[1-\mathbb{1}_{\{\Delta_i(P')<1\}}\frac{d_i(P')}{d(P')}\right],$$

strictly positive if condition (23) does not hold, which establishes the result.